Internship Report
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Competition between Electricity Providers and Consumers: a Bilevel Model

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ABSTRACT: Designing future electricity provider offers is a key component in the smart grid to effectively reduce power generation costs and user bills, and concretely describe – with fares or prices - the interaction between different actors of the electrical system - producers, network operators, providers - and consumers. A Stackelberg game between providers and consumers is considered, in its standard bilevel form. At the upper level, providers maximize their profit (possibly integrating a cost due to the sourcing part on the electricity markets), which is directly based on the providers choices of the consumers. At the lower level, consumers compare their bill with the different providers; they choose the provider with the smallest associated bill. Three different consumer classes are considered, namely non-flexible, flexible, and flexible with renewable generation and a storage capacity. Decisions of each kind of consumers are constructed by solving sub-problems like scheduling the flexible electrical consumptions, predicting renewable generation and optimizing real-time storage management decisions. We study this model theoretically and numerically.

Theoretically, an emerging lower level Nash equilibrium between consumers is characterized, when their decision interact, e.g. when the electricity fare paid by the consumers depend on the total load at the scale of the consumer pool ("endogenous price"). This allows deducing a few properties of the optimal decisions of the providers at the upper level. In particular, a supermodular game is introduced to represent the imperfect rationality of the consumers. Numerically, considering French household consumption data, a best-response like dynamics is implemented to represent the process of yearly decisions update. Choosing the base load as the non-flexible part and water-heating as the flexible one, switching decisions (from one class to another) are analyzed: if the electrical heating cost is significant, a consumer can choose gas instead (and thus becomes a non-flexible consumer, with a lower consumption); if the total energy-dependent bill is subsequent, a more competitive photovoltaic panel can be installed (such a consumer becomes a "prosumer", reducing the volume sold by the provider). In particular, when the simulated dynamics converges, this gives some insights on the structure of the provider – consumer equilibrium in the proposed setting and how this configuration differs from the current situation.

KEYWORDS: Bilevel optimization, Stackelberg multiple-leader game, water-filling, best response dynamics.

Contents

1 Introduction

A new context is emerging in electricity systems. With the integration of local production units (in particular renewable energies), the introduction of new local actors ("smart" consumers, building, cities, who want to locally control their electricity consumption profile, self-consume a local production, etc.), a collection of new problems is being gathered in a topic that can be named "decentralized management of electricity systems". Indeed, a transition is being operated from a centralized vision where production, transport, distribution and supply of electricity were operated by a unique unified actor (EDF, in France) to a new vision where many local decision-makers take a place in this system.

In this huge field of research, the topic of electricity supply is crucial. Even if this activity will still be necessary in the decentralized context to provide electricity to (small) end-consumers who could not alone source their electricity from electricity markets, it may be strongly modified. With the opening up of the competition in this sector, it may change the structure of the offers proposed to the consumers to attract them by proposing them some more adaptive and dynamic offers. In turn, a new dynamic could be expected in this sector, as observed recently in the UK. Studying the relation between providers and consumers, who react to signals (prices) could also constitute a first step to integrate demand-side-management as a tool to mitigate the impact of consumption on the impacts generated on the electricity system (production costs, power losses, network asset aging, etc.). In turn, demand-side-management is a very active topic of research in this field and could constitute a key block of decentralized electricity systems. By theoretically studying interaction mechanisms between electricity system actors (represented here by providers) and end-consumers, this work could also give some new insights on how to make the most of consumer flexibility.

Notations:

- bold text is used for vectors. For two vectors $\boldsymbol{\ell}$, $\boldsymbol{\ell}'$ indexed by t, $\boldsymbol{\ell}' \leq \boldsymbol{\ell}$ represents component-wise comparison, i.e. $\forall t, \ell'_t \leq \ell_t$;
- $\widetilde{w}$ is used for a quantity which is not deterministic in the model, and for which a stochastic approach will be needed;
- $[x]^+ = \max(x, 0)$ denotes positive part of x.

2 Model and problem formulation

Denote $\mathcal{C} = \{1, \cdots, C\}$ of consumers and a discrete set $\mathcal{F} = \{1, \cdots, F\}$ of providers are considered. Denote respectively $\mathcal{J} = \{1, \cdots, J\}$ the set of days for the period, which consists of the set of J days. Each day is itself divided into T time slots in the set $\mathcal{T} = \{1, \cdots, T\}$. At the beginning of the period, provider $f \in \mathcal{F}$ proposes a price $p_f(j,t)$ for each of the $J \times T$ time-slots. The sequence of prices $(p_f(j,t))_{j\in\mathcal{J},t\in\mathcal{T}}$ is referred to as provider f offer.

2.1 Consumer model: minimizing his bill

Electricity consumer has a consumption profile, possibly with RE (renewable energy, e.g. solar power, wind, etc.) production unit and storage. To fully describe the consumer problem, we define the following general notations. We will then distinguish three different classes of consumers, depending on their particular characteristics.

For a consumer $c \in \mathcal{C}$, denote

- $\boldsymbol{\ell}_c^0 = (\ell_c^0(1,1), \cdots, \ell_c^0(J,T)) \in \mathbb{R}_+^{J\times T}$: non-flexible consumption profile, the necessary electricity need at the time. This part is fixed, and not controllable by consumers. It can be lighting, refrigeration, etc.;
- $\boldsymbol{\ell}_c^1 = (\ell_c^1(1,1), \cdots, \ell_c^1(J,T)) \in \mathbb{R}_+^{J\times T}$: flexible consumption profile, flexible in the sense that electricity need can be scheduled during the day. This kind of consumption can be heating, washing, electric vehicles charging, etc.;
- $\boldsymbol{\ell}_c = (\ell_c(1,1), \cdots, \ell_c(J,T)) \in \mathbb{R}_+^{J\times T}$: total consumption profile, which is the sum of non-flexible and flexible consumption:

$$\boldsymbol{\ell}_c = \boldsymbol{\ell}_c^0 + \boldsymbol{\ell}_c^1, \tag{2.1}$$

Typically, total consumption profile is subject to scheduling constraints (an electric vehicle can be charged only when plugged in at home, etc.)

$$\boldsymbol{\ell}_c^0 \leq \boldsymbol{\ell}_c, \tag{2.2}$$

to an upper bound constraint imposed by consumer maximal power of breaker $\overline{\boldsymbol{\ell}}$

$$\boldsymbol{\ell}_c(j,t) \leq \overline{\boldsymbol{\ell}}, \qquad \forall (j,t) \in \mathcal{J} \times \mathcal{T}, \tag{2.3}$$

and to a daily energy need

$$\sum_{t\in\mathcal{T}} \ell_c^1(j,t) = L_c^1(j), \qquad \forall j \in \mathcal{J}, \tag{2.4}$$

where $L_c^1(j)$ is consumer's flexible energy need for day j.

Remark 1. *Flexibility cannot change the total flexible energy consumed in each day, and neither postpone energy consumption from a day to another. This is a standard notion of flexibility in smart grid literature, see e.g. reference paper [12]. When allowing to decrease total energy consumed, a utility function based on total energy consumed is often introduced.*

- $\widetilde{\boldsymbol{w}}_c = (\widetilde{w}_c(1,1), \cdots, \widetilde{w}_c(J,T)) \in \mathbb{R}_+^{J\times T}$: RE production profile;
- $\boldsymbol{s}_c = (s_c(1,1), \cdots, s_c(J,T)) \in \mathbb{R}^{J\times T}$: charging profile of storage system available to consumer c. $s_c(j,t) > 0$ (resp. $s_c(j,t) < 0$) means that c charges (resp. discharges) his storage at time (j,t);
- $\boldsymbol{e}_c = (e_c(1,1), \cdots, e_c(J,T)) \in \mathbb{R}_+^{J\times T}$: the energy contained (referred to as energy level) in the storage at the end of each time slot. Energy level at time (j,t) is the sum of energy level at time $(j,t-1)$ and charging decision taken at time (j,t), which leads to the dynamics:

  $$e_c(j,t) = e_c(j,t-1) + s_c(j,t)^{[1]}, \tag{2.5}$$

  Capacity constraints of the storage write

  $$0 \leq e_c(j,t) \leq E_c, \qquad \forall (j,t) \in \mathcal{J} \times \mathcal{T}, \tag{2.6}$$

  where E_c is storage capacity.

With these notations, the total bill - for the whole period $\mathcal{J}$ - paid by consumer c if he chooses provider f can be expressed as

Consumer total bill:
$$B_{cf} = \sum_{(j,t)\in\mathcal{J}\times\mathcal{T}} p_f(j,t) \times \Big[\ell_c(j,t) - \tilde{w}_c(j,t) + s_c(j,t)\Big], \tag{2.7}$$

while bill for day j is obtained with

Consumer day j bill:
$$B_{cf}(j) = \sum_{t\in\mathcal{T}} p_f(j,t) \times \Big[\ell_c(j,t) - \tilde{w}_c(j,t) + s_c(j,t)\Big]. \tag{2.8}$$

Consumers choose a provider (i.e. an offer). This choice is based on their electricity bill, which itself depends on consumption profile. Mathematically, consumers' problem reads,

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[1]This dynamics works only when $t \leq T-1$, in practice, we often consider a constant energy level at the end of each day, e.g. we consider at the end of each day, the storage must contain half of its capacity's energy,

$$e_c(j,T) = \frac{E_c}{2}, \qquad \forall j \in \mathcal{J}$$

.

