Erratum to Magnetic Bottles in connection with superconductivity
JFA 185, p. 604-680 (2001)

Bernard Helffer

April 18, 2005

In my above mentioned paper with A. Morame, a few misprints and inaccuracies should be mentioned.

In the appendix

There is some ambiguity in the interpretation of formula (B.18), p. 677. As observed recently (March 2005) when working with S. Fournais, the right statement should be :

In the neighborhood of $\partial\Omega$, Ω_{ϵ_0}, we have, the existence of a constant $\gamma_0 \in I\!\!R$ and of a global gauge transform such that (cf (B.8)) :

$$P^N_{h,A,\Omega} u = a^{-1}\{(hD_s - \gamma_0 + bt(1-\frac{t}{2}\kappa(s)))[a^{-1}(hD_s - \gamma_0 + bt(1-\frac{t}{2}\kappa(s)))u] + h^2 D_t(aD_t u)\}\ .$$

The constant γ_0 was forgotten. This has no consequence when using the formula locally near a point of the boundary (or in any simply connected domain) because one can then eliminate γ_0 by an additional gauge transform. This was mainly the case in the whole paper except p. 658. **Note that this error could be more problematic for other problems**.

We give the correction below and also correct the other misprints in the manuscript, which were mentioned by our student[1] A. Kachmar.

Modification of p. 658

Here is the modified text corresponding to p. 658 (Lines 1-17).

The assumption that the magnetic field is constant allows us to choose a gauge (in the adapted coordinates introduced in the appendix B), such that in the formula (B.3) (cf (B.6) and the correction above), we have

$$\widetilde{A}(w) = \gamma_0(1,0) - bt(1 - \frac{t}{2}\kappa(s), 0),$$

[1] Many thanks for his careful reading of the manuscript

which implies :

$$|\widetilde{A}(w) - (\gamma_0 - bt, 0)| \leq C\, t^2\ .$$

Using the formula (B.3), (B.4) and (9.37) for $k = 1$ and $k = 4$, we get, from (10.2) - - (10.7), and the fact that $\mu^{(1)}(h) = \mathcal{O}(h)$, the existence of $C > 0$ and $h_0 > 0$ such that, for any $h \in]0, h_0]$,

$$\begin{aligned}&|\int_{S^1\times I\!R_+}[|(hD_s - \gamma_0 + bt)(\psi_{1,\tau(h)}\, u_h^1)|^2 + h^2\,|D_t(\psi_{1,\tau(h)}\, u_h^1)|^2 - \mu^{(1)}(h)|\,\psi_{1,\tau(h)}\, u_h^1\,|^2]\, dw|\\ &\quad \leq C\, h^{3/2}\ .\end{aligned}$$

As

$$\begin{aligned}&\int_{S^1\times I\!R_+}[|(hD_s - \gamma_0 + bt)(\psi_{1,\tau(h)}(t)u_h^1(s,t))|^2 + h^2|D_t(\psi_{1,\tau(h)}(t)u_h^1(s,t))|^2]\, dw\\ &\quad \geq \Theta_0 b\, h\, \int_{S^1\times I\!R_+}|\psi_{1,\tau(h)}(t)u_h^1(s,t)|^2\, dw\ .\end{aligned}$$

Using the Fourier series in the tangential variable and the property that $\mu(\xi) \geq \Theta_0$ for any ξ, we immediately get the existence of $C_0 > 0$ such that

$$\mu^{(1)}(h) \geq \Theta_0 b\, h - C_0\, h^{3/2}\ .$$

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Other misprints
p. 663.
In (10.22), we only need to take $g^h(x) = \varphi_0(x)$.
p. 664
The formula (10.24) should read :

$$H^h = a_0^{-2}(t)[bt(1 - \frac{t}{2}\kappa_{max}) - h^{1/2}b^{1/2}\xi_0]^2 + h^2 a_0^{-1}(t)D_t[a_0(t)D_t u].$$

The formula (10.25) should read :

$$|q^N_{h,A,\Omega}(v_h^0) - (b)^{1/2}\int_{I\!R_+} H^h(U^h(g^h(t)))U^h(g^h)(t)\chi^2(t)\, dt| \leq C\, h^{7/4},$$

p. 672.

After equation (A.3), the next equation reads

$$\mu(\xi)\int \varphi_\xi(t)\varphi_{\xi+\tau}(t)dt = \int_{I\!R_+}[D_t^2 + (t-\xi)^2]\varphi_\xi\varphi_{\xi+\tau}dt = -\varphi_\xi(0)\varphi'_{\xi+\tau}(\tau) + \mu(\xi+\tau)\int \varphi_\xi(t)\varphi_{\xi+\tau}dt$$