le cnam

Master Structural Mechanics and Coupled Systems

Applied Mathematics

Student presentation 6 (course 11)

- A singular function

For $(x, y) \in \mathbb{R}^2$, we define the value $f(x, y)$ by the following way: $f(x, y) = \frac{x^5}{(y-x^2)^2+x^8}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

a) Show that for each $(x, y) \in \mathbb{R}^2$, the real number $f(x, y)$ is well defined.

b) Compute the values $f(x, 0)$ et $f(0, y)$ for every real number x and every real number y.

c) Going back to the definition of a partial derivative, deduce from the previous question that the function f admits two partial derivatives $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$ at the origin. Precise the values of these two numbers.

d) Compute $f(x, x^2)$ when $y = x^2$ for every real number x.

e) Deduce from the previous points that the function f is not continuous at the origin $(0, 0)$.

f) Prove that the function f is not differentiable at the origin, even if it has two partial derivatives at this point.

François Dubois, september 2023.