Problem 1 (Consumer)**.**

$$\min_{f\in\mathcal{F}} \sum_{j\in\mathcal{J}} \min_{\substack{\boldsymbol{\ell}^{\mathbf{1}}_{\boldsymbol{c}}(j,\cdot)\in\mathbb{R}^T_+\\ s_c(j,\cdot)\in\mathbb{R}^T_+}} \sum_{t\in\mathcal{T}} p_f(j,t)\times\left[\ell^0_c(j,t)+\ell^1_c(j,t)-\tilde{w}_c(j,t)+s_c(j,t)\right] \tag{2.9}$$

$$s.t. \begin{cases} 0\le \ell_c(j,t)\le \overline{\boldsymbol{\ell}}, & \forall (j,t)\in\mathcal{J}\times\mathcal{T},\\ \sum_{t\in\mathcal{T}} \ell^1_c(j,t)=L^1_c(j), & \forall j\in\mathcal{J},\\ 0\le e_c(j,t)\le E_c, & \forall (j,t)\in\mathcal{J}\times\mathcal{T}. \end{cases} \tag{2.10}$$

Problem 2.9 contains two subproblems. The first, represented by the second minimization, consists of consumption and storage scheduling. This problem is solved independently on a day by day basis[2]. Here, it is also assumed that it is allowed to sell energy back to the grid with same price $p_f(j,t)$, which occurs when consumption is negative

$$\ell^0_c(j,t)+\ell^1_c(j,t)-\tilde{w}_c(j,t)+s_c(j,t)<0. \tag{2.11}$$

In reality, the network may not allow this action or selling energy back to the grid will be done with a lower price. The second subproblem, represented by the first minimization, consists of provider choice. Consumer c chooses provider, who gives him the minimal total bill.

Remark 2. *As provider choice is done at the beginning of the period to come, solving scheduling problems for every J days requires having a representation of uncertain parameters involved in the model, e.g. non-flexible consumption forecast, flexible energy need, RE production forecast, etc. In the vast majority of this work, it is assumed that perfect forecast are available, so that problems are solved in a deterministic framework. As for practical application, increasing the duration of the period, i.e. J, will increase the prediction and forecasting errors involved. Studying robustness of the decisions taken in this model depending on the horizon considered could be interesting in the future. On this point, note that most studies in the literature are either day-ahead price [5, 15, 17] or real-time market, which makes the problem simpler and less subject to forecasting errors.*

With smart meters and local production systems being installed for residential consumers, new consumption behaviors are envisioned in consumer population. To integrate these innovative behaviors in the model, three consumer classes are distinguished: non-flexible (class 0), flexible (class 1) and flexible with RE and storage (class 2). These three classes are described based on aforementioned parameters:

- *non-flexible*: consumption profile is fixed and no RE production and storage systems are available, i.e. $L^1_c=0$, $\tilde{\boldsymbol{w}}_c=0$, $E_c=0$. As described a little further in Section

[2]Introducing the dependency between successive days will not changed the nature of problem solved, but make it computationally more difficult given that decisions are taken for a whole period instead of a unique day.

3.1.1, problem solved by a consumer of this class only consists in selecting the provider who minimizes his bill (scheduling subproblem in Problem (2.9) is trivial, with a unique feasible profile), which can be done by enumeration on the different providers and comparison of the corresponding bills B_{cf};

- *flexible*: a part of his consumption is flexible i.e. $L_c^1 > 0$, but no RE production and storage systems are available, i.e. $\widetilde{\boldsymbol{w}}_c = 0$, $E_c = 0$. A consumer of this class will therefore schedule his flexible part of consumption $\boldsymbol{\ell}_c^1(j,\cdot)$ (nontrivial second subproblem in Problem (2.9)) and select the provider giving him minimal bill. Section 3.1.2 explains a way of solving his problem;

- *flexible with RE and storage*: a consumer of this class not only has flexible consumption, i.e. $L_c^1 > 0$, but also an intermittent generation system, i.e. $\widetilde{\boldsymbol{w}}_c > 0$ and an associated storage system, i.e. $E_c > 0$. A consumer of this class must schedule flexible consumption $\boldsymbol{\ell}_c^1(j,\cdot)$ and charging / discharging trajectory $\boldsymbol{s}_c(j,\cdot)$ of the storage system, and select the provider who gives him a minimal bill. Section 3.1.3 provides a few elements about the resolution of his problem.

$\mathcal{C}^0$, $\mathcal{C}^1$ and $\mathcal{C}^2$ respectively denote the sets of non-flexible, flexible, and flexible with RE and storage consumers, with

$$\mathcal{C} = \mathcal{C}^1 \cup \mathcal{C}^2 \cup \mathcal{C}^3.$$

In the following, a unique representative consumer is considered for each class. In the theoretical analysis and simulations, all is done as if there was only one consumer in each class. This corresponds to a framework with a large number of consumers, where the mean behavior of a class can be approximated by a unique representative, like in nonatomic games in game theory. To obtain the aggregate behavior of the whole class, the decisions of this representative consumer are weighted by the (large) number of consumers in the class he represents.

Remark 3. *Considering consumer-dependent parameters - or utility functions - in a given class leads to more difficult game models [11]. Taking this into account could constitute an interesting direction for future research.*

2.2 Provider model: maximizing his profit

Turn now on the description of provider problem. Recall that price proposed by provider f at time (j,t) is $p_f(j,t)$. The precise structure of these prices is here presented. In accordance with [6, 12], this work allows for prices which increase with total load, referred to as endogenous prices. This type of prices are introduced with the following assumption.

Assumption 1 (Endogenous price). Providers prices are said to be endogenous, i.e. dependent on the decisions of the considered model through consumers choices, if

$$p_f(j,t) = g_f\left(\sum_{c\in\mathcal{C}} \ell_c(j,t)\right) \tag{2.12}$$

or

$$p_f(j,t) = g_f\left(\sum_{c\in\mathcal{C}_f} \ell_c(j,t)\right) \tag{2.13}$$

where g_f is a convex increasing function and $\mathcal{C}_f$ is the set of consumers who choose f as provider.

Dependency of provider f's prices can be based on the total consumption in the whole set of consumers, i.e. (2.12), or on the total load on his own pool $\mathcal{C}_f$, i.e. (2.13). Increasing and convex nature of this function is standard [12]: 1. from a practical point of view, it models the fact that sourcing costs are increasing with load to be satisfied, and very often in a convex manner because production units to be used (and consequently observed prices in electricity markets where providers source their energy) are called in the order of increasing marginal costs[3]; 2. from a theoretical standpoint, this assumption allows for strong theoretical results, in particular the solution of water-filling problems emerging for the consumers (mentioned a little further) can be easily characterized.

Remark 4. *Because the set of increasing and convex functions is not easy to be considered in optimization problems, parametrized families of functions are often used. Among them, quadratic family of the form*

$$g_f\left(\sum_{c\in\mathcal{C}} \ell_c(j,t)\right) = a_f \times \left(\sum_{c\in\mathcal{C}} \ell_c(j,t)\right)^2, \tag{2.14}$$

is often used in the literature, starting with the seminal paper [12].

More common to current consumers (rather no flexible!), a particular case will be distinguished in this work. It consists of the case where g_f is a constant function, meaning that prices do not depend on effective total consumption in consumers population.

Assumption 2 (Exogenous price). Providers prices are said to be exogenous, i.e. independent on consumer decisions described in the model, if function g_f used in (2.12) or (2.13) is constant.

Having defined the type of provider prices considered, the problem faced by these actors can now be described. As stated in the following problem, it consists of finding an optimal offer, which maximizes his benefit i.e. the summation of bills in his consumer pool.

Problem 2 (Provider)**.**

$$\max_{p_f} \sum_{c\in\mathcal{C}_f} B_{cf} \tag{2.15}$$

where B_{cf} is consumer c's bill, given by (2.7).

[3] Additionally, potential impacts on electricity systems are often increasing and convex with total load. When an electricity network operator wants to incentivize consumers in order to coordinate their consumption profiles decisions and reduce the observed impact on the network, such assumption is generally used in the mechanisms proposed in the literature, see e.g. [6, 12]

2.3 Provider-consumer model: interaction and competition

There has been a lot of works done in power systems on supply-demand balance [8], which describes the relationship between providers and consumers. In this field of research, Demand Response Management has thus become a key feature to define the interaction among multiple participants. This work is part of this field of research, consumers reacting to providers' signal (offers). Depending on the type of prices considered, this section describes the type of interaction at both upper - between providers - and lower - between consumers - levels.

2.3.1 Consumer interaction

Observe first that with the more common system of exogenous prices, there is no interaction between consumers decisions at the lower level. Indeed, prices for a given consumer c do not change if other consumers change their decisions (consumption scheduling and provider choice). The resolution of the lower level part of the model then reduces to solve independent optimization problems for the different consumers.

Now, when endogenous prices given by (2.12) or (2.13) are applied, the dependency of the price on consumers' total consumption makes all consumers' choices (both consumption scheduling and provider choice) interact. In particular, if more consumers schedule their load at the same time, it will result a higher price at this time and some of the consumers may want to change their consumption profile scheduling. As introduced in [6, 12], a natural framework to deal with the lower level interaction between consumers is game theory, by introducing a flexible consumption game. Here, this interaction has not been studied theoretically but observed in simulations, as it will be presented hereafter.

