

**Workshop « Stability of bundles Vs varieties »**  
**Brest 24th-26th June 2019**  
**ANR Emarks**

Schedule

	Monday 24th	Tuesday 25th	Wednesday 26th
9-10	Welcome	Delcroix (I)	Delcroix (II)
10-10:30	Coffee	Coffee	Coffee
10:30-11:30	Dervan (I)	Dervan (II)	Rubio (II)
12-14	Lunch	Lunch	Lunch
14-15	Garcia-Fernandez (I)	Garcia-Fernandez (II)	Rollin
15-15:30	Break	Break	Break
15:30-16:30	Legendre/Sena Dias	Rubio (I)	Discussion
16:30-18	Discussion	Discussion	Good-bye

Participants :

- Hugues Auvray (Université Paris Sud, Orsay),
- Michèle Benyounes (LMBA, Brest)
- Thibaut Delcroix (IRMA, Strasbourg),
- Ruadhair Dervan (University of Cambridge),
- Gerd Dethloff (LMBA, Brest),
- Mario Garcia-Fernandez (ICMAT, Madrid),
- Eveline Legendre (IMT, Toulouse),
- Yann Rollin (LMJL, Nantes),
- Roberto Rubio (UAB, Barcelone),
- Rosa Sena-Dias (IST, Lisbonne),
- Carl Tipler (LMBA, Brest)

Organisers : Eveline Legendre, Carl Tipler (carl.tipler@univ-brest.fr)

Lunch : Lunch will take place at « RU Armen »

Social Diner : there will be a social dinner on tuesday night, 19:30, at « Le Ruffé », 1bis Rue Yves Collet, 29200 Brest.

## Titles and abstracts :

### **Delcroix**

Title : Canonical metrics on Fano horosymmetric manifolds

Abstract : Unlike Calabi-Yau manifolds, or canonically polarized manifolds, Fano manifolds often do not admit Kähler-Einstein metrics. Several possible replacement for Kähler-Einstein metrics have been introduced over the years. In this talk I will present joint work with J. Hultgren where we characterized combinatorially the existence of several such metrics on horosymmetric manifolds. This provides in particular various examples where some canonical metrics exist while some others don't. ArXiv reference: <https://arxiv.org/abs/1812.07218>

### **Dervan**

Title: Canonical metrics on fibrations

Abstract: A fundamental link between the theory of Hermite-Einstein metrics and extremal Kähler metrics is through the study of extremal metrics on projective bundles, as initiated by Hong. One can more generally ask when an arbitrary fibred complex manifold admits an extremal metric. I will begin by describing Hong's work, and will then proceed to the state of the art on this question. Time permitting, I will also discuss the role K-stability plays in this story. This is mostly joint work with Lars Sektnan.

### **Garcia-Fernandez**

Title: Coupled equations on bundles and varieties

Abstract: In these talks I will overview the theory of the Kähler-Yang-Mills equations, coupling a hermitian metric on a holomorphic vector bundle over a compact complex manifold with a Kähler structure on the base. In the first talk I will provide a moment map interpretation of the equations and study obstructions for the existence of solutions, generalizing the Futaki invariant, the Mabuchi K-energy and geodesic stability. In the second talk I will focus on a toy example arising from dimensional reduction on  $P^1 \times P^1$ , motivated by the physics of cosmic strings. These talks are based on joint work with Luis Álvarez-Cónsul and Oscar Garcia-Prada (Geom. & Top., 2013, and Comm. Math. Phys., 2017), Julius Ross (PLMS 2013) and Carl Tipler (JLMS, 2014).

## **Legendre**

Title : TBA

## **Rollin :**

Title : Discrete geometry and isotropic surfaces

Abstract : We consider smooth isotropic immersions from the 2-dimensional torus into  $\mathbb{R}^{2n}$ , for  $n \geq 2$ . When  $n=2$  the image of such map is an immersed Lagrangian torus of  $\mathbb{R}^4$ .

We prove that such isotropic immersions can be approximated by arbitrarily  $C^0$ -close piecewise linear isotropic maps. If  $n \geq 3$  the piecewise linear isotropic maps can be chosen so that they are piecewise linear isotropic immersions as well. The proofs are obtained using analogies with an infinite dimensional moment map geometry due to Donaldson. As a byproduct of these considerations, we introduce a numerical flow in finite dimension, whose limit provide, from an experimental perspective, many examples of piecewise linear Lagrangian tori in  $\mathbb{R}^4$ .

