

## Reading seminar: Hodge Theory and o-minimality

Département de Mathématiques d'Orsay, salle 3L15 bâtiment 307

Tuesdays, 10:15–11:45

April–June, 2019

The goal of this seminar is to learn the recent advances on the applications of o-minimal geometry to Hodge theory, by Bakker, Brunenbarbe, Klingler, and Tsimerman. In particular, the first goal would be to understand the new proof for the theorem of Cattani–Deligne–Kaplan [CDK95] on the algebraicity of Hodge loci given in [BKT18]. If time permits, the second goal would be to read as well the proof of a long-standing conjecture by Griffiths on the quasi-projectivity of images of period maps [BBT18].

The first part of the seminar will be a review of Hodge Theory: in particular, we will cover (mixed) Hodge structures, their variations, degenerations, and Schmidt's  $SL_2$ -orbit Theorem. Basic references for this part are the books [Voi07, PS08] and the original articles [Del71, Del74, Sch73]. Finally, we will review the original proof of the algebraicity of the Hodge loci, by following [CDK95]. In the second part, we will cover the basics of o-minimal geometry and give an idea on a result by Wilkie that semi-algebraic sets together with the graph of the exponential function give rise to an o-minimal structure. Some references are [Wil96, PS09]. We will also need some basic facts on bi-algebraic geometry and arithmetic quotients. We will follow parts of the two survey articles [KUY17, Kli17] and reference therein. Finally, in the last meetings, we will cover the basics of the proofs of the two results on the algebraicity of Hodge loci and on Griffiths's Conjecture.

### Tentative Plan.

**Tuesday, April 9, 2019:** Introduction to (Mixed) Hodge structures. Speaker: *Céline Bonandrini*.

**Tuesday, April 16, 2019:** Variations of Hodge structures and degenerations. Speaker: *Benjamin Sung*.

**Tuesday, April 23, 2019:** Limit Hodge structures and Schmidt's  $SL_2$  Theorems. Speaker:

**Tuesday, April 30, 2019:** Ideas from the original proof of the Cattani–Deligne–Kaplan Theorem. Speaker: *Emanuele Macrì*.

**Tuesday, May 7, 2019:** Introduction to o-minimal structures:  $\mathbb{R}_{\text{alg}}$ ,  $\mathbb{R}_{\text{an}}$ ,  $\mathbb{R}_{\text{exp}}$ , and  $\mathbb{R}_{\text{an,exp}}$ . Speaker: .

**Tuesday, May 14, 2019:** Peterzil–Starchenko's Chow o-minimal Theorem. Speaker: .

- Tuesday, May 21, 2019:** Introduction to arithmetic quotients. Speaker: .
- Tuesday, May 28, 2019:** Ideas from the new proof of the Cattani–Deligne–Kaplan Theorem, part I: definability of arithmetic quotients. Speaker: .
- Tuesday, June 4, 2019:** Ideas from the new proof of the Cattani–Deligne–Kaplan Theorem, part II: definability of period maps and conclusion. Speaker: .
- Tuesday, June 11, 2019:** Ideas from the proof of Griffiths’ Conjecture, part I: definable coherent sheaves and definable analytic spaces. Speaker: .
- Tuesday, June 18, 2019:** Ideas from the proof of Griffiths’ Conjecture, part II: conclusion of the proof. Speaker: .

## REFERENCES

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- [CDK95] Cattani, E., Deligne, P., Kaplan, A., *On the locus of Hodge classes*, J. Amer. Math. Soc. **8** (1995), 483–506.
- [Del71] Deligne, P., *Théorie de Hodge. II.*, Inst. Hautes Études Sci. Publ. Math. **40** (1971), 5–57.
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- [KUY17] Klingler, B., Ullmo, E., Yafaev, A., *bi-algebraic geometry and the André–Oort conjecture*, to appear in Proceedings of 2015 AMS Summer Institute in Algebraic Geometry.
- [PS08] Peters, C., Steenbrink, J., *Mixed Hodge structures*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics **52**, Springer-Verlag, Berlin, 2008.
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