Lattice Boltzmann Method, Turbulence and Micromixing

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Introduction an Motivation

Lattice Boltzmann Method (LBM)

An alternative to classic CFD method (?)

Big progress made over the last decade

Becomes popular (?)

Advantages

Ease of boundary condition implementation

Well adapted for parallel computations

No Poisson equation for the pressure
LBM = Simplified kinetic model to simulate fluid flows

\( \rho(x,t) \)
\( u(x,t) \)

Homogenous fluid
Real fluid
Lattice fluid

Ensemble averaging
The lattice Boltzmann Method

Boltzmann equation

\[ f = f(\vec{r}, \vec{e}, t) \equiv \text{Particle probability density distribution function (PPDF)} \]

Particle velocity
Vector position

\[ \frac{df}{dt} + \vec{e}.\nabla r f + \vec{a}.\nabla e f = (\partial f)_{\text{coll}} \]

External force
collision
The lattice Boltzmann Method

Boltzmann’s (BGK) equation

\[
\frac{df}{dt} + \hat{e} \cdot \nabla_r f = -f - f_{eq} + \frac{\tau}{f_{eq}} \left( f_{eq} - f \right)
\]

If we assume: \( \nabla_r f = \frac{\tau}{f_{eq}} \left( f_{eq} - f \right) \)
The lattice Boltzmann Method

Lattice Boltzmann equation for D3Q19 (no external force)

\[ f_a(\vec{r} + \vec{e}, t + \Delta t) = f_a(\vec{r}, t) - \frac{f_a(\vec{r}, t) - f_a^{eq}(\vec{r}, t)}{\tau} \]

\[ f_a^{eq} = \rho \omega_a (1 + 3\vec{e}_a \cdot \vec{u} + \frac{9}{2} (\vec{e}_a \cdot \vec{u})^2 - \frac{3}{2} \vec{u}^2) \]

\[ \rho = \sum_{a=0}^{18} f_a \]

\[ \rho \vec{u} = \sum_{a=0}^{18} f_a \vec{e}_a \]

\[ \nu = (2\tau - 1)/6 \] (Viscosity)
Applications of LBM

Treatment of the boundary conditions

No slip condition at the wall

Outlet: Convective boundary condition or zero gradient

\[
\frac{\partial u}{\partial t} + U_{\text{mean}} \frac{\partial u}{\partial x} = 0 \quad \text{or} \quad \frac{\partial u}{\partial x} = 0
\]

Inlet: Uniform velocity \( U_0 \)

Lateral sides: Periodic conditions
Applications of LBM

Turbulent flows

- 3D Transition in a cylinder wake
- Cross-bar wake
- Grid turbulence
- 3D box turbulence
- Urban Flows

Microfluidics

- Micromixer
  - Passive mixer
  - Active mixer
Lattice Boltzmann simulation of transitional cylinder wake

\[ R_d = 200 \]

\[ d = 10 \]

\[ 70d \]
Results – 3D transitional wake (Continued)

\[ \frac{\omega_z}{\omega_{z,\text{max}}} = +/\ - 0.1 \]

\[ \frac{\omega_x}{\omega_{x,\text{max}}} = +/\ - 0.1 \]

DNS, \( \omega_x = +/- 0.25 \), \( \omega_z = +/- 0.25 \)
Persillon and Braza, JFM, 1998, 365, 23-88

Vorticity iso-contours
Results - 3D transitional wake (Continued)

\[ \frac{\omega}{\omega_{\text{max}}} = 0.07 \]


Vorticity iso-contours

LBM

Flow visualization
Results - 3D transitional wake (Continued)

Velocity spectra at $x/D = 2.5$, $y/D = 0$

$f_s = 0.129$
Results - 3D transitional wake (Continued)

Velocity spectra, DNS, x/D = 1.80, y/D = 0
Results – 3D transitional wake (Continued)

\[ u/U = 0.75 \]

\[ v/U = \pm 0.35 \]

\[ w/U = \pm 0.00006 \]

Early stage of transition
Results - 3D transitional wake (Continued)

$w/U = +/- 0.00006$

$\omega_x/\omega_{x,max} = +/- 0.1$

DNS, Braza,
FTC, 1999, 63,315-341
Lattice Boltzmann simulation of a crossbar wake

Crossbar and Computational domain

$R_d = 1600$

$12d$

$12d$

$80d$

$d = 10$
Results - Crossbar wake

Comparison between LBM and LDV

U, u', and w' along the centerline, Symb.: LVD, lines: LBM
Comparison between LBM and LDV

$u'$ and $w'$ along the centerline, Symb.: LVD, lines : LBM
Results - Crossbar wake (Continued)

w-spectrum at y/d = 3, z/d = 1.5 and x/d = 5
Comparison between LBM and PIV

Velocity field at $z = 3D$
Results – Crossbar wake (Continued)

Comparison between LBM and PIV

Velocity field at $z = 0.3D$
Results - Crossbar wake (Continued)

Instantaneous contour of $\omega / \omega_{\text{max}} = 0.3$. Left: front view, right: back view.
Results – Grid turbulence

$R_d = 1600$

Iso-surfaces of $\omega^2$ behind the grid.
Figure 8. Decay of the turbulent kinetic energy (solid line) and its components (---, $u'^2$; ----, $v'^2$; · · ·, $w'^2$) downstream of the grid. Symbols: experiments (Lavoie et al. 2005), $\times$, $u'^2$; $\circ$, $v'^2$. 
Figure 17. Longitudinal one dimensional spectra $E_{11}(k)$ in Kolmogorov units at $x/M = 20$ (...) and 60 (---). ---, experiment of Lavoie et al. (2005), symbols: experiment of Comte-Bellot & Corrsin (1971).
Lattice Boltzmann simulation of urban flows

North-East side of Newcastle

Ocean wind
Results - Urban flow (Continued)

Instantaneous surface pressure
Results - Micromixer

Passive micromixer (coaxial)
Results - Micromixer (Continued)

Passive micromixer (coaxial)
Control by Pulsed jet

Active micromixer (channel flow with pulsed jets)
Results - Micromixer (Continued)

Active micromixer (channel flow with pulsed jets)

One pulsed jet
Two pulsed jets
Conclusions

The presented LBM simulation showed:

1 - LBM is a reliable alternative DNS method to the classical procedure based on the resolution of NS-equations. (at least for incompressible flows)

2 - LBM can be used for research and development.

BUT!

Need to convince more researchers (e.g. turbulence community) to use LBM (How?)