



Improving fairness in online learning

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Collaborators and references





A unified approach to fair online learning via Blackwell approachability E. Chzhen, C. Giraud, G. Stoltz; NeurIPS 2021 (spotlight).

Small Total-Cost Constraints in CBwK, with Application to Fairness E. Chzhen, C. Giraud, Z. Li, G. Stoltz; arXiv:2305.15807

Parameter-free projected gradient descent

E. Chzhen, C. Giraud, G. Stoltz; arXiv:2305.19605

Collaborators and references





The price of unfairness in linear bandits with biased feedback

S. Gaucher, A. Carpentier, C. Giraud; NeurIPS 2022.

Machine Learning is ubiquitous in daily life, and it is used for sensitive decisions such as

- admission in university,
- bank loan,
- job recruitment,
- justice decision,

• ...

Promises of ML in decision-making

Promises of ML in decision-making

ML can be more objective and more fair than humans, as algorithms can

- incorporate more data, and more factors in a complex analysis,
- and are not subject to personal biases, tiredness, emotional factors, etc

Hungry judge effect: some historical court data for grant to parole decisions (release from jail) have revealed that, irrespective of the gravity of the crimes

- $\bullet~\approx 65\%$ of positive decisions in the early morning or afternoon,
- close to 0% of positive decisions before lunch.

Actuality of ML decision-making

Discriminations also happen in ML prediction

Many ML systems have been shown to produce unfair outcomes.

Some famous past examples:

- **Hiring AI** from Amazon was discriminating against female candidate on some jobs
- **Google Ad** was proposing higher-paying executive jobs more likely to men than women
- **COMPAS** was falsely predicting recidivism twice more likely for African-American than for Caucasian-American.

Where does the unfairness come from?

Main potential causes of unfairness in data science

- [intentional discrimination]
- historical biases in learning datasets
- inadvertent bias in evaluations (biased proxy)
- inadvertent bias from data sampling: learning dataset not representative of the target population

• inadvertent bias from algorithm objectives: focus on the benefit for majority group

How can we mitigate these issues?

This talk: some possible directions for improving fairness in online learning

- Causal Fairness
- ② Statistical Fairness
 - Adversarial setting
 - Stochastic setting

Contextual online setting

Covariate and sensitive attribute

Each request is characterized by a covariate $x \in \mathcal{X}$ (observed) and a sensitive attribute $s \in \{-1, +1\}$ (observed or not).

Informal description of a typical setting

A each epoch $t = 1, 2, \ldots$

- The Learner observes a context (x_t, s_t) or x_t only
- The Learner performs an action (or prediction) a_t
- The Learner observes a feedback y_t and suffers a regret r_t (stochastic or adversarial)

Goal of the learner

To minimize the cumulative regret $\sum_t r_t$, while complying to some fairness criteria (and possibly some other constraints).

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1- A causal point of view

Image: A math a math

Causal fairness: identifying causes of unfairness

Causal fairness: general principle

- The relations between attributes (X, S) and their influence on feedback Y is modeled by structural equations
- The objective is to remove all discriminatory influences of sensitive attribute on the action/prediction

Example: No unresolved discrimination

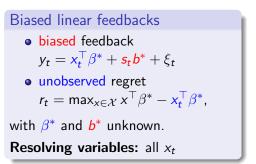
Design an action/prediction such that no path from the sensitive attribute to the action exists, except via non-discriminatory variables (resolving variables).

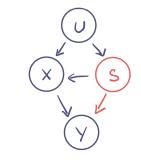
Caveat: the notions of causal fairness heavily rely on the causal model. The accuracy of this model is critical, and learning it can be problematic.

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A very (not so) simple case

The price of unfairness in linear bandits with biased feedback S. Gaucher, A. Carpentier, C. Giraud; NeurIPS 2022.





Price for learning the bias

- Worst case: the minimax regret scales as $T^{2/3}$ instead of \sqrt{T} for the unbiased case
- Asymptotic regret: scales as Δ⁻² log(T) instead of Δ⁻¹ log(T) in some cases.

