Simulation of strong nonlinear waves with vectorial lattice Boltzmann schemes

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04 march 2013 *

We have proposed in [3] to adapt the mathematical framework of dual entropy proposed by Chen *et al* [2] and extended by Bouchut [1] in order to determine equilibrium states of lattice Boltzmann schemes. Recall that with this approach, we can establish a "H-theorem" for the system of Boltzmann equations with discrete velocities and we recover the Karlin *et al* [4] minimization property. We restricted ouselves to the cases of one-dimensional scalar nonlinear waves and to nonlinear acoustics in a natural framework with a **scalar** distribution function. In this contribution, we study the hability of the dual entropy approach to simulate nonlinear waves for the shallow water equations in one a two space dimensions with **vectorial** lattice Boltzmann schemes. We have developed "D1Q3Q2" and "D2Q5Q4Q4" lattice Boltzmann schemes. A first result for a Sod type schock tube is presented in Figure 1.

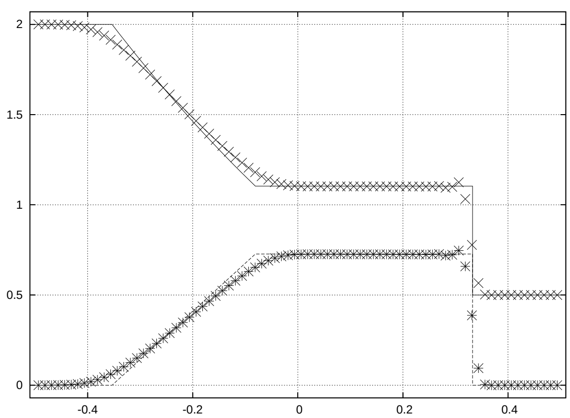

Figure 1. Density and velocity fields for a Sod type shock tube for shallow water equations ($\rho_\ell = 2$, $\rho_r = 0.5$, $u_\ell = u_r = 0$) simulated with a D1Q3Q2 vectorial lattice Boltzmann scheme. Comparison between exact and numerical solutions.

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[2] G.Q. Chen, C.D. Levermore, T.P. Liu. "Hyperbolic conservation laws with stiff relaxation and entropy", *Communications on Pure and Applied Mathematics*, vol. **47**, p. 787-830, 1994.

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[4] I.V. Karlin, A.N. Gorban, S. Succi and V. Boffi. "Maximum Entropy Principle for Lattice Kinetic Equations", *Physical Review Letters*, vol. **81**, p. 6-9, 1998.

* Contribution submitted to the conference "DSFD", Yerevan, 15-19 July 2013.