M2 Mathematics and Applications

Optimization

Optimal Design of an Electricity Network with a Community Energy Storage Device

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1 Introduction

In the following report is explained the work done during the internship corresponding to the final part of the master M2 *Optimization* of the *University Paris Saclay.* This work has been done in the Optimization Simulation Risks and Statistics Department OSIRIS of the enterprise *Électricité de France.*

The internship consisted in studying the situation of a community of users connected to a solar panel, the one has the ability of producing electricity that is storage in a battery. The optimal distribution of this electricity may mean a significant reduction of the bills paid by the users at the end of the month thanks to they locally consume the "free" electricity from the solar panel. From the point of view of the company in charge of selling the electricity, there are several important issues to determine, in order to deliver a good service to the users and at the same time to obtain as low as possible production costs.

Roughly speaking, in this problem there are two principal decisions to make: At the beginning of each day the users can decide if connecting or not to the solar panel. Once determined which users will be connected, a daily electricity scheduling has to be done to split the solar energy. These decisions will affect the daily bills of the users.

To solve this problem we have used techniques from Game Theory and Learning, to model the situation and to design algorithms through which we determine the mixed Nash Equilibria along the days. Besides the theoretical model of the problem, a Python implementation was done and the problem was solved for real past data.

Nowadays, the study of smart grids has fast increased since the multiples advantages like leveling peak energy demands and reducing energy costs that they can achieve [14]. In particular, the small-scale demand-side management as in residential gated communities has received attention [17], thanks to the cost reductions of household-distributed renewable power generation and storage technologies [6]. In France, specific mechanisms are being developed concerning the case of "collective self-consumption", which consists in locally sharing the output of a renewable production unit, in particular to avoid reverse flows[1], corresponding to cases where the net consumption at a node of the network is negative, i.e. local production exceeds consumption. Indeed, reverse flows put into question the traditional logic of (local) electricity network management. A lot of management rules of such networks have been designed for the case with only "descending" flows, from centralized production units to end-customers. The French mechanisms are then thought so that the temporal profile of local consumption, again e.g. at the scale of a residential gated communities, be as close as possible to the one of local production. The main goal of this work is to propose models and algorithms inline with this applicative setting.

In this context, *Community Energy Storage* (CES) devices can be integrated with novel small-scale *Demand Side Management Model* (DSM) approaches to efficiently utilize on-site energy generation from consumer-owned renewable power resources such as rooftop photovoltaic (PV) systems. By storing electricity when local production exceeds consumption, future needs can be satisfied with electricity coming from this local source. From the grid standpoint, this corresponds to anticipating a part of the consumption (e.g. during midday sunny hours instead of during the evening at peak), and contribute to reducing the reinjection. In turn, it can create value for the grid, part of it being shared with end-users by reducing their consumption costs without modifying their electricity demand patterns.

This kind of mechanisms could play a vital role in the future electric system, especially with the rapid growth of solar mini-grids [2]. A recent field of smart grid research is thus devoted to the relevance of studying such systems, see E. Stephens et al. in [16] and C. Mediwaththe et al. in [12] and [13]. In particular, [12] brings a novel game theory approach to solve the optimal distribution of energy obtained with a solar panel and it will be an inspiration for this work.

Compared to these existing references, this work brings the following contributions:

[1] These flows are said to be reverse because flowing from end-customer location (where RE unit is installed) to the grid.

1. To model completely the setting with users and a CES device that does not charge them for the exchange of electricity,

2. Different temporalities that allow to the users to choose between buying at a fixed price or at flexible price depending of the number of people connected to the device,

3. Different allocation mechanism through which the electricity distributor can split this energy between the users and the comparisons between them,

4. A way to select the optimal size of the solar panel for a given community.

The rest of this document is structured as follows. Section 2 explains the problem, defines a demand side model for the users stating the constraints that each one of them has to respect, as well as the PV generation and how the CES device works as a battery to then define the cost function of the users. Section 3 defines three allocation mechanisms for the Distribution Network Operator (DNO) to split the electricity between the users and then it defines a Daily Game that the users are confronted each day of a month[2]. After this, an algorithm to compute a Nash Equilibrium in mixed strategies of this daily game is presented. Section 4 shows numerical results obtained with each allocation mechanism and discuss their different pros and cons. In this section, we also discuss the viability of adding a daily ticket to be paid for being connected to the system of solar energy as well as how to determine the good size of the solar panel for each community using an heuristic rule. Finally, Section 5 summarizes the principal results of the work, gives conclusions of it and states further work to extend the one done in this document.

[2]Month taken as an example of a long time period.

2 Community Energy Storage Device

This first section is split in three subsections. Section 2.1 gives the background of the problem, state the two timescales considered and the main difference between *active* and *passive* users. Section 2.2 gives the *Demand Side Model* for the users, explain how to obtain the *grid load* and discuss the generation of energy through the solar panel. Finally, section 2.3 gives the *Energy Cost Model* of the users by defining their objective functions in their associated optimization problem.

