Feedback Stabilizability of Delay Systems in Banach Spaces

Part I: Reformulation of State-Input Delay Systems

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The state-input delay system

$$(SIDS) \begin{cases} \dot{x}(t) = Ax(t) + \int\limits_{-r}^{0} x(t+\theta)\,\mathrm{d}\mu(\theta) + \int\limits_{-r}^{0} u(t+\theta)\,\mathrm{d}\nu(\theta)\,, \quad t \geq 0 \\ x(0) = z \in X \\ \quad x_0 = \varphi \in L^p([-r,0],X) \\ \quad u_0 = \psi \in L^p([-r,0],U) \end{cases}$$

Here

- X, U Banach spaces, $r > 0$, $p \in (1,\infty)$, $u \in L^p_{loc}([-r,\infty),U)$.
- A generates a C_0-semigroup $(T(t))_{t\geq 0}$ on X.
- $\mu : [-r,0] \to \mathcal{L}(X)$, $\nu : [-r,0] \to \mathcal{L}(U,X)$ of bounded variation and no mass at 0.
- $x_t \in L^p([-r,0],X)$, $x_t = x(t+\cdot)$ the history function of x at time t. u_t defined analogously.

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Reformulation of the state-delay equation

Consider

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(SDE) \begin{cases} \dot{x}(t) &= Ax(t) + Lx_t\,, \quad t \geq 0 \\ x(0) &= z \in X \\ x_0 &= \varphi \in L^p([-r,0],X) \end{cases}
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Goal

Reformulate (SDE) as an abstract Cauchy problem.

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Reformulate (SDE) as an abstract Cauchy problem.

Idea

- Study $w(t) := \begin{pmatrix} x(t) \\ x_t \end{pmatrix}$ on $\mathcal{X} := X \times L^p([-r,0],X)$.
- Which differential equation satisfies w?

Let

- $x : [-r, \infty) \to X$ a classical solution to (SDE).
- $t \geq 0$ fixed, $\widetilde{x} \in W^{1,p}(\mathbb{R}, X)$ an extension of $x_{|[-r,t+1]}$.

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For $|h|$ small enough

$$\left\| \frac{x_{t+h} - x_t}{h} - \frac{\partial}{\partial\theta} x_t \right\|_{p,[-r,0]} = \left\| \frac{x(t+h+\cdot) - x(t+\cdot)}{h} - \frac{\partial}{\partial\theta} x(t+\cdot) \right\|_{p,[-r,0]}$$

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$$\leq \left\| \frac{\widetilde{x}(t+h+\cdot) - \widetilde{x}(t+\cdot)}{h} - \frac{\partial}{\partial \theta} \widetilde{x}(t+\cdot) \right\|_{p,\mathbb{R}}$$

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For $|h|$ small enough

$$\begin{aligned}\left\| \frac{x_{t+h} - x_t}{h} - \frac{\partial}{\partial\theta} x_t \right\|_{p,[-r,0]} &= \left\| \frac{\widetilde{x}(t+h+\cdot) - \widetilde{x}(t+\cdot)}{h} - \frac{\partial}{\partial\theta}\widetilde{x}(t+\cdot) \right\|_{p,[-r,0]} \\ &\leq \left\| \frac{\widetilde{x}(t+h+\cdot) - \widetilde{x}(t+\cdot)}{h} - \frac{\partial}{\partial\theta}\widetilde{x}(t+\cdot) \right\|_{p,\mathbb{R}} \xrightarrow{h\to 0} 0\end{aligned}$$

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We conclude for $t \geq 0$

$$\dot{w}(t) = \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x(t) \\ x_t \end{pmatrix} = \begin{pmatrix} A & L \\ 0 & \frac{\partial}{\partial\theta} \end{pmatrix} w(t)\,, \quad w(0) = \begin{pmatrix} z \\ \varphi \end{pmatrix}$$
$$w(t) \in \left\{ \begin{pmatrix} z \\ \varphi \end{pmatrix} \in D(A) \times W^{1,p}([-r,0], X) \mid \varphi(0) = z \right\} \subset \mathcal{X}$$

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$$\dot{w}(t) = \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x(t) \\ x_t \end{pmatrix} = \overbrace{\begin{pmatrix} A & L \\ 0 & \frac{\partial}{\partial \theta} \end{pmatrix}}^{:= \mathcal{A}_L} w(t), \quad w(0) = \begin{pmatrix} z \\ \varphi \end{pmatrix}$$
$$w(t) \in \underbrace{\left\{ \begin{pmatrix} z \\ \varphi \end{pmatrix} \in D(A) \times W^{1,p}([-r,0], X) \mid \varphi(0) = z \right\}}_{:= D(\mathcal{A}_L)} \subset \mathcal{X}$$

(SDE) as an abstract Cauchy problem on $\mathcal{X}$

Theorem

Let

$$\mathcal{A}_L := \begin{pmatrix} A & L \\ 0 & \frac{\partial}{\partial\theta} \end{pmatrix}, \quad D(\mathcal{A}_L) := \left\{ \begin{pmatrix} z \\ \varphi \end{pmatrix} \in D(A) \times W^{1,p}([-r,0],X) \mid \varphi(0) = z \right\}.$$

$\mathcal{A}_L$ generates a C_0-semigroup $(\mathcal{T}_L(t))_{t\geq 0}$ on $\mathcal{X}$.

