.

The small denominators associated with Celestial Mechanics and, more generally, with Hamiltonian systems, are of a quite distinct nature, and a cause for much confusion.

The so-called Lindstedt-Poincare series – formally quasi-periodic – that describe the motion in the Many Body problem are, depending on the initial conditions, sometimes convergent, sometimes divergent. In either case, there is nothing to 'compensate'. The confusion arises from the fact that even in the convergent case, the coefficients c_ω of the L-P series that emerge from the calculations involve finite sums of the form:

$$c_{\omega,r} = \sum_{1\leqslant k\leqslant K} \frac{a_{\omega,k}}{\omega_{k,1}\,\omega_{k,2}\dots\omega_{k,r}} \quad \textit{with} \quad \begin{cases} r=r(\omega)\,,\ K=K(\omega) \\ \lambda_1,\dots,\lambda_\nu \ \textit{fixed} \\ \omega_{k,i}=\langle \boldsymbol{n^i},\boldsymbol{\lambda}\rangle \quad \textit{with} \quad \boldsymbol{n^i}\in\mathbb{Z}^\nu \end{cases}$$

which may carry prohibitively small denominators, and therefore call for some 'compensation' mechanism. However, the finite expansion (**??**) for $c_{\omega,r}$ is by no means unique, and if we conduct the calculations (inductively on r) with deftness, by observing a neat set of rules, we can keep these abnormally small denominators at bay.

More interesting is the case of Hamiltonian vector fields that, on top of the intrinsic resonance $\lambda_i+\lambda_{i+\nu}=0$, exhibit some extrinsic resonance, say $\lambda_1=0$ or $\sum n_i\lambda_i=0$. Attached to this extrinsic resonance, we have *resurgence*, carried by a critical variable z and described as usual by the Bridge equation, but with two eye-catching peculiarities:

- Each differential operator $\mathbf{A}_\omega$ now derives from a potential $\mathcal{A}_\omega$ – an 'alien potential', so to speak.

- The shortest cut for calculating the potentials $\mathcal{A}_\omega$ involves writing the original Hamiltonian H as $H_2+\mathcal{H}$, with a quadradic part H_2 and a 'perturbation' $\mathcal{H}$, and subjecting $\mathcal{H}$ to a remarkable involution $\mathcal{H}\mapsto\mathcal{K}$:

  $$\mathcal{K} = -\mathcal{H} - \frac{1}{2!}\{z,\mathcal{H}\}_P - \frac{1}{3!}\{z,\{z,\mathcal{H}\}_P\}_P - \frac{1}{4!}\{z,\{z,\{z,\mathcal{H}\}_P\}_P\}_P\dots$$

  where $\{.,.\}_P$ denotes the Poisson bracket, expressed in any map that isolates the critical variable z.