Anisotropic distributions and compactness by compensation

Mardi 13 juin 2017 14 :00-15 :00

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Résumé : H-measures, also called microlocal defect measures, have been introduced independently by Luc Tartar and Patrick Gérard. They are matrix Radon measures defined on the co-spherical bundle describing the behaviour of weak limits of quadratic quantities of L^2 sequences. They, together with a variant of compactness by compensation theory with variable continuous coefficients, have been successfully applied in many problems involving asymptotic limits of quadratic quantities. However, they turned insufficient for nonlinear problems with solutions in L^p spaces. H-distributions were introduced by Antonić and Mitrović as an extension of H-measures to the $L^p - L^q$ setting. Their variants have been successfully applied to problems in velocity averaging and compactness by compensation with variable coefficients.
The two basic steps used in the construction of H-measures and H-distributions are the First commutation lemma and the Schwartz kernel theorem. The First commutation lemma allowed Tartar to consider bilinear functionals instead of trilinear, while the Schwartz kernel theorem (or its variant) identified the bilinear functional with an element of an appropriate dual space of functions.
In the first part of the talk we will shortly discuss the extension of the First commutation lemma to the L^p setting. For that, we need a variant of the Krasnosel'skij type result for the interpolation of operators (on unbounded domains). The significance is in the fact that the regularity of symbol of the Fourier multiplier operator happens to be the same as required by the Hörmander-Mihlin theorem. This is an improvement over the existing result of Heinz Otto Cordes for the L^p case.
Unlike the H-measures, which are nonnegative Radon measures, H-distributions are distributions in the Schwartz sense, which follows from the standard Schwartz kernel theorem. To give a precise description of H-distributions, we will introduce the notion of anisotropic distributions — the distributions of different order with respect to different coordinate directions. In order to show that H-distributions are anisotropic distributions of finite order with respect to every coordinate direction, we will prove a variant of the Schwartz kernel theorem.
In the last part of the talk, we will show a variant of compensated compactness using a variant of H-distributions. Namely, we will investigate conditions under which, for two sequences (u_r) and (v_r) weakly converging to u and v in $L^p(R^d; R^N)$ and $L^q(R^d; R^N)$, respectively, $1/p + 1/q \leq 1$, a quadratic form $q(x; u_r; v_r) = \sum_{j,m=1}^{N} q_{jm}(x) u_{jr} v_{mr}$ converges toward $q(x; u; v)$ in the sense of distributions. The conditions involve fractional derivatives and variable coefficients, and they represent a generalization of the known compensated compactness theory. We will apply the developed techniques to a nonlinear (degenerate) parabolic equation.
This talk will present results of joint works with Nenad Antonic, Marko Erceg and Darko Mitrovic.
References

— N. Antonic, M. Erceg, M. Misur, On H-distributions, in progress 25 pp.
— N. Antonic, M. Misur, D. Mitrovic, On the First commutation lemma, submitted 17 pp.
— M. Misur, D. Mitrovic, On a Generalization of Compensated Compactness in the $L^p - L^q$ setting, Journal of Functional Analysis, 268, (2015) 1904–1927.

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