Elements of complex analysis and geometry

M2 AAG, September 2016 – H. Auvray

Programme:

I - *Holomorphic functions of several variables and complex manifolds*

1) Definition (continuity and separate holomorphy), Cauchy formula and straightforward consequences (analogous to complex dimension 1: power series expansions, analytic extension, local inversion).
2) Complex manifolds
   a) definition *via* an atlas (remark: alternative/sheaf-theoretic defintion); tangent and cotangent spaces, (bi)degree, complex structure.
   b) Integral and differential calculus on (complex) manifolds: real case: currents (in particular, formula $F^*[Z] = [F^{-1}(Z)]$, F submersion, Z oriented submanifold); complex case: ∂ and $\overline{\partial}$ operators; (remark: differential forms, integration, Poincaré lemma, de Rham cohomology: done the week before by F. Paulin).

II - *Local study of holomorphic functions: extensions*

1) Hartogs' extension theorem
   a) Statement of the theorem, proof modulo resolution of $\overline{\partial}$ equation with compact support in dimension ≥ 2.
   b) Kernels (Newton, Bochner-Martinelli), Koppelman formula and resolution of $\overline{\partial}$ equation with compact support.
   c) Dolbeault-Grothendieck lemma.
2) Hartogs' phenomenon
   a) Hartogs' figure; extension *via* Cauchy formula.
   b) Riemann extension theorem.

III - *Domains of holomorphy I*

1) Definition; examples (convex open sets – exercise/digression on unit ball and polydisc as a counterexample of the Conformal Mapping in dimension ≥ 2).
2) Holomorphic hull and holomorphic convexity, basic properties; first characterisation of domains of holomorphy.

IV - *Domains of holomorphy II and psh functions*

1) Definition of exhaustion functions. Second characterisation of domains of holomorphy – and of Stein manifolds – *via* the existence of smooth plurisubharmonic exhaustion functions (*i.e.* functions with positive complex Hessian).
2) Harmonic functions. Green kernel and Dirichlet problem.
3) (Pluri-)Subharmonic functions.
4) Proofs of the statements "a domain of holomorphy is (weakly) pseudo convex", and "a Stein manifold is strongly pseudoconvex"; Levi problem.

There will be no typewritten notes; most of these contents may however be found in J.-P. Demailly's book, first chapter. Prerequisites are: Complex analysis (in one complex variable), Basic differential geometry, Distribution theory (see e.g. Th. Ramond's notes, chapters 1 to 4).