Conditions limites numériques pour les systèmes hyperboliques ; application en mécanique des fluides.

An elementary test case

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- We consider the advection equation on the interval $[0, 1]$:

$$(1) \qquad \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \,, \qquad 0 < x < 1 \,, \quad t > 0 \,,$$

with an homogeneous initial condition

$$(2) \qquad u(x, 0) = 0 \,, \qquad 0 < x < 1 \,,$$

with a discontinuous boundary condition for the incoming boundary :

$$(3) \quad u(0, t) = \begin{cases} 1 & \text{if} \quad 0 < t < 1 \,, \\ 0 & \text{if} \quad t > 1. \end{cases}$$

The exact solution of this problem (1)(2)(3) is elementary and is given by the method of characteristics :

$$(4) \quad u(x, t) = \begin{cases} 1 & \text{if} \quad 0 < t - x < 1 \,, \\ 0 & \text{if} \quad t < x \quad \text{or} \quad t > x + 1 \,, \quad 0 < x < 1. \end{cases}$$

- The difficulties of this test case are of purely numerical origin and the following questions are natural : *(i)* How many mesh points are used for the capture of this moving linear discontinuity ? *(ii)* What is the exact definition of the numerical scheme near the incoming **and** the out-coming boundary ? *(iii)* Does the numerical approximation satisfies the maximum principle ? We suggest to use at least second order schemes and meshes with a number N of points satisfying the conditions

$$(5) \qquad N = 10, 20, 40, 80, 160, 320$$

and the participants to present the following results :

$$(6) \qquad u(0^+, t) \,, \quad u(1^-, t) \,, \quad 0 < t < 2 \,,$$

$$(7) \qquad u(x, t) \,, \qquad 0 < x < 1 \,, \quad t = \frac{j}{4} \,, \; 1 \leq j \leq 7 \,.$$

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