2- Statistical fairness

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Statistical fairness

General principle

To comply to some fairness criteria at the sub-population level (statistical notions)

Example 1: equalized Odds

$$A \perp\!\!\!\perp S \mid Y$$

- Equalized Odds encodes a notion of meritocracy
- There are many variants

Caveat: strongly subject to biases in learning datasets

Statistical fairness: Demographic parity

Example 2: Demographic parity	h
$A \perp\!\!\!\perp S$	J

Demographic parity promotes *diversity* and can be related to affirmative action policies.

Caveat: the feedback Y is not taken into account

Finding a balance between different notions

Relaxing fairness criteria

- Fairness criteria are imperfect mathematical transposition of qualitative ideas;
- Evaluations of fairness criteria are subjected to uncertainties;
- Some fairness criteria are incompatible;
- so, it is wise to
 - introduce some quantitive measures of violation of the fairness criteria;
 - seek for a good trade-off between different fairness criteria and regret (Pareto frontier).

An instantiation in online learning

Fairness cost

Fairness criteria can be encoded as vector valued cost constraints.

Example: demographic parity

The empirical demographic parity criteria (for $a_t \in \{0,1\}$)

$$\left|\frac{1}{p_1 T} \sum_{t \leq T; s_t = 1} a_t - \frac{1}{p_{-1} T} \sum_{t \leq T; s_t = -1} a_t \right| = \tilde{O}(T^{-1/2})$$

can be encoded as

$$\sum_{t \leq T} c_t = \tilde{O}(\sqrt{T}) \quad \text{with} \quad c_t := \begin{bmatrix} a_t s_t / p_{s_t} \\ -a_t s_t / p_{s_t} \end{bmatrix}$$

Our contributions

Informal objective

 $\min_{\substack{\sum_{t < T} c_t \le \tilde{O}(\sqrt{T})}} \sum_{t < T} r_t.$

Two points of view

In adversarial setting:

- the fair learning problem can be formulated as a contextual approachability problem,
- Blackwell theory can be adapted to handle this setting.

In stochastic bandit setting:

- the fairness objective falls into the Contextual Bandit with Knapsack (CBwK) framework,
- the theory for CBwK must be improved to handle $\tilde{O}(\sqrt{T})$ constraints (and signed cost).

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Adversarial Setting :

a Contextual Blackwell Approachability Perspective

A unified approach to fair online learning via Blackwell approachability. E. Chzhen, C. Giraud, G. Stoltz; NeurIPS 2021 (spotlight).

Online learning setting: formal description

We model our fair online learning problem as a <u>contextual learning game</u> between the Learner and Nature.

Stochastic attributes (context)

At each time t, the attributes (x_t, s_t) are sampled according to **Q**, independently from the past.

Nature (un)awareness

Let G denotes Nature (un)awareness mapping

- Nature awareness G(x, s) = (x, s),
- Nature unawareness: G(x, s) = x.

Nature is an adverse player

At each time t, Nature observes $G(x_t, s_t)$ and outputs an adversarial feedback y_t .

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Fair online learning as a contextual approachability problem

Encoding the objectives of the learner

We can encode the learning objectives (vanishing-regret, demographic parity, etc) via

- a vector-valued payoff function $\mathbf{m}(a_t, y_t, x_t, s_t)$
- and a target set \mathcal{C} .

The learning objective is to comply to $\frac{1}{T}\sum_{t=1}^{T} \mathbf{m}(a_t, y_t, x_t, s_t) \longrightarrow C$.

Examples of targets (to be combined)