2.3.2 Provider interaction

Let us now consider $\mathcal{C}_f$, the set of consumer choosing f as provider. Observe that, whatever the type of prices considered, the alternative offers proposed by the providers other than f, denoted by[4]

$$\boldsymbol{p}_{-f} = (\boldsymbol{p}_1, \cdots, \boldsymbol{p}_{f-1}, \boldsymbol{p}_{f+1}, \cdots, \boldsymbol{p}_F), \tag{2.16}$$

will interact with the choice $\boldsymbol{p}_f$ of provider f through the provider choice of the consumers. If an alternative provider proposes a very low offer (i.e., with low prices), $\mathcal{C}_f$ should be reduced. To make this dependency explicit, the following notation will be considered in the following

$$\mathcal{C}_f(\boldsymbol{p}_f, \boldsymbol{p}_{-f}) \tag{2.17}$$

Setting a proper offer against the other providers offers (i.e. $\boldsymbol{p}_{-f}$) to attract more consumers becomes a key task for each provider, then in competition. In turn, providers are playing a game (in the sense of game theory), where utility functions are difficult to express, given that they depend on $\mathcal{C}_f(\boldsymbol{p}_f, \boldsymbol{p}_{-f})$ for which an analytical expression with providers

[4] Standard notation in game theory to define the strategy of all the players but one, i.e. his opponents.

strategies $\boldsymbol{p}_f$ is not available. Thus, this upper level game will be here analyzed through simulations.

2.3.3 Provider-consumer interaction

The relation between providers and consumers (see Fig. 1) can be modeled as a bilevel competition game named Stackelberg multiple-leader game [16] where:

- Leaders (upper level): electricity providers $f \in \mathcal{F}$;
- Followers (lower level): electricity consumers $c \in \mathcal{C}$.

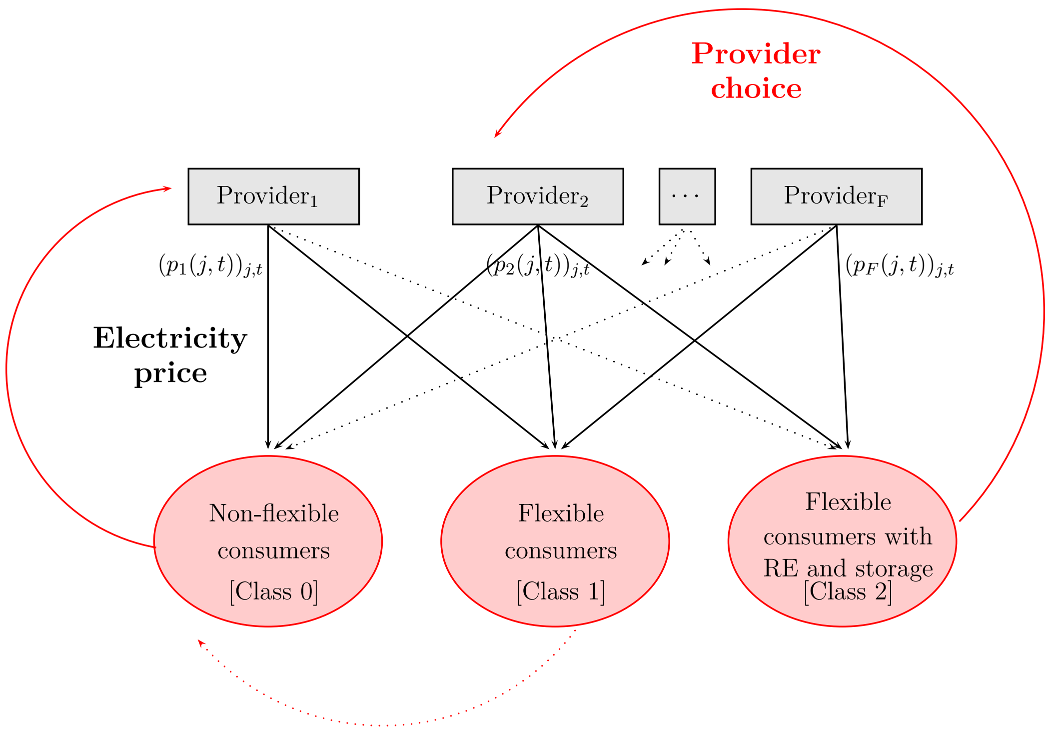

Figure 1. Illustration of the proposed model for the competition between electricity providers and consumers. *In each period, providers propose an offers to the consumers. Different classes of consumers will react to the prices by scheduling their flexible consumption profile (for classes* 1 *and* 2*), scheduling their storage (dis)charging profile (for class* 2 *only) and choosing their provider (for the three classes). In a dynamic setting (when multiple periods are considered), depending on consumers reactions and current alternative providers offers, providers will propose new offers to their consumers in next period to maximize their profits to come.*

3 Theoretical analysis: resolution of the model

In this section, we analyse the solutions for the different classes of consumers and then propose an iterative approach to obtain a Nash equilibrium between providers by using a standard game theoretical dynamics named best response dynamics.

3.1 Resolution of consumers problem

3.1.1 Non-flexible consumer: enumeration and comparison

As briefly introduced before, a non-flexible consumer $c \in \mathcal{C}^0$ only has to select the provider (i.e. offer) who gives him a minimal bill. A simple enumeration and comparison scheme of all bills with the different providers can solve this problem, as described in Algorithm 1.

Algorithm 1 Non-flexible consumer - Enumeration and comparison

1: **for** $f \in \mathcal{F}$ **do**
2: Calculate total bill: B_{cf}
3: Select minimal bill and corresponding provider.

3.1.2 Flexible consumer: scheduling problem then enumeration and comparison

As for a flexible consumer $c \in \mathcal{C}^1$, given that $\tilde{\boldsymbol{w}}_c = 0$ and $E_c = 0$, his decision Problem (2.9) reduces to solve

$$\min_{f\in\mathcal{F}} \min_{\boldsymbol{\ell}_c^1 \in \mathbb{R}_+^{J\times T}} \sum_{j\in\mathcal{J}} \sum_{t\in\mathcal{T}} p_f(j,t) \times \left[\ell_c^0(j,t) + \ell_c^1(j,t)\right]$$

$$\text{s.t.} \begin{cases} 0 \leq \ell_c(j,t) \leq \bar{\ell}, & \forall (j,t) \in \mathcal{J}\times\mathcal{T}, \\ \sum_{t\in\mathcal{T}} \ell_c^1(j,t) = L_c^1(j), & \forall j \in \mathcal{J}. \end{cases} \tag{3.1}$$

Depending on the type of prices considered, this leads to two standard problems for the daily scheduling subproblem: a Knapsack problem or a water-filling problem as stated in the next proposition.

Proposition 1. *For a flexible consumer $c \in \mathcal{C}_1$ in provider f's pool:*

- *if exogenous prices are considered, daily scheduling problem is a Knapsack problem and the optimality conditions for this problem can be expressed with a single critical price p_f^*, such that*

$$\forall (j,t) \in \mathcal{J}\times\mathcal{T},\ \ell_c^{1,*}(j,t) = \begin{cases} \bar{\ell} - \ell_c^0(j,t) & \text{if } p_f(j,t) < p_f^*(j,t) \\ \ell^{1,*} \in \left[0, \bar{l} - \ell_c^0(j,t)\right] & \text{if } p_f(j,t) = p_f^*(j,t) \\ 0 & \text{else} \end{cases} ; \tag{3.2}$$

- *if endogenous prices are considered, (3.1) is a water-filling problem with individual peak power constraints problem, it thus has the same optimality conditions as (3.2) where the critical level represents the optimal "water-level".*

Remark 5. *For endogenous price, even problem is more complex, but as for the price function g_f is convex and regular, we can still give the optimality conditions for flexible consumers, in Vicente and Calamai [19], the authors generalized the first and second order optimality conditions to the case of bilevel programs involving quadratic strictly convex lower-level problems.*

Using Prop. 1, each daily flexible load scheduling problem can be solved with the iterative procedure described in Algorithm 2. The idea of Algorithm 2 used for consumption scheduling problem is simple: at every iteration (indexed by k), find the lowest price, put the energy need on that time slots until it reaches the limit, continue until the energy need is fulfilled.

Algorithm 2 Flexible consumer daily scheduling problem

for $j \in \mathcal{J}$ **do**
  k=0: set $\ell_c^{1(k=0)}(j,t) = \ell_c^0(j,t), \forall t \in \mathcal{T}$;
  while $\exists t, \ell_c^{1(k)}(t) < \overline{\ell}_c$ and $\sum_t \ell_c^{1(k)}(j,t) < L_c^1(j)$ **do**
    for $\ell_c^{1(k)}(j,t) < \overline{\ell}_c$ **do**
      select one (only one) of the non-saturated time slot(s) $\tilde{t}$ with the smallest value for

$$p_f(j,t) \quad \text{for exogenous price;} \tag{3.3}$$

or

$$\frac{\partial\left(\ell_c(j,t) \times g_f(\ell_c(j,t))\right)}{\partial \ell_c(t)} \quad \text{for endogenous price;} \tag{3.4}$$

      Set $\ell_c^{1(k+1)}(j,\tilde{t}) = \min\{\overline{\ell}_c,\ \ell_c^{1(k)}(j,\tilde{t}) + [L_c^1(j) - \sum_{t\in\{1,\cdots,T\}} \ell_c^{1(k)}(j,t)]^+\}$;
  if $\sum_{t\in\{1,\cdots,T\}} \ell_c^{(k+1)}(j,t) = L_c^1(j)$ **then**
    Stop;
  else
    $k = k+1$;

Once the scheduling problem is solved for each day and the total bill obtained by summation, a flexible consumer select the provider who gives him the minimum bill. This part is done in the same way than for non-flexible consumers.

3.1.3 Flexible consumer with RE and storage: dynamic programming then enumeration and comparison

The last part is about flexible consumer with an intermittent production profile $\widetilde{\boldsymbol{w}}$ and an associated storage (energy level $\boldsymbol{e}$). Due to the uncertainty of the RE production, in this section, we first consider the simpler case where the production profile is known (perfect forecast for the whole period to come), then the case it is not.