2.1 The Problem

Consider the situation of a residential neighborhood where the residents can decide whether to connect or not to a CES device, which is itself connected to a PV production unit for DSM as in Figure 1. The CES device is charged by a solar panel installed and its energy is used to satisfy the individual consumption of the connected users.

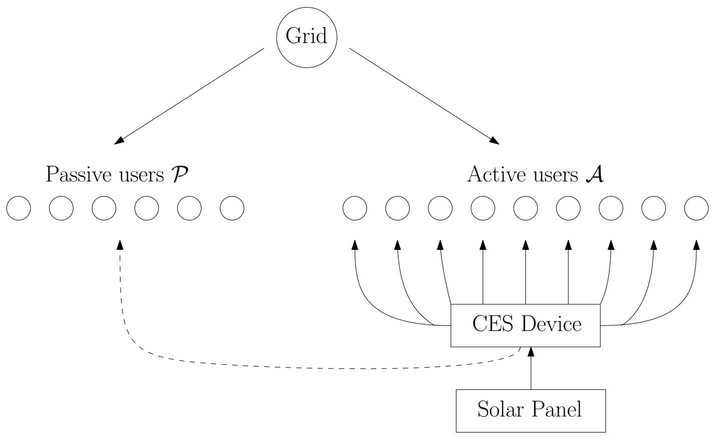


Figure 1: Basic design of a residential neighborhood with a CES device. *Active users get electricity from the CES device, the one is charged by a solar panel. Besides all the users can buy electricity to the grid and they can decide whether to be active or not at the beginning of each day.*

If a connected resident has a deficit of electricity (*after* having consumed a part of PV production), he/she can buy electricity "from the grid"[3] in order to fulfill his/her consumption need. On the other hand, the residents not connected to the CES device buy all their electricity from the grid. **Throughout this document, connected (resp. not connected) residents are named *active users* (resp. *passive users*).**

There are two timescales in the considered problem:

1. A **set of days**, typically a month: At the beginning of each day, users decide whether to be active or passive for the day to come.
2. **Each day is divided into a discrete set of time-slots**, usually 24 hours. The intraday consumption profile for each of the active consumers has to be scheduled, because his/her (exogenous) demand is jointly satisfied by the CES and the grid. This work considers the case where the split of the PV generation is done by the CES operator according to a given allocation mechanism, as explained in Section 3. The demands of passive users are completely satisfied by the grid.

[3] To be realistic, electricity is bought from a provider and then transmitted through the grid. However, the key assumption used here of a fixed price remains true with this refinement.

Therefore, consumers make decisions only at the beginning of each day. During the day, both passive and active players have no more to say in how their electricity demands are satisfied.

A key difference between active and passive users concerns the electricity price they are paying. Passive users are charged at a fixed price which is exogenous to the grid. On the contrary, active users are charged at a unit price set by the grid, which depends on the total net load demanded by these active users, so that it is endogenous to their choices. Section 2.3 describes these cost models. At a given time slot, the bigger the net load from active users, the higher the unit price at this instant. This corresponds to a congestion effect in the grid, which is commonly assumed in the smart grid literature following the seminal paper [14]. A recurrent topic in the literature in this field consists in how to avoid large variations in load profile seen from the grid. When a local production unit is available, as considered in this work, the ideal net load profile is the one being constant and as small as possible and hopefully zero.

Remark 2.1. *Usually in electricity industry the models consider the passive load and active load aggregated to determine prices and costs. Our cost model makes the distinction between these two kind of loads to impose a desired profile over the active users. Although this cost model has no physical basis it obtains good results in cost terms.*

To conclude the presentation of the background, let us point out the intuition of how the players' utilities are influenced by their choices of being active or passive. If all players are active, then the PV production will not be enough to satisfy them. In turn, the net load will be (probably highly) positive, and the price paid by active users become significant. In this case, it may be better for some of them to take the alternative choice of being passive and charged at a fixed unit price. This work aims to propose mechanisms leading to find an stable distribution of active players; the ones whose characteristics are the most favorable for that.

2.2 Demand Side Model

The Demand side model is divided in four parts. We start by explaining the model of the users and their electricity needs. Once this done, we define the grid load, that is, the electricity purchased by the users from the grid. The last two parts correspond to the PV generation from the solar panel and finally the CES device itself, with the constraints that it has to satisfy.

2.2.1 Users: an electricity consumption need

We consider a discrete **set of days** $\mathcal{D}$ (typically a month), where each day is identically divided into a discrete **set of time-slots** $\mathcal{T}$ (typically 24 hours). Let $\mathcal{N}$ denote the set of N residents in the network.