Criterion	Vector payoff function m	Closed convex target set $\ensuremath{\mathcal{C}}$	
Demographic parity	$m_{\mathrm{DP}}(a,s) = \left(\frac{a}{p_{-1}} 1_{s=-1}, \frac{a}{p_1} 1_{s=1}\right)$	$\mathcal{C}_{\mathrm{DP}} = \left\{ (u, v) \in \mathbb{R}^2 : u - v \le \delta \right\}$	
No-regret	$\mathbf{m}_{\mathrm{reg}}(a, y, x, s) = \left(f(a, y, x, s) - f(a', y, x, s)\right)_{a' \in \mathcal{A}}$	$\mathcal{C}_{\mathrm{reg}} = [0, +\infty)^N$	
Group-calibration	$\mathbf{m}_{\text{gr-cal}}(a, y, s) = \left(\left(a' - y \right) 1_{s=s'} / \gamma_{s'} \right)_{a' \in \mathcal{A}, \ s' \in \mathcal{S}}$	$\mathcal{C}_{\text{gr-cal}} = \left\{ \mathbf{v} \in \mathbb{R}^{N \mathcal{S} } : \ \mathbf{v}\ _1 \le \varepsilon \right\}$	
Equalized payoffs	$\mathbf{m}_{\text{eq-pay}}(a, y, x, s) = \left(\frac{f(a, y, x, s')}{\gamma_{s'}} 1_{s=s'}\right)_{s' \in S}$	$C_{\text{eq-pay}} = \left\{ (u, v) \in \mathbb{R}^2 : u - v \leq \varepsilon \right\}$	

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Online learning setting

Learning setting

- For t = 1, 2, ...
 - Simultaneously,
 - $\begin{array}{l} \quad \text{the Learner chooses } (\mathbf{p}_t^{\mathsf{x}})_{\mathsf{x}\in\mathcal{X}} \text{ based on } (\mathbf{m}_{\tau}, \mathsf{x}_{\tau}, \mathsf{s}_{\tau})_{\tau\leq t-1} \\ \quad \text{Nature chooses } \left(\mathbf{q}_t^{\mathsf{G}(\mathsf{x},\mathsf{s})}\right)_{(\mathsf{x},\mathsf{s})\in\mathcal{X}\times\mathcal{S}} \text{ based on } (\mathsf{a}_{\tau}, \mathsf{y}_{\tau}, \mathsf{x}_{\tau}, \mathsf{s}_{\tau})_{\tau\leq t-1} \end{array}$
 - ${f Q}$ (x_t,s_t) are sampled according to ${f Q}$, independently from the past
 - Simultaneously
 - the Learner observes x_t , and picks an action $a_t \in \mathcal{A}$ according to $\mathbf{p}_t^{\mathsf{x}_t}$
 - Nature observes $G(x_t, s_t)$, and picks $y_t \in \mathcal{Y}$ according to $\mathbf{q}_t^{G(x_t, s_t)}$
 - The Learner observes the payoff $\mathbf{m}_t = \mathbf{m}(a_t, y_t, x_t, s_t)$ and (x_t, s_t) , while Nature observes (a_t, y_t, x_t, s_t) .

Aim: The Learner wants to ensure that $\bar{\mathbf{m}}_T := \frac{1}{T} \sum_{t=1}^{T} \mathbf{m}_t \to \mathcal{C}$ a.s. for

some target set ${\mathcal C}.$

Online learning setting

Learning setting

- For t = 1, 2, ...
 - Simultaneously,
 - $\begin{array}{l} \quad \text{the Learner chooses } (\mathbf{p}_t^{\mathsf{x}})_{\mathsf{x}\in\mathcal{X}} \text{ based on } (\mathbf{m}_{\tau},\mathsf{x}_{\tau},\mathsf{s}_{\tau})_{\tau\leq t-1} \\ \quad \text{Nature chooses } \left(\mathbf{q}_t^{G(\mathsf{x},\mathsf{s})}\right)_{(\mathsf{x},\mathsf{s})\in\mathcal{X}\times\mathcal{S}} \text{ based on } (a_{\tau},y_{\tau},\mathsf{x}_{\tau},\mathsf{s}_{\tau})_{\tau\leq t-1} \end{array}$
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 - The Learner observes the payoff $\mathbf{m}_t = \mathbf{m}(a_t, y_t, x_t, s_t)$ and (x_t, s_t) , while Nature observes (a_t, y_t, x_t, s_t) .

Aim: The Learner wants to ensure that $\bar{\mathbf{m}}_{\mathcal{T}} := \frac{1}{\mathcal{T}} \sum_{t=1}^{\prime} \mathbf{m}_t \to \mathcal{C}$ a.s. for some target set \mathcal{C} .