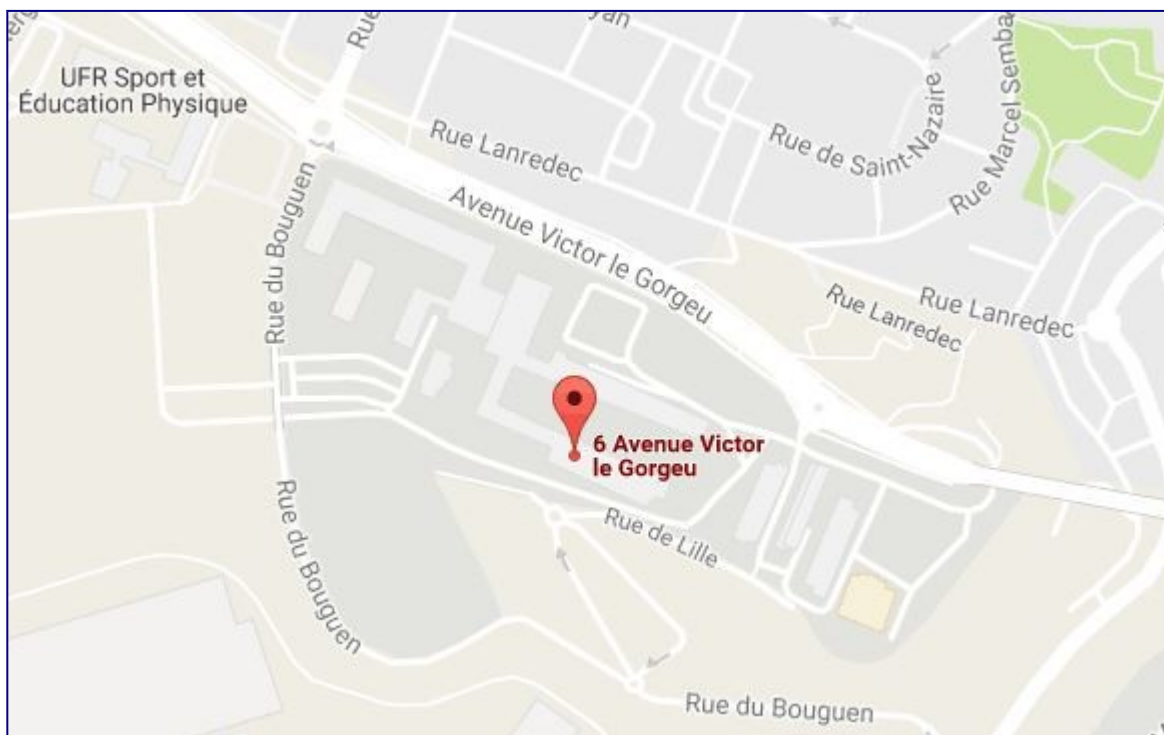
## **Rubio**

Title : HIGGS BUNDLES, THE TOLEDO INVARIANT, AND THE CAYLEY CORRESPONDENCE

Abstract : For  $X$  a compact Riemann surface of genus  $g \geq 2$ , unitary representations of its fundamental group,  $\rho: \pi_1 X \rightarrow U(n)$ , correspond, up to equivalence, to the moduli space of polystable holomorphic bundles over  $X$  of rank  $n$  and degree zero. When the representation is into an arbitrary real Lie group  $G$  and moreover reductive,  $\rho: \pi_1 X \rightarrow G$ , then the holomorphic counterpart is the moduli space of polystable  $G$ -Higgs bundles. To surface group representations into real Lie groups of Hermitian type (whose corresponding symmetric space  $G/H$  is Hermitian) is associated, by means of a normalized Kähler form on  $G/H$ , the Toledo invariant. This numerical invariant is bounded, by the Milnor-Wood inequality, constant on connected components of the moduli space, and singles out the maximal representations, of special geometrical significance. The purpose of these two lectures is to introduce the Toledo invariant for Higgs bundles, by using the Jordan triple structure of the isotropy representation, and show how this approach helps us describe the rigidity of maximal Higgs bundles. This will be based on the work of my PhD thesis, which became a joint article with O. Biquard and O. Garcia-Prada (J. Topol. 2017). I will start by motivating the definition of a polystable  $G$ -Higgs bundle, and recalling what a group of Hermitian type is and how the Toledo invariant was defined for surface group representations. I will then introduce the Toledo invariant for Higgs bundles, using  $SU(n,n)$  as an example and highlighting the role of the underlying Jordan algebra structure. This structure, together with polystability, will allow us to sketch the proof of a general version of the Milnor-Wood inequality for Higgs bundles. Finally, I will look at the moduli space of maximal polystable  $G$ -Higgs bundles for  $G$  of tube type, where a phenomenon of rigidity is described by the Cayley correspondence. Namely, by choosing a  $|Z(G)|$ -root of the canonical bundle  $K$ , the moduli space of polystable maximal  $G$ -Higgs bundles is isomorphic to the moduli space of polystable  $K^2$ -twisted  $H^*$ -Higgs bundles, where  $H^*$  is a non-compact dual of the maximal compact subgroup  $H$  of  $G$ .

### Travelling to Brest :

The workshop will take place at UBO, 6 avenue Victor Le Gorgeu, Brest, and the talks will be in room H 118, H building. Please contact Carl Tipler for any information.



### **Par avion :**

Grâce à l'aéroport international Brest-Bretagne, il est facile de se rendre à l'UBO : 12 lignes régulières, 20 lignes saisonnières, et 12 liaisons quotidiennes avec Paris !

Pour tous renseignements : <http://ww.brest.aeroport.fr>

### **Par le train :**

Pour vous rendre à Brest, il existe de nombreuses liaisons quotidiennes en TGV avec Paris.

Pour tous renseignements : <http://www.voyages-sncf.com/>

### **Réseau Bibus :**

Depuis la gare prendre le bus n° 1 ou 2 en direction "Provence" ou "Hopital Cavale" et descendre à l'arrêt "Universités".

Pour tous renseignements : <http://www.bibus.fr>