For each day $d \in \mathcal{D}$ and each time-slot $t \in \mathcal{T}$, user $n \in \mathcal{N}$ has a given **individual electricity consumption demand** to be satisfied, denoted by $e_n^d(t) \geq 0$. **In this section, unless we explicitly make the distinction, we continue the description for an arbitrarily given day** d**. All the definitions and properties presented hereafter hold for each day in** $\mathcal{D}$.

On day d, each user in $\mathcal{N}$ chooses to belong to one, and only one, of the following subsets of $\mathcal{N}$:

- the set of **active users** $\mathcal{A}^d$, who share electricity from the CES device which is connected to the PV panel, and buy electricity from the grid if they have a residual demand;
- the set of **passive users** $\mathcal{P}^d$, who buy all their electricity directly from the grid.

Note that, because residents can decide each day to be active or passive, these sets are day-dependent, and hence indexed by d.

In contrast to passive users, active users can trade electricity with the CES device. The quantity of **electricity traded** by user n with the CES device at time t on day d is denoted by $x_n^d(t)$. We assume by convention that the trades are made from the CES to the active users[4]. The temporal profile, $(x_n^d(t))_{n\in\mathcal{A}^d}$ for $t \in \mathcal{T}$, corresponds to the solution of an optimization problem on behalf of the CES operator[5] which allocates the electricity among the active users by an allocation rule. This model is stated in Section 3.1. The variables of electricity trading are subject to the following constraints:

$$0 \leq x_n^d(t) \leq e_n^d(t), \quad \forall n \in \mathcal{A}^d,\ \forall t \in \mathcal{T}. \tag{1}$$

The non-negativity constraint (lower bound) means that active users are discharging the CES device by obtaining electricity from it, but cannot recharge it since they do not have devices capable of reinjection, like electrical vehicles. The upper bound constraint means that active users cannot get more electricity from the battery than their current individual consumption demand at each time slot, since they do not have their own storage capacity.

2.2.2 Grid Load

The quantity of individual consumption demand to be satisfied by the grid is derived from the demand satisfied by the CES device. Indeed, a passive user's total demand is satisfied by the grid, while an active user purchases electricity from the grid in case of a deficit of supply from the CES device. Explicitly, the quantity of load that each user $n \in \mathcal{N}$ takes from the grid, called his/her **Individual Grid Load**, is denoted by:

$$\begin{aligned} \ell_n^d(t) &:= e_n^d(t) - x_t^d(t), \quad \forall n \in \mathcal{A}^d,\ \forall t \in \mathcal{T}, \\ \ell_n^d(t) &:= e_n^d(t), \qquad\qquad\quad \forall n \in \mathcal{P}^d,\ \forall t \in \mathcal{T}. \end{aligned} \tag{2}$$

The individual grid load corresponds to the electricity traded between users and the grid, and it highlights the fundamental difference between active and passive users concerning their relation with the grid. Note that from (1) we deduce:

$$\ell_n^d(t) \geq 0, \quad \forall n \in \mathcal{N},\ \forall t \in \mathcal{T}, \tag{3}$$

and for the active users it is positive if and only if the electricity taken from the CES device is not enough to satisfy their individual consumption.

The **Active and Passive Grid Loads** for time slot t on day d, defined as the aggregate loads on the grid respectively from the active and passive users and denoted by $\ell_{\mathcal{A}}^d(t)$ and $\ell_{\mathcal{P}}^d(t)$, are derived from the above individual grid loads as follows:

$$\begin{cases} \ell_{\mathcal{A}}^d(t) := \sum_{n\in\mathcal{A}^d} \ell_n^d(t), & \forall t \in \mathcal{T}, \\ \ell_{\mathcal{P}}^d(t) := \sum_{n\in\mathcal{P}^d} \ell_n^d(t), & \forall t \in \mathcal{T}. \end{cases} \tag{4}$$

Finally, let us define the **Total Grid Load** as the aggregated load from all the users:

$$L^d(t) := \ell_{\mathcal{A}}^d(t) + \ell_{\mathcal{P}}^d(t). \tag{5}$$

Remark 2.2. *The daily profile of* (5) *is fundamental regarding the management of (local) grid, which is carried out by the Distribution Network Operator (at the Medium Voltage - Low Voltage level of distribution grid). It is directly related to the losses, equipment aging (e.g. transformer, as studied in [1], [9]). The rough idea of the management is that the more "constant" this profile, the better for the grid[6]. This is typically measured using the **Peak to Average Ratio** (PAR) [11], [15], defined by*

[4]This way, a positive value of this variable means that users are getting electricity from the device.
[5]We can assume that the CES device is administrated by someone who allocates the electricity obtained by the solar panel to the active users.
[6]This is also true regarding high-level generation costs, which is out of the scope of this work.

$$\text{PAR} := \frac{\max_{t\in\mathcal{T}} L^d(t)}{\overline{L}^d(t)}, \ \textit{where } \overline{L}^d(t) := \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} L^d(t).$$

2.2.3 PV Generation: a simple forecasted daily scenario

At each time-slot $t \in \mathcal{T}$, the solar panel produces a **PV generation**, denoted by $g^d(t)$. This amount of electricity is distributed to the active users in $\mathcal{A}^d$ in order to satisfy at most their individual consumption demand $(e_n^d(t))_{t\in\mathcal{T}}$. In this work, we assume that, at the beginning of day d, the PV generation $(g^d(t))_{t\in\mathcal{T}}$ is obtained through a **forecast** based on the past data of electricity generation, and the forecast profile is known by all the users. Then, the approaches proposed in the following (Section 3) are applied *off-line* according to this forecasting.