Online learning setting

Learning setting

- For t = 1, 2, ...
 - Simultaneously,

 - ► the Learner chooses $(\mathbf{p}_t^x)_{x \in \mathcal{X}}$ based on $(\mathbf{m}_{\tau}, x_{\tau}, s_{\tau})_{\tau \leq t-1}$ ► Nature chooses $(\mathbf{q}_t^{G(x,s)})_{(x,s) \in \mathcal{X} \times S}$ based on $(a_{\tau}, y_{\tau}, x_{\tau}, s_{\tau})_{\tau \leq t-1}$
 - (x_t, s_t) are sampled according to **Q**, independently from the past
 - Simultaneously
 - the Learner observes x_t , and picks an action $a_t \in \mathcal{A}$ according to $\mathbf{p}_t^{x_t}$
 - Nature observes $G(x_t, s_t)$, and picks $y_t \in \mathcal{Y}$ according to $\mathbf{q}_t^{G(x_t, s_t)}$
 - The Learner observes the payoff $\mathbf{m}_t = \mathbf{m}(a_t, y_t, x_t, s_t)$ and (x_t, s_t) , while Nature observes (a_t, y_t, x_t, s_t) .

Aim: The Learner wants to ensure that $\bar{\mathbf{m}}_{\mathcal{T}} := \frac{1}{\mathcal{T}} \sum_{t=1}^{t} \mathbf{m}_{t} \to \mathcal{C}$ a.s. for some target set \mathcal{C} .

Assumption: fast enough sequential estimation of **Q**

The Player can build estimators $(\hat{\mathbf{Q}}_t)_{t\geq 1}$ of the unknown distribution \mathbf{Q} such that $\mathbb{E}\left[\mathsf{TV}^2(\hat{\mathbf{Q}}_t, \mathbf{Q})\right] \leq c \ (\log(t))^{-3} \quad \forall t \geq 2 \tag{1}$

Theorem : Contextual Blackwell approachability

If $\mathcal{C} \subset \mathbb{R}^d$ is closed convex, **m** is bounded, and (1) is satisfied, then $\exists (\mathbf{p}_t^x)_{x \in \mathcal{X}, t \ge 1}$ such that $\forall (\mathbf{q}_t^{G(x,s)})_{(x,s) \in \mathcal{X} \times \{0,1\}, t \ge 1}$ we have $\bar{\mathbf{m}}_T \stackrel{a.s.}{\to} \mathcal{C}$ if and only if $\forall (\mathbf{q}^{G(x,s)})_{(x,s) \in \mathcal{X} \times \{0,1\}} \exists (\mathbf{p}^x)_{x \in \mathcal{X}}$ such that $\mathbf{m}(\mathbf{p}, \mathbf{q}, \mathbf{Q}) := \int_{\mathcal{X} \times \mathcal{S}} \mathbf{m}(\mathbf{p}^x, \mathbf{q}^{G(x,s)}, x, s) d\mathbf{Q}(x, s) \in \mathcal{C}$

Contextual Blackwell strategy

Set $\mathbf{m}(\mathbf{p}, \mathbf{q}, \hat{\mathbf{Q}}_t) := \int \mathbf{m}(\mathbf{p}^x, \mathbf{q}^{G(x,s)}, x, s) d\hat{\mathbf{Q}}_t(x, s)$. At stage t + 1, choose $(\mathbf{p}_{t+1}^x)_{x \in \mathcal{X}} \in \underset{(\mathbf{q}^{G(x,s)})_{x,s}}{\operatorname{argmin}} \max_{(\mathbf{q}^{G(x,s)})_{x,s}} \langle \bar{\mathbf{m}}_t - \Pi_{\mathcal{C}} \bar{\mathbf{m}}_t, \mathbf{m}(\mathbf{p}, \mathbf{q}, \hat{\mathbf{Q}}_t) \rangle$

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Caveats

Caveat 1: the target set $\ensuremath{\mathcal{C}}$ has to be known

The results can be extended (at the price of some technicalities) to the case where we only have a consistent super-estimate \hat{C}_t of C.