The classical solution to the abstract Cauchy problem

$$(ACP) \begin{cases} \dot{w}(t) = \mathcal{A}_L w(t), & t \geq 0 \\ w(0) = \begin{pmatrix} z \\ \varphi \end{pmatrix} \in D(A) \end{cases}$$

is given by $w(t) = \begin{pmatrix} x(t) \\ x_t \end{pmatrix}$, where x is a classical solution to (SDE).

Properties of the delay semigroup $(\mathcal{T}_L(t))_{t\geq 0}$

Remark

For $\lambda \in \mathbb{C}$ define

$$\mathrm{e}_\lambda : X \to L^p([-r,0],X)\,, \quad (\mathrm{e}_\lambda x)(\theta) = \mathrm{e}^{\lambda\theta} x.$$

The spectrum of $\mathcal{A}_L$ can be characterized by

$$\lambda \in \sigma(\mathcal{A}_L) \Longleftrightarrow \lambda \in \sigma(A + L\mathrm{e}_\lambda).$$

The set $\sigma^+ := \sigma(\mathcal{A}_L) \cap \mathbb{C}_+$ is called unstable set of $\mathcal{A}_L$.

Theorem

If $(T(t))_{t\geq 0}$ is immediately compact, $(\mathcal{T}_L(t))_{t\geq 0}$ is compact for $t > r$.

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Reformulation of the state-input delay system

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Now

- $f(t) := \begin{pmatrix} Bu_t \\ 0 \end{pmatrix}$ well-defined only for $u \in W^{1,p}_{loc}([-r,\infty),U)$.

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- In this case $f \in C([0,\infty),\mathcal{X})$ and the mild solution to (SIDS) is

$$w(t) = \mathcal{T}_L(t)\begin{pmatrix} z \\ \varphi \end{pmatrix} + \int_0^t \mathcal{T}_L(t-\tau)f(\tau)\,\mathrm{d}\tau.$$

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- How to extend this definition to $u \in L^p_{loc}([-r,\infty),U)$?

Extension of B to general inputs

Let

- $(S(t))_{t\geq 0}$ the nilpotent left-shift semigroup on $L^p([-r,0],U)$ with generator Q.
- $\Phi_t \in \mathcal{L}(L^p([-r,0],U))\,, \quad (\Phi_t u)(\theta) = \begin{cases} u(t+\theta) & \theta \geq -t \\ 0 & \theta < -t \end{cases}.$

Proposition

(S,Φ) is control system on $L^p([-r,0],U)$, U represented by $\mathbb{B}_U := (\lambda - Q_{-1})\mathrm{e}_\lambda$.

Furthermore $B \in \mathcal{O}^p_U(S)$ and $(Q,\mathbb{B}_U,B)$ generates RLS on $L^p([-r,0],U),U,X$.

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Mild solution for $u \in L^p_{loc}([-r,\infty),U)$ is

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(SIDS) as a delay-free open loop system

Theorem (Hadd, Idrissi 2005)

The system (SIDS) can be reformulated as an open loop system

$$(OLS) \quad \dot{\xi}(t) = \mathcal{A}_{L,B}\,\xi(t) + \mathcal{B}u(t)\,, \quad \xi(0) = \begin{pmatrix} z & \varphi & \psi \end{pmatrix}^{\mathrm{T}}$$

with state space $\mathcal{Z} := \mathcal{X} \times L^p([-r, 0], U)$ *and control space* U*.*

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For $u \in L^p_{loc}([-r,\infty),U)$ *the state trajectory of (OLS) is*

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Moreover

$$R(\lambda,(\mathcal{A}_{L,B})_{-1})\mathcal{B}v = \begin{pmatrix} R(\lambda,\mathcal{A}_L)\begin{pmatrix} B\mathrm{e}_\lambda v \\ 0 \end{pmatrix} \\ \mathrm{e}_\lambda v \end{pmatrix}, \quad v \in U.$$

Feedback Stabilizability of (SIDS)

Stabilize (OLS) via feedback

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$$(OLS)\begin{cases} \dot{\xi}(t) &= \mathcal{A}_{L,B}\xi(t) + \mathcal{B}u(t)\,, \quad t \geq 0 \\ \xi(0) &= \begin{pmatrix} z & \varphi & \psi \end{pmatrix}^{\mathrm{T}} \\ y(t) &= \mathcal{C}\xi(t) \end{cases}$$

Definition

(SIDS) is called feedback stabilizable if $\mathcal{C} \in \mathcal{O}_U^p(\mathcal{A}_{L,B})$ exists, s.t.

1. $(\mathcal{A}_{L,B}, \mathcal{B}, \mathcal{C})$ generates a RLS $\Sigma = (\mathcal{T}_{L,B}, \Phi_{L,B}, \Psi, \mathbb{F})$ on $\mathcal{Z}, U, U$
2. $\mathrm{Id} - \mathbb{F}_\infty(t)$ has in $t \geq 0$ uniformly bounded inverse.
3. The semigroup generated by $(\mathcal{A}_{L,B})_{-1} + \mathcal{B}\mathcal{C}_\Lambda$ with domain

$$D((\mathcal{A}_{L,B})_{-1} + \mathcal{B}\mathcal{C}_\Lambda) = \{\zeta \in \mathcal{Z} \mid ((\mathcal{A}_{L,B})_{-1} + \mathcal{B}\mathcal{C}_\Lambda)\zeta \in \mathcal{Z}\}$$

   is exponentially stable.

In this case we say that $\mathcal{C}$ stabilizes (SIDS).