# TD 09 : Complex dynamics and dimensions.

We consider the quadratic family $P_c(z) = z^2 + c$ with $|c| < 1/6$.
Let $A = \{z : \frac{3}{4} < |z| < \frac{4}{3}\}$ and let $J(P_c) = \partial\{z : P_c^n(z) \not\to \infty\}$ be the Julia set of $P_c$.

Let $L\phi(w) = \sum_{z:P_c^n z=w} \phi(z)$ be the Ruelle transfer operator with weight zero.

Pick $w \in A$ and define for $n \geq 1$ the empirical measure : $\mu_n = \frac{1}{2^n} \sum_{z:P_c^n z=w} \delta_z.$

1. Show that $\mu_* = \text{weak}-\lim_n \mu_n$ is the Gibbs measure associated with $L$. In particular, it is invariant with respect to $P_c$ and independent of the choice of $w$. It is called the 'harmonic' measure of $P_c$.
2. Write $(P_c)^{-n}(w) = \{z_k\}_{1\leq k\leq 2^n}$ and show that $P_c^n(0) - w = \prod_{k=1}^{2^n} z_k$.
3. Show that $\int \log |P_c'| \, d\mu_n = \log 2 + \frac{1}{2^n} \log |P_c^n(0) - w|$.
4. Show that $\int \log |P_c'| \, d\mu_* = \log 2$.
5. Conclude that $\dim_H(\mu_*) = 1$.

   (Result by Anthony Manning, published in Annals of Math, 1984).