3.1.3.1 Deterministic case

Suppose first that intermittent production profile is perfectly known with $\widetilde{\boldsymbol{w}} = \boldsymbol{w}$. It can either represent the idealistic case of a perfect forecast or the forecast without noise. It can also consists of a given scenario used among multiple ones to calculate then an empirical expectation. It is then to solve Problem (2.9) with $\widetilde{\boldsymbol{w}} = \boldsymbol{w} > 0$ and $E_c > 0$.

A method to solve this problem is dynamic programming. Consider only one day $j \in \mathcal{J}$ problem, denote

- $L_c(j,t)$ the residual flexible need for consumer c of day j from time t to the end of the day. This need must be met with consumption in $t' = t+1, \ldots, t' = T$:

$$\sum_{t'=t+1}^{T} \ell_c^1(j,t') = L_c(j,t) \quad ; \tag{3.5}$$

  In particular $L_c(j,0) = L_c(j)$;

- $B_{j,t}$ the bill for consumer c of day j from time t to the end of the day;

- $e_c(j,T) = \frac{E}{2}$ the final state of the storage.

With state $(L_c(j,t), e_c(j,t))$, we have the Bellman equation

$$\begin{aligned}
&\forall (j,t) \in \mathcal{J} \times \mathcal{T}, \\
&B_{j,t-1}(L_c(j,t-2), e_c(j,t-2)) = \\
&\quad \min_{\ell^1 \in \mathbb{R}_+, s \in \mathbb{R}} \; p_f(j,t-1) \times \left[\ell_c^0(j,t-1) + \ell^1 - w_c(j,t-1) + s\right] \\
&\quad + B_{j,t}(L_c(j,t-2) - \ell^1, e_c(j,t-2) + s)
\end{aligned} \tag{3.6}$$

Once this dynamic programming problem is solved for each day[5], the total bill for period $\mathcal{J}$ is calculated and provider choice is obtained in the same way than for non-flexible consumers (enumeration and comparison of different providers).

3.1.3.2 Stochastic case: MDP

In this part, consider the case where the intermittent production profile is uncertain, which makes sense because renewable generation are strongly depends on the conditions of the nature(wind [9], sun, etc.). Thus, a stochastic dynamic programming approach is to be preferred.

By considering the RE production as a Markov chain, we can cast optimal storage management as a Markov decision process (MDP) [17]. Giving the specificity of this type

[5] Recall that all these consumption and battery scheduling problems are daily independent.

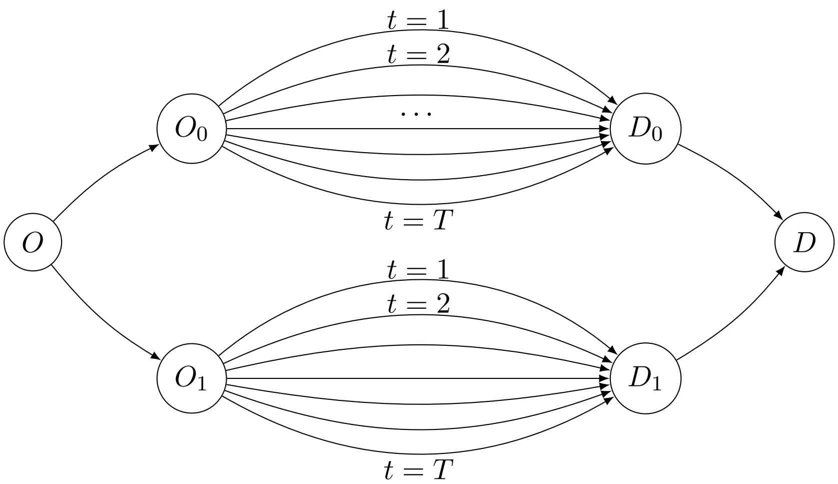

Figure 2. Network of the equivalent routing model. *The original arc O is the provider set, final arc D is the scheduled result for consumers, links from O to D are consumptions at each time slots $t \in \mathcal{T}$. Flows are atomic on the two first and last arcs and non-atomic on the two providers' sub-networks.*

of problem, heuristic rules have been preferred during my internship, as detailed in the simulation part.

3.2 Resolution of the Problem: Provider

If we can characterize the equilibrium between consumers at the lower level, we can take the equilibrium conditions at the lower level as an input for the interaction between providers at the upper level. In the works of Orda et al. [13] and Altman et al. [3], the authors characterized some properties of the Nash equilibrium in competitive routing network links depending on the congestion function on the links, and the conditions for the existence and uniqueness of the Nash equilibrium, in particular the case with polynomial congestion cost functions.

Interestingly, these results can be used in our model considering time slot (j, t) as a network link, and parametrized (polynomial) endogenous provider prices as congestion cost functions. Fig. 3.2 proposes an illustrative example for two providers ($F = 2$) and a unique day ($J = 1$).

This reinterpretation of scheduling problem allows applying the following Theorems to study the competition between consumers in a given provider pool (when endogenous prices with intrapool dependency are considered applying (2.13)). First, uniqueness of the equilibrium at the lower level is obtained.

Theorem 1 (Result from Orda et al. [13])**.** *Consider only nonflexible and flexible consumer classes. For a provider f who has endogenous prices of type (2.13) at each time slot, with price function g_f increasing in each and the total load, then NE between flexible consumers at the lower level is unique.*

Furthermore, if linear endogenous prices are considered, it can be explicitly characterized.

Theorem 2 (Result from Altman et al. [3]). *Consider only nonflexible and flexible consumer classes. For a provider f who have linear price of type (2.13) at each time slot, i.e. of the form*

$$g_f(\ell_f(j,t)) = a_f(j,t) \times \ell_f(j,t) + b_f \tag{3.7}$$

where $\ell_f(j,t) = \sum_{c \in \mathcal{C}_f} \ell_c(j,t)$ is total consumption in provider f's pool at time (j,t), then, if all consumers use all time-slots

$$\forall c \in \mathcal{C}_f, \forall (j,t) \in \mathcal{J} \times \mathcal{T}, \ell^*_c(j,t) > 0 \ . \tag{3.8}$$

Then the unique Nash equilibrium between consumers in provider f's pool is

$$\forall c \in \mathcal{C}_f, \forall (j,t) \in \mathcal{J} \times \mathcal{T}, \ell^*_c(j,t) = \frac{1/a_f(j,t)}{\sum_{t'} 1/a_f(j,t')} \times \sum_{j \in \mathcal{J}} L_c(j). \tag{3.9}$$

The results above are the first step to integrate the equilibrium between consumers in the profit function of the providers (through $\mathcal{C}_f(\boldsymbol{p}_f, \boldsymbol{p}_{-f})$ to be determined based on the scheduling equilibrium between flexible consumers) and to formally study the interaction between providers as a game. In such a game, player set will be $\mathcal{F}$, action set $(\mathcal{P}_f)_{f \in \mathcal{F}}$ (it can correspond to exogenous or endogenous prices), and the payoff function is defined as in (2.15), also we put it as

$$\Pi_f(\boldsymbol{p}_f, \boldsymbol{p}_{-f}) = \sum_{c \in \mathcal{C}(p_f, p_{-f})} \sum_{(j,t) \in \mathcal{J} \times \mathcal{T}} p_f(j,t) \times \ell_c(j,t) \tag{3.10}$$

3.2.1 Approach of the solution: best response dynamics

As explained above, for now we can not characterize the equilibrium directly at the lower level, except for linear endogenous price case. In turn, the complexity of our model make it even harder for a theoretical study of the equilibrium between providers at the upper level. However, game theory offers us a way to approach the solution for our problem with an iterative procedure, which is best response dynamics. In evolutionary game theory [10], best response dynamics represents a class of strategy updating rules, where players strategies in the next iteration are determined by their best responses.

For our defined providers game $< \mathcal{F}, (\mathcal{P}_f), (\Pi_f) >$, adaptation dynamics of the new-period offer for provider $f \in \mathcal{F}$ is

$$BR_f(p_{-f}) = arg \max_{p_f \in \mathcal{P}_f} \Pi_f(p_f, p_{-f}) \tag{3.11}$$

In practice, provider f may not get access to the offers proposed by the alternatives $-f$ for next period, in this case, his next period offer will be the best response to the current period offers of the other providers. In the simulation section, one period is a year, and providers are updating their new-year offers (for year y) against the other providers' current year offers (of year $y-1$), which we refer to as a yearly dynamics. Formally, provider f's price-update strategy for the coming year y reads,

$$\boldsymbol{p}^{f,y} \in arg \max_{\boldsymbol{p}^f \in \mathcal{P}^f} \Pi^f(\boldsymbol{p}^f, \boldsymbol{p}^{-f,y-1}) \tag{3.12}$$

with y year index.

Pseudo-code for the yearly dynamics is shown in Algorithm 3[6].

Algorithm 3 Yearly Dynamics

Initial year $p^{f,y=0}$ randomly chosen
  Calculate initial consumers consumption + provider choice
while $\sum_{f\in\mathcal{F}} \|p^{f,y} - p^{f,y-1}\| >= \epsilon$ **or** $y \leq Y$ **do**
  Next year $y = y + 1$
  for $f \in \mathcal{F}$ **do**
    for $p^f \in \widetilde{\mathcal{P}}_f$ **do**
      Simulate consumers reaction:
        choose classes + scheduling consumption + provider choice
      Calculate $\Pi^f(p^f, \boldsymbol{p}^{-f,y-1})$
    Choose $p^{f,y}$ maximizing $\Pi^f(\cdot, \boldsymbol{p}^{-f,y-1})$

Observe that fixing the decisions of all providers but one leads to a bilevel model with a unique leader, which can be written as such and could offer some theoretical perspectives, as described in the next section.

3.3 Problem of a given Provider casted as a Bilevel program

3.3.1 Best-Response of a given Provider written as a standard bilevel problem

In our model, by fixing the decisions of all other providers but f, and considering how provider f best-responds to the others, we can define a bilevel model with only one leader f, the others seen as parameters.