Remark 2.3. *Obtaining an accurate forecast of local PV generation is not an easy task [10] and remains to be an important issue in practice in such systems. As is usually the case in the literature ([7, 8, 12, 13, 16]), we start by the case of perfect forecast as a benchmark. The case of noisy forecast can then be studied as an extension. In particular, running the proposed optimization/game approach several times a day (e.g., in a Model Predictive Control fashion, as suggested in [7]) could improve the decision-making in a more realistic setting that takes into account forecast errors.*

Now let us introduce the CES device in support of the PV production panel.

2.2.4 CES Device: the flexibility mean of active users

The CES device is a mean of storage of electricity. It allows saving the excess of PV production so as to use it later in case of demand from active users. Conventionally, its behavior is determined by three coefficients: the **Leakage Rate** $\alpha \in [0,1]$, the **Charging Conversion Loss** $\beta^+ \in [0,1]$ and the **Discharging Conversion Loss** $\beta^- \geq 1$. With these three parameters, the dynamics of the **Level of electricity** q of the CES is given by:

$$q^d(t) = \min\left\{\alpha q^d(t-1) + \beta^+ g^d(t) - \beta^- \sum_{n\in\mathcal{A}^d} x_n^d(t),\ Q_{\max}\right\}. \tag{6}$$

Roughly, (6) states that the level of electricity at each time slot depends on the level at the previous time slot, plus the generation through the solar panel, and less the electricity traded to the active users during that time slot, while it must not exceed the capacity of the battery Q_{max}. In particular, if

$$\alpha q^d(t-1) + \beta^+ g^d(t) - \beta^- \sum_{n\in\mathcal{A}^d} x_n^d(t) > Q_{\max},$$

i.e. after each active user has got his/her share of the electricity from the battery, there is still an excess of PV generation, then the excessive quantity cannot be stored and one sets $q^d(t) = Q_{\max}$ for the next time slot.

Remark 2.4. *Theoretically if the PV generation is bigger than Q_{Max} we just set the level of electricity in the battery as Q_{Max}, what may be seen as to trash the excess. In practice this is different since if the PV generation starts to exceed Q_{Max} is just needed to turn off the solar panel.*

Besides, the level of electricity of the CES must be nonnegative at any time, i.e. $q^d(t) \geq 0$. Note that this implies in particular that

$$\sum_{n\in\mathcal{A}^d} x_n^d(t) \leq \frac{\alpha q^d(t-1) + \beta^+ g^d(t)}{\beta^-}, \tag{7}$$

i.e. one cannot take more electricity from the CES device than the available amount.

2.3 Electricity Cost Model

Every user wants to satisfy his/her own demand of electricity consumption. The grid operator determines a per-unit price for each of the two types of users in order to satisfy their needs while covering its cost. In turn, these per-unit prices determine the cost functions of the users. Let us study the cost functions set by the grid for the two kind of users respectively.

Remark 2.5. *When we talk about grid costs we are doing it in a rough way since we are not defining an optimization problem for the grid operator to minimizes its costs.*

A passive user buys electricity from the grid at the following fixed price:

$$p_{\text{Pass}}(t) := \begin{cases} 18.5 \text{ c\$/kWh} & \text{if } t \in \text{On-Peak hours,} \\ 9.1 \text{ c\$/kWh} & \text{if } t \in \text{Off-Peak hours.} \end{cases} \tag{8}$$

The On-Peak (corresponding Off-Peak hours) are the time slots of the day where there is a higher (corresponding a lower) electricity demand from the users to the grid. For these periods with a higher electricity demand the grid charges highest prices in order to be able to cover the costs of producing and transporting all this demand. The prices in (8) as well as the On-Peak, Off-Peak hours were obtained from [4], that corresponds to the ones used in Texas for the year 2018. The On-Peak hours are given by:

- **May to October:** From 3PM to 8PM,
- **November to April:** From 6AM to 8AM and from 3PM to 8PM,

and the Off-Peak hours by the rest of the day. The cost function for passive users is thus given by

$$C_n^d(t) := p_{\text{Pass}}(t)\, e_n^d(t), \qquad \forall n \in \mathcal{P}^d,\ \forall t \in \mathcal{T}, \tag{9}$$

as $\ell_n(t) = e_n(t)$ for passive users during all time slots.

