le cnam

Master Structural Mechanics and Coupled Systems

Applied Mathematics

Student presentation 4 (course 09)

- Harmonic oscillator excited by its own frequency

We introduce $\omega > 0$. We study an harmonic oscillator $x(t)$. It satisfies the second order differential equation $\frac{d^2x}{dt^2} + \omega^2 x(t) = f(t)$ and the initial conditions $x(0) = x_0$ and $\frac{dx}{dt}(0) = v_0$.

a) When the external force $f(t)$ is null, recall the algebraic expression $x(t)$ of the solution of the previous problem.

We introduce a constant $X \neq 0$ and we suppose now that the external force admits the expression $f(t) = \omega^2 X \sin \omega t$.

b) Find an algebraic expression for the harmonic oscillator $x(t)$, composed by the differential equation (with the nontrivial external force) and the boundary conditions. A particular solution is obtained by taking $x(t)$ equal to a polynomial of degree one relative to time, mutiplied by a sinus function, plus an other polynomial of degree one mutiplied by a cosine function.

c) Draw the general appearance of the trajectory $t \longmapsto x(t)$ of the harmonic oscillator with the external force $f(t) = \omega^2 X \sin \omega t$.

d) What do you remark ? Can you make a mechanical comment about this property ?