We assume consumers are purely rational, i.e. they always choose the provider who give them the minimum bill. We consider only two representative consumers - one for non-flexible class (indexed by 0), one for flexible class (indexed by 1). It is sufficient to theoretically describe the competition among the providers, since each class has a same action to take - choose the provider with the minimum bill if this provider is unique (with equal bills, the consumer has the same probability to choose any of the providers).

For simplicity, we only consider a unique day (index j is omitted), we assume that consumption for consumers are daily independent and they can change their providers day

[6]The iteration end with either a high accuracy ϵ or a limited loops Y.

by day. There are only two providers ($F = 2$) and they propose different exogenous prices 2 to the two classes (more general than the main model with a common offer to all classes). Then provider, f's problem reads

Problem 3 (Bilevel Model)**.** *Denote:*

- $\ell_c(1,t) = \ell_c(t)$ *since it is a one day problem;*
- x_0^f *(resp.* x_1^f*) the assignment of non-flexible (resp. flexible consumer) to provider* f *(*1 *if consumer chooses* f*,* 0 *otherwise);*
- $\ell_1^{1,f}$ *(resp.* $\ell_1^{1,f'}$*) the flexible consumption profile for flexible consumer if he chooses* f *(resp.*f'*);*
- ℓ_1^0 *the non-flexible part consumption of flexible consumer;*
- ℓ_0^0 *the consumption of non-flexible consumer;*
- $p_{f,0}$ *(resp.* $p_{f,1}$*) the offer proposed by provider* f *to non-flexible class (resp. flexible class).*

Then,

$$\max_{\substack{p_{f,0},p_{f,1}\\ x_0^f,x_1^f,\boldsymbol{\ell}_1^{1,f},\boldsymbol{\ell}_1^{1,f'}}} \quad x_0^f \sum_{t\in\mathcal{T}} p_{f,0} \times \ell_0^0(t) + x_1^f \sum_{t\in\mathcal{T}} p_{f,1}(t) \times \left[\ell_1^0(t) + \ell_1^{1,f}(t)\right]$$

$$s.t. \begin{cases} p_{f,0}, p_{f,1} \in \mathcal{P}_f \\ x_0^f, x_1^f, x_0^{f'}, x_1^{f'} \in \{0,1\} \\ \min\limits_{x_1^f\in\{0,1\}} x_1^f \left(\min\limits_{\boldsymbol{\ell}_1^{1,f}\in\mathbf{R}_+^{\mathcal{T}}} \sum\limits_{t\in\mathcal{T}} p_{f,1}(t) \times \left[\ell_1^0(t) + \ell_1^{1,f}(t)\right]\right) + x_1^{f'} \left(\min\limits_{\boldsymbol{\ell}_1^{1,f'}\in\mathbf{R}_+^{\mathcal{T}}} \sum\limits_{t\in\mathcal{T}} p_{f',1}(t) \times \left[\ell_1^0(t) + \ell_1^{1,f'}(t)\right]\right) \\ \qquad s.t. \begin{cases} \sum\limits_{t\in\mathcal{T}} \ell_1^{1,f}(t) \leq \bar{\ell} - \ell_1^0(t), & \forall t\in\mathcal{T} \\ \ell_1^{1,f}(t) = L^1 \\ \sum\limits_{t\in\mathcal{T}} \ell_1^{1,f'}(t) \leq \bar{\ell} - \ell_1^0(t), & \forall t\in\mathcal{T} \\ \ell_1^{1,f'}(t) = L^1, & \forall t\in\mathcal{T} \\ x_1^f + x_1^{f'} = 1 \end{cases} \\ \min\limits_{x_0^f\in\{0,1\}} x_0^f \sum\limits_{t\in\mathcal{T}} p_{f,0}(t) \times \ell_0^0(t) + x_0^{f'} \sum\limits_{t\in\mathcal{T}} p_{f',0}(t) \times \ell_0^0(t) \\ s.t. \quad x_0^f + x_0^{f'} = 1 \end{cases} \tag{3.13}$$

The structure above can be extended for $F > 2$ by introducing an assignment variable by provider for consumers 0 and 1, with associated constraints

$$\sum_{f \in \mathcal{F}} x_0^f = 1 \qquad \text{and} \qquad \sum_{f \in \mathcal{F}} x_1^f = 1 \quad . \tag{3.14}$$

Then it suffices to introduce summation at the lower level for flexible class

$$\sum_{f \in \mathcal{F}} x_1^f \min_{x_1^f \in \{0,1\}} \left(\min_{\boldsymbol{\ell}_1^{1,f} \in \mathbf{R}_+^T} \sum_{t \in \mathcal{T}} p_1^f(t) \times \left[\ell_1^0(t) + \ell_1^{1,f}(t) \right] \right) \tag{3.15}$$

to solve flexible consumer problem. Same summation applies to the non-flexible one.

3.3.2 Basis for the resolution of bilevel problem

The bilevel problem defined in Problem 3 is a standard BiLevel Programming Problem (BLPP) [7]. As a branch of mathematical programming, BLPP has been studied for both practical and theoretical interest in the past 30 years, and Stackelberg problems, due to the similar structure to BLPP, can be treated as BLPP, although we are looking for the equilibrium at the lower level rather than solve the optimization problems.

Among the existing methods, when the lower level problem is convex and regular, it can be replaced by its optimality conditions (sometimes with so-called "big-M constraints"). In our model, the optimality conditions of flexible consumers' scheduling problem are described in Section 3.1.2. The optimal provider choice can be obtained by comparing the bills with f with the fixed (given that other providers are seen as parameters) alternative bills.

Once the problem is solved at the lower level i.e. the lower level solution is well defined, then we can put the lower level solution as a parameter into the upper level optimization problem, then upper level problem will be no special but a classical optimization problem.

4 Focus on Consumers' Imperfect Rationality: study with a supermodular game

The perfect rationality of consumers in previous section can not reflect the reality. In practice, consumers is not that sensible to new offers bill differences (especially if the cost difference is very small), but also on taste-dependent preferences. Discrete choice models are often introduced to describe this imperfect rationality of consumers [22]. The proportion in provider f consumers pool for the period to come is then expressed as

$$\gamma_f^y = \frac{e^{-\alpha_{prov} B_f^y}}{\sum_{i \in \mathcal{F}} e^{-\alpha_{prov} B_i^y}} \tag{4.1}$$

with y index of period, i index of the providers, B_i (2.7) the bill with provider i[7], α_{prov} the bill sensibility for consumers. Small α_{prov} means lower sensibility where the consumers

[7] This bill is the aggregated bill for all classes.

will more likely stay with their provider with small change of their bills: for $\alpha_{prov} = 0$, the consumers are randomly distributed in each provider pool; for large α_{prov}, consumers will be more sensible to price and change their providers even with small benefit in their bills (e.g. when $\alpha_{prov} \to \infty$, the consumers will be perfect rational and choose the lowest price).

From year to the next one, consumers can also change their classes to fit their need, the proportion of consumers in each class's pool can be defined similarly with another sensibility parameters we refer to as α_{cons}.

The normalized payoff[8] for a provider f for the period to come can be written as:

$$\Pi_f(p_f^y, p_{-f}^y) = B_f^y \times \frac{e^{-\alpha_{prov} B_f^y}}{\sum_{i \in \mathcal{F}} e^{-\alpha_{prov} B_i^y}} \tag{4.2}$$

Consider a game $< \mathcal{F}, (\mathcal{P}_f)_f, (\Pi_f)_f >$ where $\mathcal{F}$ is the provider set, $\mathcal{P}_f$ is the price set, and Π_f is the payoff function set. Interestingly, this game has the property of being super modular, which is now defined.

Definition 1 (Asu [4]). *The strategic game $< \mathcal{F}, (\mathcal{P}_f), (\Pi_f) >$ is supermodular if for all $f \in \mathcal{F}$:*

- *$\mathcal{P}_f$ is a compact subset of $\mathbb{R}^{J \times T}$;*
- *Π_f is upper semicontinuous in p_f, continuous in p_{-f};*
- *Π_f is a supermodular function in (p_f, p_{-f}).*

It is easy to check $\Pi^f(p_f^y, p_{-f}^y)$ is log-supermodular (see proof in Appendix Section A), thus $\Pi^f(p_f^y, p_{-f}^y)$ is a supermodular function. The game defined above is then a supermodular game. In turn, it inherits strong properties of this class of games, described in the following proposition.

Proposition 2. *Given that $< \mathcal{F}, (\mathcal{P}_f), (\Pi_f) >$ is a symmetric strict supermodular game, only symmetric equilibria exist and best response dynamics will converge to the largest equilibrium.*

The property of convergence of best response dynamics is observed by simulation in Fig. 3.

5 Simulation results

In this section, we present the simulation results with the model defined in previous sections using yearly dynamics 3.2.1. The simulation setting is as follows. We consider one year French household consumption data taken from "Recoflux" ERDF, $J = 364$ days, each day

[8]suppose the total population is fixed, the true payoff is the payoff in 4.2 multiplied by the total population.