Caveat 2: computational cost of projection

Computing the projection $\Pi_{\mathcal{C}}$ can be computationally expensive.

Caveat 3: pessimistic Pareto frontier and slow rates

- The adversarial setting leads to pessimistic Pareto frontier (trade-off) between the different criteria;
- The rates are governed by the estimation rate TV(Q
 ^ˆt, Q), which is typically slow outside the finite case.

Stochastic setting:

A Contextual Bandit with Knapsack perspective

Small Total-Cost Constraints in CBwK, with Application to Fairness E. Chzhen, C. Giraud, Z. Li, G. Stoltz; arXiv:2305.15807

Stochastic setting

Learning problem

- The learner observes $\tilde{x}_t = (x_t, s_t) \stackrel{\text{i.i.d.}}{\sim} \mathbf{Q}$
- The learner chooses a policy $\pi_t : \tilde{\mathcal{X}} \to \mathcal{P}(\mathcal{A})$, and picks an action $a_t \sim \pi_t(\tilde{x}_t)$,
- The learner receives a feedback y_t and a fairness cost c_t such that

$$\mathbb{E}[y_t|\mathcal{F}_t] = f(\tilde{x}_t, a_t) \text{ and } \mathbb{E}[c_t|\mathcal{F}_t] = c(\tilde{x}_t, a_t).$$

• The learner suffers a regret $r_t = OPT - y_t$ (described below).

Example: Demographic Parity

$$c(ilde{x}_t, a_t) = \left[egin{array}{c} a_t s_t / p_{s_t} \ -a_t s_t / p_{s_t} \end{array}
ight].$$

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Optimal policy and regret

Optimal static policy and regret

The optimal static feedback is

$$\mathsf{OPT}(\mathbf{Q}, f, c) := \max_{\pi \ : \ \mathbb{E}_{\mathbf{Q}}\left[\sum_{a \in \mathcal{A}} c(\tilde{X}, a) \pi_{a}(\tilde{X})\right] \leq \delta_{T}} \mathbb{E}_{\mathbf{Q}}\left[\sum_{a \in \mathcal{A}} f(\tilde{X}, a) \pi_{a}(\tilde{X})\right]$$

and the regret is

$$r_t = \mathsf{OPT}(\mathbf{Q}, f, c) - y_t.$$

Learning Objective

Minimize the cumulative regret $\sum_{t \leq T} r_t$ while complying to the fairness constraint $\sum_{t \leq T} c_t \leq T \delta_T$.

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Fairness as CBwK

CBwK problem

• we recognize a Contextual Bandit with Knapsack (CBwK) problem

2 but state of the art theory can only handle $\delta_T = T^{-1/4}$ (or \mathcal{X} finite), which is too large for fairness constraints, where we typically wish to have $\delta_T = \tilde{O}(T^{-1/2})$

Learning assumption

Assumption: UCB and LCB

We can built UCB and LCB such that with probability $\geq 1-\delta$

$$egin{aligned} \hat{f}_t^{\mathsf{UCB}}(.,.) &pprox f(.,.) + ilde{O}_\delta(1/\sqrt{t}) \ \hat{c}_t^{\mathsf{LCB}}(.,.) &pprox c(.,.) + ilde{O}_\delta(1/\sqrt{t}) \end{aligned}$$

Examples

Linear or logistic model : when

$$f(x,a) = \eta(\varphi(x,a)^T heta_a)$$
 and $c(x,a) = \eta(\psi(x,a)^T eta_a),$

with $\eta(u) = u$ or $\eta(u) = e^u/(1 + e^u)$, we can use variant of LinUCB or LogisticUCB1.

A first idea

Idea1: playing empirical optimal static policy

Choose a_t according to a policy $\hat{\pi}_t$ maximizing $OPT(\hat{\mathbf{Q}}_t, \hat{f}_t^{UCB}, \hat{c}_t^{LCB})$.

Issues

The analysis of

$$\mathsf{OPT}(\mathbf{Q}, f, c) - \mathsf{OPT}(\hat{\mathbf{Q}}_t, \hat{f}_t^{\mathsf{UCB}}, \hat{c}_t^{\mathsf{LCB}})$$

produces some $TV(\hat{\mathbf{Q}}_t, \mathbf{Q})$ terms, leading to slow rates / large fairness violation.