For active users, the grid operator determines per-unit prices $p^d(t)$ according to the active grid load $(\ell_{\mathcal{A}}^d(t))_{t\in\mathcal{T}}$ that they generate. Explicitly,

$$p^d(t) := \phi_t^d \ell_{\mathcal{A}}^d(t) + \delta_t^d, \qquad \forall t \in \mathcal{T},$$

where nonnegative constants $\phi_t^d, \delta_t^d \in \mathbb{R}_+$ are determined depending on the maximum and minimum grid load level reached by the users as in [7, 8]. Different from [12], we assume that active users do not incur a cost by trading electricity with the CES device. Therefore, the cost function for active users is

$$C_n^d(t) := p^d(t)\ell_n^d(t) = \left(\phi_t^d \ell_{\mathcal{A}}^d(t) + \delta_t^d\right) \ell_n^d(t), \qquad \forall n \in \mathcal{A}^d, \forall t \in \mathcal{T}. \tag{10}$$

Having the *Demand Side Model* and the *Electricity Cost Model* defined we are done with the model of the problem. The next section defines allocation mechanisms for the CES operator to split the PV generation between the active players and explain the game theory approach used in the work.

3 Game Modeling

In this section, we define three different allocation mechanisms used by the CES operator to split the PV generation between the active users and a variation to the On-Peak/Off-Peak hours given before. These mechanisms determine the variables $x_n^d(t)$ for each active user, at each hour of the day. This allows us to compute the daily bills of the users (active and passive) that correspond to the final costs derived from the decision done by each user at the beginning of the day of being active or not. Due to this, each day can be seen as a game where players decide if connecting or not to the CES device.

Once explained the allocation mechanisms, an algorithm based on the work of Cominetti, Melo, Sorin [3] and Cominetti, Dumett [5] is proposed to determine a Nash Equilibrium in mixed strategies for each day, so players decides if being active or passive.

3.1 Allocation mechanisms

In this section we define three allocation mechanisms, we explain the reasons to use each of them plus their pros and cons. Recall that these electricity allocation mechanisms are used each day, once that the set of active players is determined. After to define the three allocation mechanisms we define an alternative On-Peak/Off-Peak schedule based in the level of daily PV generation.

3.1.1 Splitting Allocation Mechanism "SAM"

The first of the three mechanisms is the *Splitting Allocation Mechanism* (SAM). The CES operator considers the available electricity (AE) at each $t \in \mathcal{T},\ d \in \mathcal{D}$ given by

$$\mathrm{AE}^d(t) := \frac{\alpha q^d(t-1) + \beta^+ g^d(t)}{\beta^-}, \tag{11}$$

and divide it into $\left|\mathcal{A}^d\right|$ equal parts to be allocated to each active user. If some of them have a demand less than their share, i.e.

$$e_n^d < \frac{\mathrm{AE}^d(t)}{|\mathcal{A}|^d} = \frac{\alpha q^d(t-1) + \beta^+ g^d(t)}{\beta^- |\mathcal{A}^d|},$$

then the CES operator allocates the excessive electricity to those whose demands e_n^d have not been satisfied yet uniformly. This process is carried on until every active user's demand is satisfied or until there is no more electricity left in the battery.

Algorithm 1 summarizes the mechanism SAM during a fixed day $d \in \mathcal{D}$. For each hour $t \in \mathcal{T}$, the while loop corresponds to the re-splitting of electricity. By considering all the active users at the beginning as players with deficit, at each iteration of this while loop the mechanism recompute the set of active players with deficit by comparing their individual consumption with the electricity obtained until the moment. The loop continues until the set of active players with deficit is empty or there is no PV generation left to split. It is important to recall that at step **7** the electricity split is added to the electricity that each player already has. Because of this it is necessary to set the variables equal to zero at the beginning of the algorithm.

Remark 3.1. *In practice line **16**, that corresponds to the criterion to end the while loop, is not enough. Numerical issues may provoke to the algorithm to loop without ending. Indeed, if there is not enough PV generation to satisfy all the active demands, the algorithm continues looping by splitting smaller and smaller remaining of PV generation because of problem with numerical precision. To avoid these problems it is enough with setting $AE^d(t)$ equal to zero once that it reaches certain small quantity ε.*

Input: Active Players $\mathcal{A}^d$, Daily PV Generation $(g^d(t))^{t\in\mathcal{T}}$, Daily Individual Consumption $(e_n^d(t))_{t\in\mathcal{T}}^{n\in\mathcal{N}}$, Initial Electricity in the Battery q_0^d