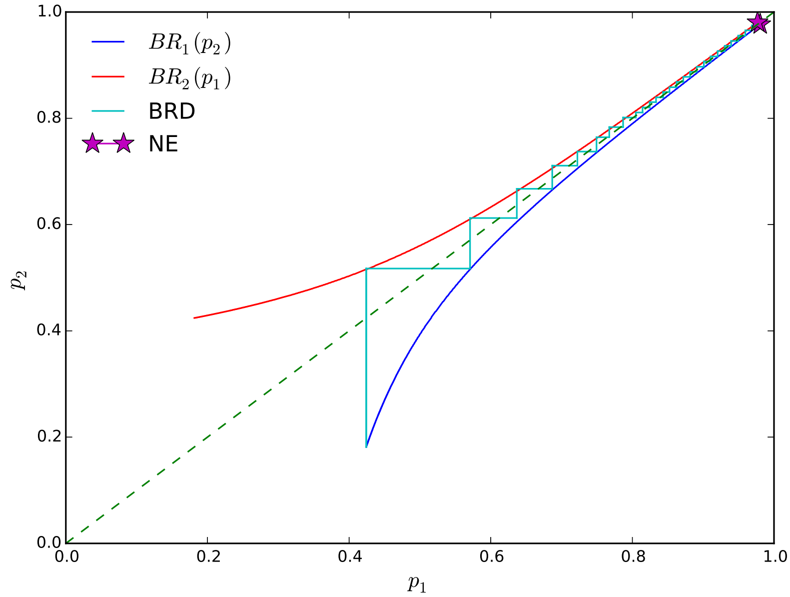

Figure 3. Providers' best responses in game with imperfect rationality. *"BRD" = best response dynamics, it converges to the largest equilibrium.*

with $T = 48$ time slots - 0.5h. Non-flexible consumption is base consumption (yearly usage: 6,400 kWh), and flexible consumption is water-heating (yearly usage: 9,000 kWh). For the yearly dynamics, we consider the same data for every year. We focus on two providers $F = 2$, two classes of consumers - non-flexible and flexible - with consumer population: 1000,

$$n_{\mathcal{C}^0} + n_{\mathcal{C}^1} = 1000. \tag{5.1}$$

In yearly dynamics, at the end of each year y, calculate the aggregated bill $\{B_c\}_{c\in\{0,1\}}$ of all providers for each class, the proportion of class $c \in \{0,1\}$ for the year to come is

$$\gamma_c^{y+1} = \frac{e^{-\alpha_{\text{cons}} B_c^y}}{\sum_{i\in\{0,1\}} e^{-\alpha_{\text{cons}} B_i^y}} \tag{5.2}$$

The population in each class is then

$$n_{\mathcal{C}^0} = \gamma_0^{y+1} \times (n_{\mathcal{C}^0} + n_{\mathcal{C}^1}) \qquad \text{and} \qquad n_{\mathcal{C}^1} = \gamma_1^{y+1} \times (n_{\mathcal{C}^0} + n_{\mathcal{C}^1}) \tag{5.3}$$

The problem of third class is treated separately and presented at the end. The offer is daily on/off peak fare, on peak price is chosen in a discrete set

$$\mathcal{P}_{\text{on}} = \{0.12, 0.18, 0.24\}, \tag{5.4}$$

off peak price is choosen in a discrete set

$$\mathcal{P}_{\text{off}} = \{0.04, 0.08, 0.12\}, \tag{5.5}$$

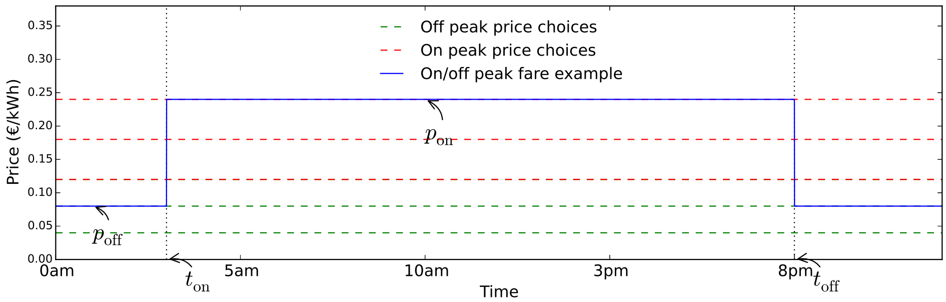

Figure 4. On/off peak fare example.

such a fare is represented as a four dimension vector[9]

$$\boldsymbol{P} = (t_{\text{on}}, t_{\text{off}}, p_{\text{on}}, p_{\text{off}}) \in \mathcal{T} \times \mathcal{T} \times \mathcal{P}_{\text{on}} \times \mathcal{P}_{\text{off}}, \tag{5.6}$$

where $t_{\text{on}} \leq t_{\text{off}}$ are the start and end time of on peak period of the day, see Fig. 4. The discrete choice model is used in the simulation, in most cases, we consider the sensibility parameters for consumer are $\alpha_{\text{prov}} = \alpha_{\text{cons}} = 0.05$ (α_{prov}, α_{cons} are discrete choice parameters for provider choice and class choice respectively.), flexible consumption limit is $\bar{\ell} = 2.5$ kW, and γ_{flex} is the flexibility we give to consumer c for the flexible consumption part ℓ_c^1, such that, for a given γ_{flex}, consumer c's consumption profile is modified to:

$$\ell_c^1 = \gamma_{\text{flex}} \times \ell_c^1 \tag{5.7}$$

$$\ell_c^0 = \ell_c^0 + (1 - \gamma_{\text{flex}}) \times \ell_c^1 \tag{5.8}$$

.

5.1 Convergence of yearly dynamics

Recall from Section 4, consumer sensibility to bill is a main factor for practical study. Real parameters are usually estimated from existing data[10], and can highly affect the obtained equilibrium by best response dynamics in our model. However, in simulation, we can consider several choices for both α_{prov} and α_{cons}, see Fig. 5. Typically, smaller sensibility gives us convergence of yearly dynamics. In Fig. 7 and Fig. 8, we put flexible consumers' bill as an example to distinguish the convergent and divergent cases.

5.2 Performance profits for providers and bills for consumers

Next, we look at the resulting profits for providers and bills for consumers when we use Algorithm 2 and yearly best response dynamics Algorithm 3. With small sensibility pa-

[9]In France, EDF propose on/off peak fares in -"Tarif Bleu"- for consumers. For example, in these offers, the constant fare is 14.49 cts €TTC/kWh, the on/off peak fare is 15.6/12.7 cts €TTC/kWh, the off peak period last 8 hours in the night.

[10]In EDF, we only have the data for residential users of UK.

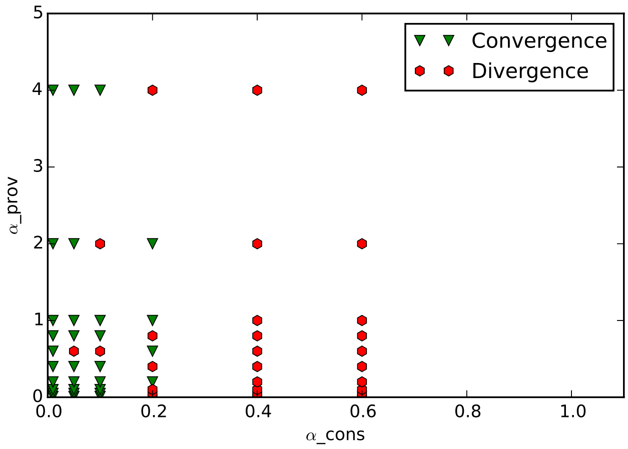

Figure 5. Convergence of the dynamics. *There is no monotonicity of the convergence with increasing sensibilities, however, with small sensibility, the dynamics are more likly to converge.*

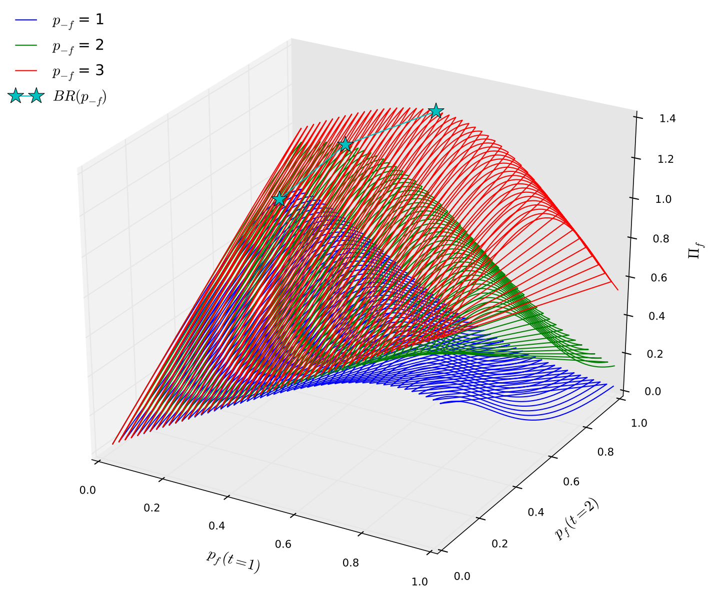

Figure 6. Supermodular model with two time slots ($T = 2$) daily offer ($J = 1$). p_{-f} *is the alternative offers.*

rameters, providers always choose the maximal fare [11] (see Fig. 9) and their profits are nearly doubled; at the point of the equilibrium, for the same price, flexible consumer can

[11]In our case, providers choose the maximal on/off allowed prices and minimum off-peak duration.

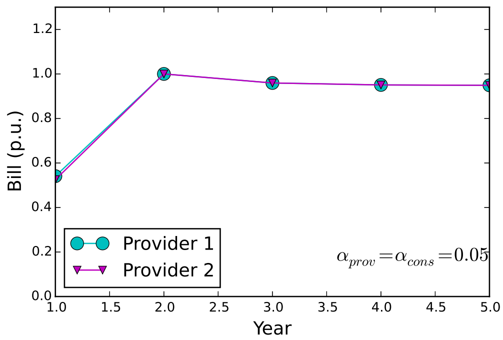

Figure 7. Convergent case with small sensibility to bill.

