Formulaire de trigonométrie

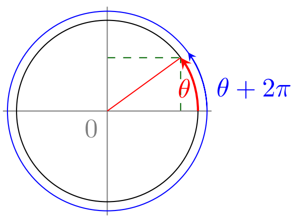
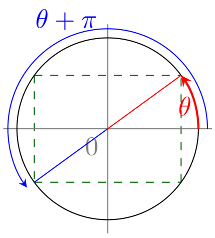
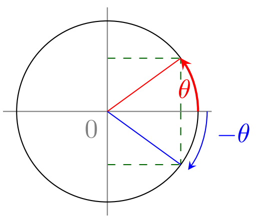
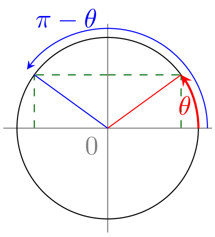
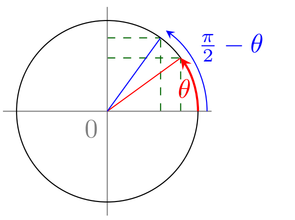
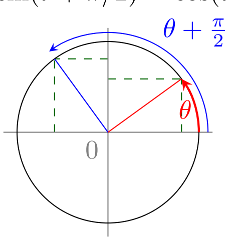
Dans la suite θ, x, a et b désignent des réels.

Valeurs usuelles.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
$\cos(\theta)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
$\sin(\theta)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0

Identité fondamentale. $\cos^2(\theta) + \sin^2(\theta) = 1$

Arcs associés.

Tour complet	*Demi-tour*
$\cos(\theta + 2\pi) = \cos(\theta)$ $\sin(\theta + 2\pi) = \sin(\theta)$ 	$\cos(\theta + \pi) = -\cos(\theta)$ $\sin(\theta + \pi) = -\sin(\theta)$ 
Angles opposés	*Angles supplémentaires*
$\cos(-\theta) = \cos(\theta)$ $\sin(-\theta) = -\sin(\theta)$ 	$\cos(\pi - \theta) = -\cos(\theta)$ $\sin(\pi - \theta) = \sin(\theta)$ 
Angles complémentaires	*Quart de tout direct*
$\cos(\pi/2 - \theta) = \sin(\theta)$ $\sin(\pi/2 - \theta) = \cos(\theta)$ 	$\cos(\theta + \pi/2) = -\sin(\theta)$ $\sin(\theta + \pi/2) = \cos(\theta)$ 

Résolution d'équations.

$$\cos(x) = \cos(a) \iff \begin{cases} x \equiv a\,[2\pi] \\ \text{ou} \\ x \equiv -a\,[2\pi] \end{cases}$$

$$\sin(x) = \sin(a) \iff \begin{cases} x \equiv a\,[2\pi] \\ \text{ou} \\ x \equiv \pi - a\,[2\pi] \end{cases}$$

Formules d'addition.

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

Formules de duplication.

$$\cos(2a) = \cos^2(a) - \sin^2(a)$$
$$= 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$$
$$\sin(2a) = 2\cos(a)\sin(a)$$

$$\cos(3a) = 4\cos^3(a) - 3\cos(a)$$
$$\sin(3a) = -4\sin^3(a) + 3\sin(a)$$

Formules de linéarisation.

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$
$$\sin(a)\sin(b) = -\frac{1}{2}\left(\cos(a+b) - \cos(a-b)\right)$$
$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

$$\cos^2(a) = \frac{1}{2}(1 + \cos(2a))$$
$$\sin^2(a) = \frac{1}{2}(1 - \cos(2a))$$

Formules de factorisation.