$q^d(0) \leftarrow q_0^d$, $x_n^d(t) \leftarrow 0, \forall n \in \mathcal{A}^d,\ \forall t \in \mathcal{T}$;
for $t \in \mathcal{T}$ **do**
  Compute the available electricity $\text{AE}^d(t)$ at time t using (11);
  Initialize the set of active users with deficit $\mathcal{A}_t^d \leftarrow \mathcal{A}^d$;
  while *True* **do**
    for $n \in \mathcal{A}_t^d$ **do**
      Allocate the split amount of electricity $x_n^d(t) := x_n^d(t) + \min\left\{e_n^d(t),\ \frac{\text{AE}^d(t)}{|\mathcal{A}_t^d|}\right\}$;
      if $x_n^d(t) = e_n^d(t)$ **then**
        $\mathcal{A}_t^d = \mathcal{A}_t^d \setminus \{n\}$;
      end
      else
        Redefine the individual consumption of n as his/her deficit $e_n^d(t) := e_n^d(t) - x_n^d(t)$;
      end
    end
    Compute the excess of PV generation by $\text{AE}^d(t) = \text{AE}^d(t) - \sum_{n\in\mathcal{A}_t^d} x_n^d(t)$;
    if $\mathcal{A}_t^d = \emptyset$ *or* $AE^d(t) = 0$ **then**
      End while loop
    end
  end
end

Output: Daily electricity scheduling $(x_n^d(t))_{n\in\mathcal{A}^d, t\in\mathcal{T}}$

Algorithm 1: SAM algorithm. *Routine to allocate the PV generation between active users. SAM considers re-splitting of the electricity remaining.*

We propose this mechanism since the CES operator, by utilizing SAM, is minimizing the *total cost* of the active users, i.e. she chooses a profile $x^d(t) := (x_n^d(t))_{n\in\mathcal{A}^d}$ to minimize

$$\begin{aligned}
\text{TC}(x^d(t)) &:= \sum_{n\in\mathcal{A}^d} C_{n,t}^d(x^d(t)) \\
&= \sum_{n\in\mathcal{A}^d} p^d(t)\,\ell_n^d(t) \\
&= p^d(t)\ell_{\mathcal{A}^d}^d(t) \\
&= \left[\phi_t^d \ell_{\mathcal{A}^d}^d(t) + \delta_t^d\right]\ell_{\mathcal{A}^d}^d(t) \\
&= \phi_t^d \left(\ell_{\mathcal{A}^d}^d(t)\right)^2 + \delta_t^d \ell_{\mathcal{A}^d}^d(t).
\end{aligned} \tag{12}$$

All the elements in (12) being positive, the minimum is attained for the smallest possible value of $\ell_{\mathcal{A}}^d(t)$. This corresponds to distributing all the PV generation to the active users. Clearly, the more their demands are satisfied by the CES, the less they need to buy from the grid and consequently the lower the per-unit price they are charged by the grid.

The aim of this allocation mechanism is to share in a fair[7] way the PV generation of the panel between the active users, in the sense that all of them, at the same time, reduce their bills as much as possible. The idea of

[7] In other words, fair in the sense that all the players get the same electricity from the solar panel if they cannot satisfy their demands.

re-splitting the excess of electricity between the users with higher individual consumption is to take advantage as much as possible of the PV generation. However, this re-splitting process may increase the execution time considerably as it will be observed numerically.

3.1.2 One-time Splitting Allocation Mechanism "OtSAM"

The second allocation mechanism is the *One-time Splitting Allocation Mechanism* (OtSAM). It corresponds to the allocation mechanism SAM without re-splitting, that is, to Algorithm 1 without a while loop, and therefore, making the split of electricity only once and storing all the excess of PV generation for the next hour.

Even though it can be thought that this mechanism does not minimize *Total Cost* (12), numerical results show that the final bills obtained for the users with this mechanism are similar to the ones obtained with SAM. The intuition behind this is that, for a small solar panel with a low PV generation, in reality there is not re-splitting of electricity since this one is all distributed in one iteration, while for a big solar panel with a high PV generation, all the individual consumptions are quickly satisfied thanks that the electricity given to each active user at the first iteration of the while loop of Algorithm 1 cover almost all or eventually all the demand, making no big difference between having or not re-splitting. This discussion is continued in Section 4.

3.1.3 Proportional Allocation Mechanism "PAM"

The third and last allocation mechanism is the *Proportional Allocation Mechanism* (PAM). In this case, and as the name suggests, the CES operator distributes the available electricity in a proportional way considering the individual active consumption. Given the set of active players $\mathcal{A}^d$, each active player $n \in \mathcal{A}^d$ receives:

$$x_n^d(t) := \min\left\{ e_n^d(t),\ \mathrm{AE}^d(t) \cdot \frac{e_n^d(t)}{\sum_{m\in\mathcal{A}^d} e_m^d(t)} \right\}, \tag{13}$$

where AE is defined in (11). The minimum in (13) is considered to satisfy constraint (1). As Section 4 shows, this mechanism achieves as good results as the SAM and OtSAM in bills term. However, there is an important issue to consider. When active users have the faculty to announce their individual consumption $(e_n^d(t))_{t\in\mathcal{T}}$, for example at the beginning of the day, this kind of proportional mechanism has the problem that users have incentives to lie by announcing a higher individual consumption than their real need with the aim of receiving more electricity and to reduce their individual bills not truly telling. In order to avoid this problem, it is important to carefully develop an allocation mechanism with a proportional splitting rule.

