Département de Mathématiques d'Orsay

Variational approach to the regularity of the singular free boundaries

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Résumé :In this talk we will present some recent results on the structure of the free boundaries of the (local) minimizers of the Bernoulli problem in R^d,

$$(*) \qquad \min\Big\{\int_{B_1}\big(|\nabla u|^2+1_{\{u>0\}}\big)\Big\}_{:u\in H^1(B_1)+\,Dirichlet\ boundary\ conditions\ on\ \partial B_1}.$$

In 1981 Alt and Caffarelli proved that if u is a minimizer of the above problem, then the free boundary $\partial\{u>0\}\cap B_1$ can be decomposed into a regular part, $Reg\big(\partial\{u>0\}\big)$, and a singular part, $Sing\big(\partial\{u>0\}\big)$, where

— $Reg\big(\{u>0\}\big)$ is locally the graph of a smooth function ;

— $Sing\big(\{u>0\}\big)$ is a small (possibly empty) set.

Recently, De Silva and Jerison proved that starting from dimension $d=7$ there are minimal cones with isolated singularities in zero. In particular, the set of singular points $Sing\big(\{u>0\}\big)$ might not be empty.

The aim of this talk is to describe the structure of the free boundary around a singular point. In particular, we will show that if u is a solution of (*), x_0 is a point of the free boundary $\partial\{u>0\}$ and there exists one blow-up limit $u_0=\lim_{n\to\infty}\frac{u(x_0+r_nx)}{r_n}$, which has an isolated singularity in zero, then the free boundary $\partial\{u>0\}$ is a C^1 graph over the cone $\partial\{u_0>0\}$.

Our approach is based on the so called logarithmic epiperimetric inequality, which is a purely variational tool for the study of free boundaries and was introduced in the framework of the obstacle problem in a series of works in collaboration with Maria Colombo and Luca Spolaor.

IMO ; salle 3L8.

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