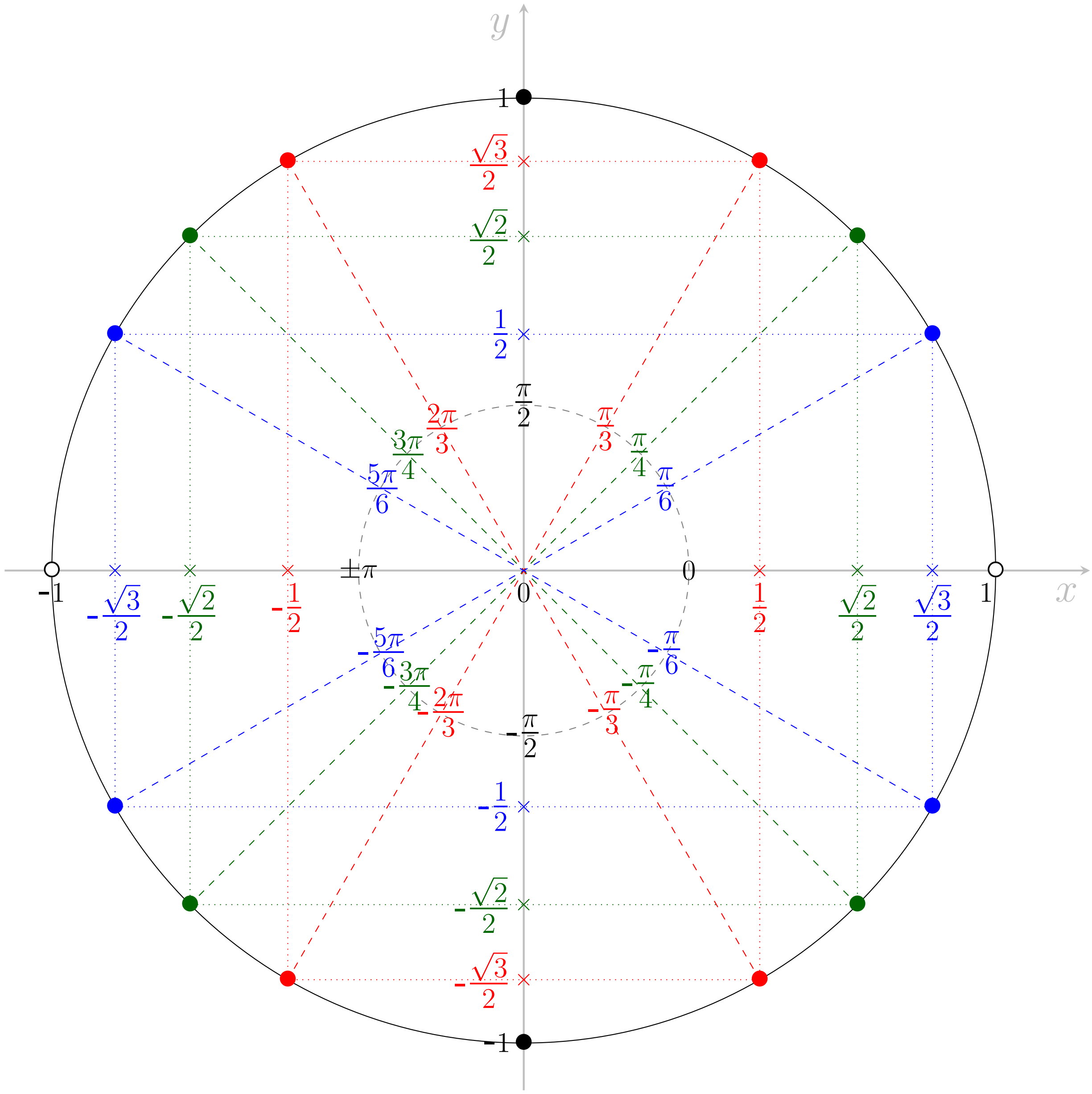
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Formules d'Euler.

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad \text{et} \quad \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

Formule de Moivre. Si $n \in \mathbb{Z}$, alors

$$\big(\cos(\theta) + i\sin(\theta)\big)^n = \cos(n\theta) + i\sin(n\theta).$$



y
x
1
-1
0
$\frac{\sqrt{3}}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{1}{2}$
$-\frac{1}{2}$
$-\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
$\pm\pi$
0
$-\frac{5\pi}{6}$
$-\frac{\pi}{6}$
$-\frac{3\pi}{4}$
$-\frac{\pi}{4}$
$-\frac{2\pi}{3}$
$-\frac{\pi}{3}$
$-\frac{\pi}{2}$