2 Solving $OPT(\hat{Q}_t, \hat{f}_t^{UCB}, \hat{c}_t^{LCB})$ is computationally expensive

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Lagrangian version

Lagrangian formulation

$$OPT(\mathbf{Q}, f, c) = \max_{\pi : \mathbb{E}_{\mathbf{Q}}\left[\sum_{a \in \mathcal{A}} c(\tilde{X}, a) \pi_{a}(\tilde{X})\right] \leq \delta_{T}} \mathbb{E}_{\mathbf{Q}} \left[\sum_{a \in \mathcal{A}} f(\tilde{X}, a) \pi_{a}(\tilde{X})\right]$$
$$= \min_{\lambda \geq 0} \max_{\pi} \mathbb{E}_{\mathbf{Q}} \left[\sum_{a \in \mathcal{A}} \pi_{a}(\tilde{X}) \left(f(\tilde{X}, a) - \langle \lambda, c(\tilde{X}, a) - \delta_{T} \rangle\right)\right]$$
$$= \min_{\lambda \geq 0} \mathbb{E}_{\mathbf{Q}} \left[\max_{a \in \mathcal{A}} \left\{f(\tilde{X}, a) - \langle \lambda, c(\tilde{X}, a) - \delta_{T} \rangle\right\}\right]$$

Two immediate benefits

① for a fixed λ the problem is separable, and **Q** can be forgotten;

2 we only need to learn the optimal $\lambda^* \in \mathbb{R}^d \Longrightarrow$ parametric rates. \odot

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High-level algorithm: Primal-dual descent-ascent

Iterate

• full optimisation on primal variable: pick

$$a_t \in \operatorname*{argmax}_{a \in \mathcal{A}} \left\{ \widehat{f}_t(\tilde{X}, a) - \langle \lambda_{t-1}, \widehat{c}_t(\tilde{X}, a) - \delta_T
angle
ight\}$$

• projected subgradient step on dual variable: update

$$\lambda_t = \left(\lambda_{t-1} + \gamma\left(\hat{c}_t(\tilde{X}, a) - \delta_T\right)\right)_+$$

Issues

• Benign issue: we must replace δ_T by $\delta'_T = \delta_T - \tilde{O}(1/\sqrt{T})$ to prevent from violation of the fairness criteria due to random fluctuations

2 Major issue: to satisfy the constraints, we need to set $\gamma \approx ||\lambda^*||/\sqrt{T}$

Adaptative algorithm

Adaptive version

Iterate: for $t \ge 1$ • Pick $a_t \in \operatorname{argmax}_{a \in \mathcal{A}} \left\{ \hat{f}_t(\tilde{X}, a) - \langle \lambda_{t-1}, \hat{c}_t(\tilde{X}, a) - \delta'_T \rangle \right\}$ • Update $\lambda_t = \left(\lambda_{t-1} + \frac{2^k}{\sqrt{T}} \left(\hat{c}_t(\tilde{X}, a) - \delta'_T \right) \right)_+$ Until $\left\| \left(\sum_{\tau=T_k}^t c_\tau - (t - T_k + 1)\delta'_T \right)_+ \right\| > \tilde{O}(\sqrt{T})$

Then: increase k by one, set $T_k = t + 1$, and **iterate** again.

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Theory

Regret bound For $\delta_T \ge \tilde{O}(T^{-1/2})$, the above algorithm fulfills with probability at least $1 - \delta$ $\sum_{t \le T} r_t \le \tilde{O}_{\delta}\left((1 + \|\lambda^*\|)\sqrt{T}\right) \qquad \sum_{t \le T} c_t \le \delta_T T.$

Suitable for fairness constraints 🙂

Optimality?

A proof scheme suggests that this regret is optimal.

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Concluding remarks

- Fairness in decision-making is an important topic;
- The statistical community has an important role to play for providing
 - conceptual ideas
 - competitive algorithms with provable performances
 - theoretical insights
 - education of the next generation of data scientists