Anomalous advection in LBE simulations*

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LBE models whether based on a description in terms of "populations" f_i or in the d'Humières approach [1] with moments m_i include non linear terms of the form $v_x^2 + v_y^2$, $v_x^2 - v_y^2$, $v_x v_y$ in 2-D (with more expressions of the same type in 3-D). It is usually taken for granted that this leads to correct advection in the simulated flows. Using the equivalent equation technique proposed in [2], one can easily analyze any LBE model and obtain macroscopic equations that include third order space derivatives (or higher if needed).

In an initial use of these extended dynamic equations one considers a situation periodic in space with initial condition : uniform large speed V plus a plane wave of small amplitude defined by a wave vector k (making an angle θ with V) and a "polarization" either parallel (sound wave) or perpendicular (shear wave) to k. The solution is $V + f(t)\, u(x)$ with $f(t) = \exp -\Gamma t$. The (complex) relaxation rate Γ can be obtained by solving the full dispersion equation [3], or the deduced one derived from the macroscopic equivalent equations. It can also be measured from simulations (through the evolution of the correlation with the initial condition).

For D2Q9 with one conservation, with relevant tuning, one gets $\Gamma = i\, k.V + \kappa\, k^2 + i\, A(V,\theta)\, k^3 + \mathcal{O}(k^4)$ allowing to define an "anomalous" advection coefficient $g_T = 1 + A/(k.V)\, k^2$, with $A = a_0 + a_2 \cos^2\theta$ and κ is the diffusivity. For D2Q9 or D2Q13 with three conservations (with relevant tuning of D2Q13 to eliminate the velocity dependence of the shear and bulk viscosities) one gets similar results. For shear waves : $\Gamma = i\, k.V + \nu(V)\, k^2 + i\, B(V,\theta)\, k^3 + \mathcal{O}(k^4)$ allowing to define an "anomalous" advection coefficient $g_\nu = 1 + B/(k.V)k^2$, with $B = b_0 + b_2 \cos^2\theta + b_4 \cos^4\theta$ and ν is the kinematic shear viscosity. Expressions for A and B will be discussed for 2-D models together with results of analysis of simulations. In previous work in the Stokes regime, we concentrated on the order 4 contribution [4].

A second case is a gaussian vortex of initial radius r_0 on top of a uniform velocity V. Simulations show loss of rotational invariance due to the angular dependence of the advection factor. In the simpler 1-conservation case, we show how to choose parameters of the model to get $a_2 = 0$ and thus recover circular symmetry. Getting $b_2 = b_4 = 0$ is possible but the model is no longer stable. Results of simulations will be compared to direct computation of a proper sum of plane waves taking into account the angular dependence of the phase factor (Im Γ).

References

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