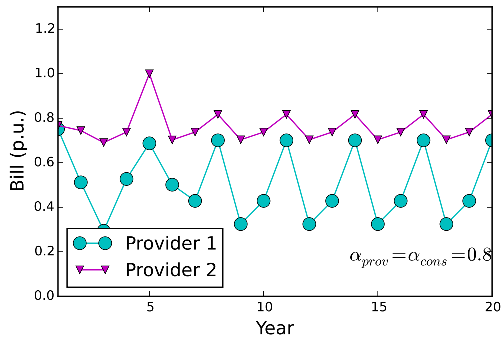

Figure 8. Divergent case with high sensibility to bill.

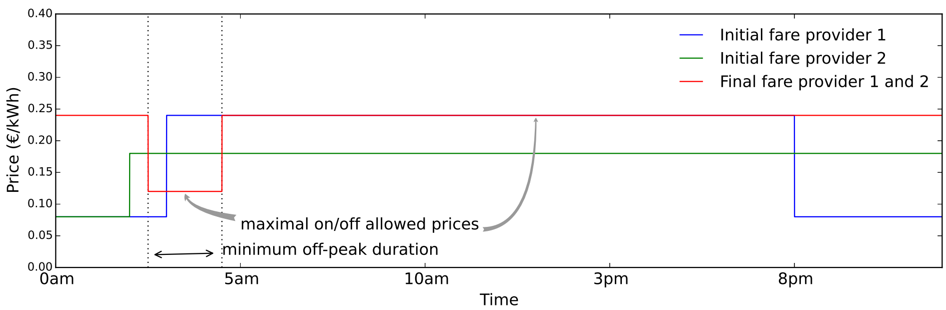

Figure 9. The final on/off peak fare for both providers. *The maximal fare is reached.*

save more money compared with non-flexible one.

As we saw in supermodular game model, the simulation setting fits very well the assumptions, providers have symmetric information and update strategies, the best response dynamics converge to the same maximum fare, see example in Fig. 6).

In practice, the providers may offer different prices to different classes, for example, in France, EDF proposes two different prices to residential consumers, fixed price and on/off peak fare, the former is normally lower than the latter one i.e. $p_{\text{off}} < p_{\text{fixed}} < p_{\text{on}}$, which shall reduce the non-flexible consumers' bills.

5.3 Impact of initial proportion and different flexibility of flexible consumers

The equilibrium may be different for different initial settings. In this section, we first want to show that initial proportion for flexible consumers (i.e. the initial population in flexible consumer class $n_{\mathcal{C}_1}$.) does not affect the final equilibrium, see Fig. 10.

Being more flexible i.e. having more flexible consumption to be scheduled in total consumption profile for a flexible consumer, has a strong impact on his bill, see Fig. 11, consumer's bill is decreasing with increasing flexibility, which makes sense when provider always choose the maximal price at the equilibrium.

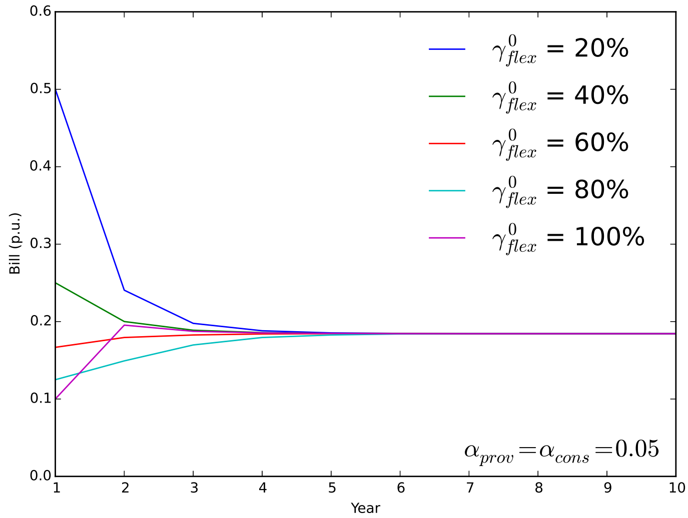

Figure 10. Flexible consumers bill dynamics with different initial proportions of flexible consumers. They converge to the same equilibrium after approximately 6 years.

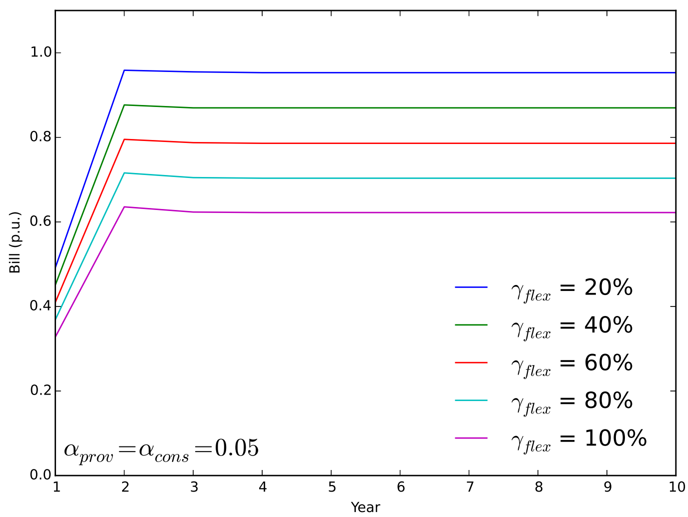

Figure 11. Dynamic bills for flexible consumers with different level of flexible. The bill for consumers is decreasing respect to the increasing flexibility. The final price proposed by providers keep the same.

5.4 Class 2: flexible consumer with RE and storage

The third class is introduced highly due to the potential economic and environmental advantages compared to current residential energy consumption mode. A consumer of

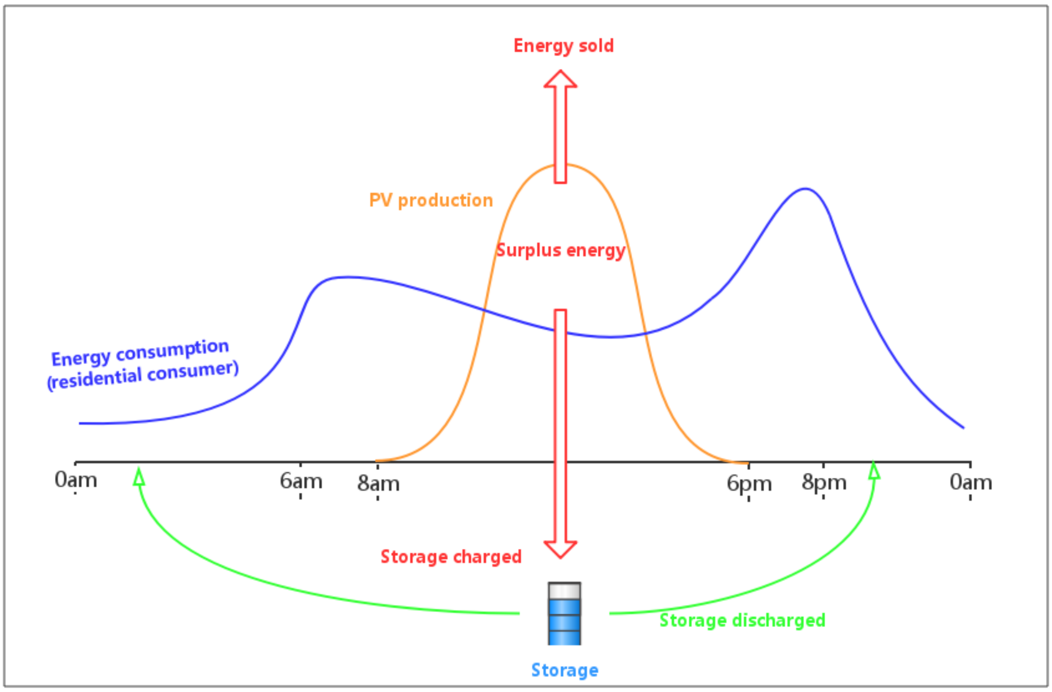

Figure 12. Daily energy consumption pattern of flexible consumer with PV and storage.

class 3 has a daily consumption pattern described as in Fig. 12.

For third class of consumers, we assume:

1. for energy production, local use is preferable than put it in the storage battery, storage battery is preferable than selling it back to the grid;
2. for storage battery, scheduling the consumption is preferable than discharging the storage battery;
3. the case consumer buying energy from grid only for the storage battery is not allowed.

Given decision rules above, at time $t \in \mathcal{T}$, if there is energy production $w(t) > 0$, we put it to local use first, if the production has more left i.e. $w(t) - \ell(t) > 0$, we put it into the storage battery, if there is still more i.e. the storage is fully filled up $w(t) - \ell(t) - s(t) > 0$, we consider selling it back to the grid; after dealing with the energy production, we schedule the consumption left for the day i.e. being flexible with the consumptions, then we discharging the storage battery slot by slot. For now, we does not consider the scheduling problem of the storage battery.

Also for simplification, we does not count the installation and maintenance cost of the PV and storage for the consumers, then follow the assumptions above, we have the normalized bills for different classes, see Fig. 1.

