Further difficulties for simulation of convective flows with D2Q13

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• Introduction.

In previous communications [1, 2], the 2-d standard lattice Boltzmann model D2Q13 based on the velocities $((0,0), (\pm1,0), (\pm1,\pm1), (\pm2,0))$ with 4 conserved moments (density, linear momentum and a scalar T that we call "Temperature") has been studied. It was shown that one can set the expressions for the equilibrium values of the 9 non-conserved moments to achieve correct advection of the 4 basic hydrodynamic modes and choose relaxation rates such that the spurious coupling between diffusive modes can be neglected at long wave length. Here we add features to perform a simulation of the simple de Vahl Davis [3] problem (square cavity with differentially heated lateral walls). This involves setting boundary conditions and introducing a buoyancy force.

• Boundaries.

We use the fact that velocity transforms like a vector and "Temperature" and density as scalars. The velocity of the fluid is 0 on the 4 walls of the cavity and this can be achieved by the usual "bounce-back" condition for the distribution functions f_i. It turns out that this has to be done only for f_i that correspond to unit velocities that have a non-zero component along the boundary (for $\{\pm1,\ \pm1\}$). On the vertical walls the temperature is imposed and the density is estimated by linear extrapolation from the bulk. These values are imposed by "anti-Bounce-back". On the horizontal walls adiabaticity is achieved by Bounce-back. (for $\{\pm1,\ 0\}$, $\{\pm2,\ 0\}$).

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- External force.

Buoyancy forces are simply introduced by modifying the conservation of linear momentum in the vertical direction proportionally to T.

- Tests.

Several tests have been performed on a square domain with various boundary conditions. The fully periodic case allows to compare attenuation and advection of plane waves to theoretical predictions. Solid vertical boundaries and periodic condition on the horizontal plates allows to verify that temperature can be set on the edges. It is however found that the de Vahl Davis case gives unsatisfactory results as the steady state temperature field is not symmetric with respect to the center of the domain, as is found when simulating the same case with D2Q9-D2Q5[4] or with a simple finite-difference implementation of the Navier-Stokes equations.

- Unsolved features.

A simple problem has been considered : We consider a domain of size N_x, N_y: periodic in y direction and with solid walls at $x = 1$ and $x = N_x$ set at temperature $T_0 \pm \delta T$ leading to a linear profile $T(x)$. The walls can move in the vertical direction (Couette flow) or a uniform body force can be introduced (Poiseuille flow). It is found that the linear temperature profile $T(x)$ is non-linearly modified by a flow $V_y(x)$, even though the heat production term proportional to the shear viscosity is not included in the model. The simulation of the same problem with D2Q9-D2Q5 or Navier-Stokes does not show this modification. To eliminate possible incorrect boundary conditions, we study the normal modes of the linearized problem with a given non uniform $V_y(x)$ and obtain similar results.

References

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