3.1.4 Solar On-Peak/Off-Peak hours and Varied SAM "VSAM"

Finally, we propose alternative On-Peak/Off-Peak hours than the ones in Section 2.3 (but with the same values). The grid operator in charge of selling the lack of electricity to the users may have the incentive to make them consuming more electricity when there is a higher PV production. This way, the users may take full advantage of the solar PV generation and only consume electricity from the grid the days with a very low PV generation. Because of this, we define new On-Peak and Off-Peak hours according to the following rule:

$$\forall d \in \mathcal{D}, \forall t \in \mathcal{T}, \begin{cases} t \in \text{On-Peak hours, if } g^d(t) \leq \frac{1}{|\mathcal{T}|} \sum\limits_{t'\in\mathcal{T}} g^d(t'), \\ t \in \text{Off-Peak hours, otherwise.} \end{cases}$$

This way, an hour $t \in \mathcal{T}$ belongs to the set of On-Peak hours if and only if the PV generation at that moment is lower than the average PV generation of the day. Considering the solar On-Peak/Off-Peak hours we define the variation of SAM (VSAM).

3.2 Daily Game

In this section we give the normal form of the daily game where players decide if being active or passive and obtain a daily bill, being their costs in this game. Consider a fixed day $d \in \mathcal{D}$, we define the N-persons game:

$$\begin{aligned}\Gamma_d &:= \left(\mathcal{N}, (\mathcal{S}_n)_{n\in\mathcal{N}}, \left(\overline{C}_n(d)\right)_{n\in\mathcal{N}}\right), \\ \mathcal{N} &:= \text{Set of users}, \qquad (14)\\ \mathcal{S}_n &:= \{\text{Active}, \text{Passive}\}, \forall n \in \mathcal{N}, \\ \overline{C}_n(d) &:= \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} C^d_{n,t}(x^d(t)), \forall n \in \mathcal{N}. \qquad (15)\end{aligned}$$

Each player's payoff function is defined as his/her average stage cost during that day. The variables $x^d = (x^d(t))_{t\in\mathcal{T}}$ with $x^d(t) = (x^d_n(t))_{n\in\mathcal{A}^d}$ are determined by the CES operator with one of the allocation mechanism presented previously: SAM, OtSAM, PAR or VSAM. Then, the functions $C^d_{n,t}(\cdot)$ are obtained with (9) and (10).

Remark 3.2. *The daily game Γ_d is different for each day since it depends on the daily PV generation $\{g^d(t)\}_{t\in\mathcal{T}}$ and the individual consumption $\{(e^d_n(t))_{t\in\mathcal{T}}, n \in \mathcal{N}\}$ of the players that day. Recall that players cannot decide how much electricity to get from the CES device and for them the game only consists in deciding if being connected or not to the CES device during the day d.*

With a finite number of players and a finite number of time slots, each game Γ_d is a finite game so it has at least a mixed Nash Equilibria. As Remark 3.2 mentions the daily game is parameterized by $\{g^d(t)\}_{t\in\mathcal{T}}$ and $\{(e^d_n(t))_{t\in\mathcal{T}}, n \in \mathcal{N}\}$ and therefore, the equilibria depends on the day.

In this work, we are interested in taking *optimal* decisions from the point of view of the grid operator[8] to fix the size of the solar panel, the allocation mechanism and the viability of a daily ticket to be paid by the users to become active. We study these objectives under the assumption that players are playing on equilibrium. For that, we propose an algorithm to compute a Nash Equilibrium in mixed strategies each day. This will allow to assess the performance of each allocation mechanism and the computation of the solar panel size when players take their decisions under the probabilities defined by the equilibrium. Section 3.3 describes the algorithm to compute this Nash equilibrium.

3.3 Computation of a daily Nash Equilibrium

At the beginning of each day the players have to decide if being active or passive. To solve this, we consider each day independently and compute a N.E. in mixed strategy for each day. To compute this strategy, we use the idea of Cominetti, Melo, Sorin in [3] and Cominetti, Dumett in [5] with their learning mechanism. It is important to point out that in our case the player do not learn how to play during the month, rather the learning algorithm is a way to calculate a Nash Equilibrium in each daily game Γ_d.

Remark 3.3. *The reason to solve each day separately is due to the learning mechanism of [3] and [5] is well defined only for repeated games where at each iteration the players have to play exactly the same stage game. However, in Contextual Learning exist learning mechanisms where the stage game changes from one iteration to another. This may be a good extension to our work since it can deal with the different settings that each day presents, given by the daily PV production and the individual consumption of the users.*

We present now the mechanism to solve each daily game Γ_d. In order to do it, we make the players to play the daily game several times until to have convergence of the probabilities to the equilibrium. From now, let

[8]Other interpretations of this role would be possible: city operator, building manager, etc.

