# The systematic quest for exotic derivations.

Interesting *exotic derivations* appear to fall into three broad classes, of which the first two are well understood, the third one hardly at all. They are:

- The *alien derivations* $\Delta_\omega$: it is their pull-backs $\widehat{\Delta}_\omega$ in the $\zeta$-plane, or Borel plane, that make direct sense. These $\widehat{\Delta}_\omega$ are derivations relative to the convolution product $*$ and can tackle *any type* of isolated singularity over the points $\omega \in \mathbb{C}$.

- The *foreign derivations* $\nabla_\omega$: they are defined directly in the multiplicative $z$-plane; act as derivations relative to ordinary point-wise multiplication; but can tackle only mild singularities of type $exp\big(o(logz)\big)$ over $\omega \in \mathbb{C}$.

- The *arithmetical derivations* $\square_\tau$ : these are derivations that act trivially on the ring $\mathbb{A}$ of algebraic numbers, and non-trivially on some larger ring $\mathbb{B}$, with $\mathbb{A} \subset \mathbb{B} \subset \mathbb{C}$.

  A *useless instance* would be a ring $\mathbb{B} = \mathbb{A} \otimes \mathbb{Q}[\cup_\tau x_\tau]$ generated by a set of complex numbers $x_\tau$ *known* to be transcendental and algebraically independent, with derivations $\square_\tau$ acting as follows:

  – $\square_\tau(x\,y) \equiv (\square_\tau x)\,y + x\,(\square_\tau y)$
  – $\square_\tau\ \mathbb{A} = \{0\}$
  – $\square_{\tau_1} x_{\tau_2} = \delta_{\tau_1,\tau_2}$ (= Kronecker symbol)

  A *useful instance* would be the exact reverse: it would be a ring $\mathbb{B}$ consisting of numbers whose arithmetical nature is *a priori* unknown, plus a system $\boxed{\square}$ of derivations $\square_\tau$ whose action is defined *directly*, based on some universal representation of the numbers in $\mathbb{B}$ (say, some generalisation of continued fractions) so that the arithmetical nature of the elements of $\mathbb{B}$ (transcendence + algebraic dependence or independence) could be inferred from the action of $\boxed{\square}$ on $\mathbb{B}$. The theory is still in its infancy – it is actually more of a dream than even an infant theory – but the multizetas, with the perinomal representation of their irreducibles, might offer a promising start.