Consumer class	Class 0	Class 1	Class 0 + PV + storage	Class 2
Bill (p.u.)	1	0.85	0.45	0.26

Table 1. Normalized bills for different classes. *All the bill is divided by the bill of Class 0. The bills is decreasing with more flexibility and with PV + storage, which will encourage consumers to choose Class 2.*

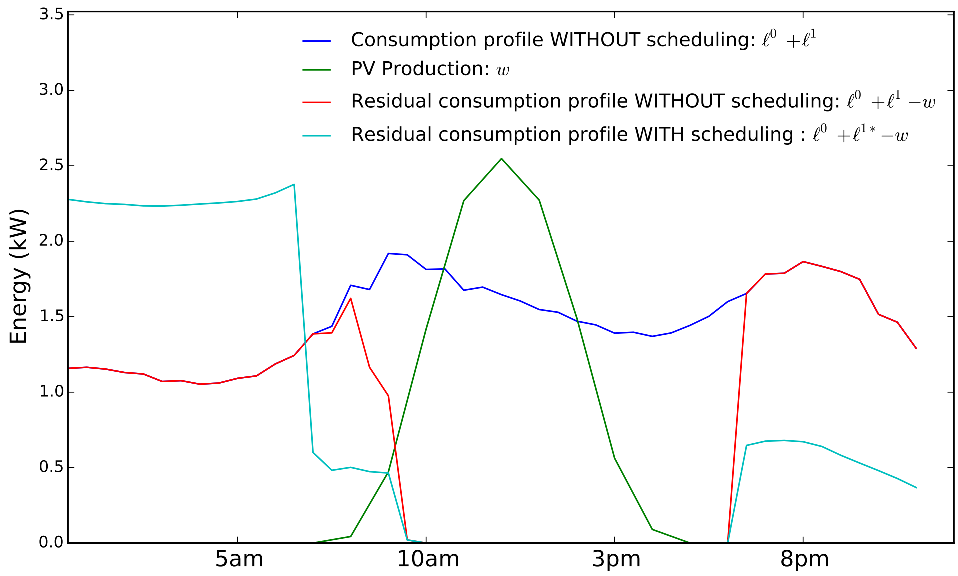

Figure 13. Daily energy consumption of flexible consumer with PV and storage. 8 PV panel are installed, the storage battery capacity is 7kWh. ℓ^{1*} *is the optimal scheduled flexible consumption profile. Most of the energy need is removed to the late night.*

An example of scheduled energy for a daily use is shown in Fig. 13. Without considering the installation and maintenance cost for consumers, as shown in Table 1, class 2 can save more money, more than 70 percent compare with class 0. In fact, we can still looking for the solution of the best parameters for the installed solar panels and storage battery, in the best case, a consumer is self-consumption i.e. all the energy need is coming from PV production. However, since the energy production is uncertain, even in the case the consumer is self-consumption, he still has to stay in the whole network system to manage the worst case i.e. 0 production in several days but still has energy need, so he should still contribute to the network.

In practice, the installation and maintenance cost for solar panel or storage are still expensive with current technology. For example, for a PV power with 3 kWc [12], the bill

[12]kWc (nominal power) : the nameplate capacity of PV devices; an installation of 1 kWc is performed by $5 \sim 10\ m^2$ of solar modules with current technology.

will be 3 €/Wc, for average 20-23 years' use; for a Tesla power-wall with 3 kWh [13], the cost is 500 €/kWh, for average 10 years' use. However, this part of cost will largely decreased in the future [1][14], [2][15].

5.5 Two extensions: escape to gas and sourcing from the Market

To fully describe the complexity of the residential energy use, gas must considered as a big role. In France, residential electricity price kept increasing[16] in the last 10 years and it is much expensive than gas (doubled), since the lower and stable price for gas, the residential consumers will much likely to choose gas rather than electricity if it is available. Despite all the disadvantages for gas i.e. maintenance every year; lower efficiency; limited use for energy need (for now, only the equipments involving heating can use gas.), it is still a big part in real life energy consumption[17].

Because the much lower price for gas, in our model, if we consider gas can be used in flexible consumption energy need i.e. water-heating $\cdots$, in perfect rationality, all consumers will put their flexible energy need in gas, which will decrease the consumers' bills. Even in imperfect rationality, because of the much lower prices, almost all the consumers will choose gas too.

In practice, to be more competitive, energy providers offer not only electricity or gas but both of them for consumers. In France, EDF and ENGIE are this kind of providers.

Also, an energy provider may not the energy producer in real life, thus they must buy energy from the market and then provide the energy to residential consumers, they must also define their strategies for sourcing from the market which will affect the whole supply system. In simulation, we considered linear sourcing (i.e. buy energy need for next year linearly in current year) strategies for providers, the only difference will be the unstable profits (not a equilibrium anymore) for providers in yearly dynamics.

6 Conclusion

In this report, supply-demand management between consumers and electricity providers is modeled as a bilevel problem, which can also be seen as a non-cooperative Stackelberg

[13]kWh (Kilowatt hour): the unit of energy equivalent to one kilowatt (1 kW) of power sustained for one hour(3.6 MJ).

[14] "that average costs for electricity generated by solar and wind technologies could decrease by between 26 and 59 percent by 2025".

[15]"70 percent Decrease In Energy Storage Costs By 2030".

[16]From "Ministère du développement durable and Commissariat du Développement Durable et Base Pégase." See http://www.kelwatt.fr/guide/chauffage-gaz-ou-electricite

[17]"France is the fourth-largest gas market in Europe, with consumption of around 50 billion Gm3, or 10 percent of EU demand."; "According to the International Energy Agency (IEA), natural gas consumption will double by 2030. It is expected to account for 25 percent of the global energy portfolio by 2035, compared with 21 percent today, giving production of 5,100 billion m3 (Gm3). " See http://www.grtgaz.com/en/the-advantages-of-natural-gas/

game. We characterized the solution to the decision problem faced by different classes of consumers at the lower level and proposed an iterative approach to obtain an equilibrium between providers at the upper level by using best response dynamics. To take into account consumers imperfect rationality, a supermodular game is considered in an approximated model of the main setting.

This work can be extended in various ways. As shown in the section of theoretical analysis, the resolution of the problem of flexible consumer with RE and storage is done alone but not integrated in the whole bilevel model in simulation, an interesting extension would be to combine the resolution of this class with the two other classes. Another compelling addition would be to find the NE at the lower level for more general type of endogenous prices than the linear ones. Furthermore, introducing gas and sourcing market will make the work more applicable in reality.

A Proof of the supermodularity of the game

Consider first two providers, a single price p_f for each year (period is year) for provider f, the bill for the year is

$$B_f = L \times p_f$$

where $L = \sum_{j \in \mathcal{J}} L(j)$ is the total consumption of the year.

Thus the normalized payoff for a provider f for year y is

$$\Pi^f(p_f^y, p_{-f}^y) = p_f^y \times \frac{e^{-\alpha B_f^y}}{\sum_{i \in \mathcal{F}} e^{-\alpha B_i^y}} = p_f^y \times \frac{e^{-\beta p_f^y}}{\sum_{i \in \mathcal{F}} e^{-\beta p_i^y}}$$

with $\beta = L \times \alpha$, since L is a constant, β can still represent the sensibility to the price.

The optimal solution is

$$p_f^{y*} = arg \max_{p_f^y} p_f^y \times \frac{e^{-\beta p_f^y}}{\sum_{i \in \mathcal{F}} e^{-\beta p_i^y}}$$

Since $p_f^y \geq 0$, then

$$p_f^{y*} = arg \max_{p_f^y} \{\log(p_f^y \times \frac{e^{-\beta p_f^y}}{\sum_{i \in \mathcal{F}} e^{-\beta p_i^y}})\} = arg \max_{p_f^y} \{\log p_f^y - \beta p_f^y - \log(e^{-\beta p_f^y} + e^{-\beta p_{-f}^y})\}$$

It is easy to check $\Pi^f(p_f^y, p_{-f}^y)$ is a log-supermodular,

$$\frac{\partial^2 \log(\Pi^f)}{\partial p_f \partial p_{-f}} = \frac{\beta^2 e^{-\beta p_f} e^{-\beta p_{-f}}}{(e^{-\beta p_f} + e^{-\beta p_{-f}})^2} \geq 0$$

thus $\Pi^f(p_f^y, p_{-f}^y)$ is a supermodular function.

In complex offer case i.e. more time slots, more providers, the supermodularity can still be verified in the same way.

B Properties of supermodular game

Proposition 3 (Topkis [18]). *If $< \mathcal{F}, (\mathcal{P}_f)_f, (\Pi_f)_f >$ is a supermodular game, let*

$$BR_f(p_{-f}) = arg \max_{p_f \in \mathcal{P}_f} \Pi^f(p_f, p_{-f})$$

Then:

- *$B_f(p_{-f})$ has a greatest and least element, denoted by $\bar{B}_f(p_{-f})$ and $\underline{B_f}(p_{-f})$;*
- *If $p'_{-f} \geq p_{-f}$, then $\bar{B}_f(p'_{-f}) \geq \bar{B}_f(p_{-f})$ and $\underline{B_f}(p'_{-f}) \geq \underline{B_f}(p_{-f})$.*

Theorem 3 (Paul and John [14]). *In a supermodular game, the set of strategies that survive iterated strict dominance in pure strategies has greast and least elements i.e. the greast and least elements of best response function, coinciding with the greatest and the least pure strategy NE.*

Proposition 4 (Vives [21]). *In a symmetric supermodular game, the extremal equilibrium are symmetric and, if strategy spaces are completely ordered and the game is strictly supermodular, then only symmetric equilibrium exist.*

Proposition 5 (Paul and John [14], Vives [20]). *In a supermodular game if there are positive spillovers (i.e. the payoff of a player is increasing in the strategies of the others), then the greatest (least) equilibrium point is the Pareto best (worst) equilibrium.*

Proposition 6. *In a supermodular game,*

- *Best-response dynamics approach the interval defined by the smallest and the largest equilibrium points of the game. Therefore, if the equilibrium is unique it is globally stable. This provides the iterative procedure to find the largest (smallest) equilibrium Topkis [18].*
- *The extremal equilibria correspond to the largest and smallest serially undominated strategies Paul and John [14]. Therefore, if the equilibrium is unique, the game is dominance solvable.*

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