$d \in \mathcal{D}$ be a fixed day of the month. For each player $n \in \mathcal{N}$, we define an initial score of being active and passive during day d respectively by:

$$u_{n,d}^{\text{Act}}(0) := \textbf{Score} \text{ of player } n \text{ of being } \textbf{active}, \tag{16}$$

$$u_{n,d}^{\text{Pass}}(0) := \textbf{Score} \text{ of player } n \text{ of being } \textbf{passive}. \tag{17}$$

The players update this scores until they converge to a fixed value. For this, they start to play the daily game repeatedly and to observe the payoff obtained at each iteration. Formally, let $r \in \mathbb{N}$ be an iteration, that is, suppose the players have played the same daily game $r-1$ times. Let $u_{n,d}^{\text{Act}}(r-1), u_{n,d}^{\text{Pass}}(r-1)$ be the scores of player n at the previous iteration $r-1$. The update is done following the rule (4) of Section 2 in article [5]:

$$u_{n,d}^{\text{Act}}(r) = \begin{cases} (1-\gamma_r^d)u_{n,d}^{\text{Act}}(r-1) + \gamma_r^d \overline{C}_{n,d}^{\text{Act}}(r-1) & \text{if } n \in \mathcal{A}_d^{r-1}, \\ u_{n,d}^{\text{Act}}(r-1) & \text{if } n \in \mathcal{P}_d^{r-1}, \end{cases} \tag{18}$$

$$u_{n,d}^{\text{Pass}}(r) = \begin{cases} u_{n,d}^{\text{Pass}}(r-1) & \text{if } n \in \mathcal{A}_d^{r-1}, \\ (1-\gamma_r^d)u_{n,d}^{\text{Pass}}(r-1) + \gamma_r^d \overline{C}_{n,d}^{\text{Pass}}(r-1) & \text{if } n \in \mathcal{P}_d^{r-1}, \end{cases} \tag{19}$$

where γ_r^d can by any sequence such that

$$\sum_{r \geq 1} \gamma_r^d = \infty \text{ and } \sum_{r \geq 1} (\gamma_r^d)^2 < \infty, \tag{20}$$

for example $\gamma_r^d = 1/r$, $\overline{C}_n^{\text{Act}}(r-1)$ and $\overline{C}_n^{\text{Pass}}(r-1)$ correspond to the last daily payoff observed by player n, and $\mathcal{A}_d^{r-1}, \mathcal{P}_d^{r-1}$ are the set of active and passive players respectively of the daily game at the $r-1$ iteration.

Intuitively, at each iteration r, players decide if being active or passive with the settings of day d, that is, with the same PV generation and individual consumption of day d for all the iterations. Once decided the set of active players, using one of the allocation mechanisms of Section 3.1, the electricity is split and the average costs $\overline{C}_n$ are computed. Note that at each iteration, each player updates one and only one of the scores depending if he/she was active or passive at that current iteration. The score of the option not taken remains constant for the next iteration.

Remark 3.4. *The fact that players only update the scores with the payoffs observed makes them to play a repeated game with incomplete information since they only know their current cost but not the ones of the other players. Moreover, players cannot know the possible costs that they would have got if they had chosen the other option in previous iterations. In particular, this lack of information gives to the users the privacy of not having to reveal their personal needs to the others.*

Equations (18) and (19) define a recurrence and for that, Equation (16) and (17) work as initial cases. Usually these initial scores are set equal to zero.

Before explaining how players compute their mixed strategy at each iteration, let us mention an interesting case of the updating rule corresponding to consider $\gamma_r^d = 1/r$. Let $j \in \{\text{Act}, \text{Pass}\}$ be a fixed strategy. Suppose that for $r' < r$ we have

$$u_{n,d}^j(r') = \frac{1}{r'} \sum_{r'' < r'} \overline{C}_{n,d}^j(r''), \tag{21}$$

that is, for all the previous iterations the scores correspond to the average of the daily payoffs. Then,

$$\begin{aligned} u^j_{n,d}(r) &= \left(1 - \frac{1}{r}\right) u^j_{n,d}(r-1) + \frac{1}{r}\overline{C}^j_{n,d}(r-1) \\ &= \left(\frac{r-1}{r}\right) u^j_{n,d}(r-1) + \frac{1}{r}\overline{C}^j_{n,d}(r-1) \\ &= \frac{1}{r}\left[(r-1)u^j_{n,d}(r-1) + \overline{C}^j_{n,d}(r-1)\right] \\ &\overset{(21)}{=} \frac{1}{r}\left[(r-1)\left(\frac{1}{r-1}\right)\sum_{r'<r-1}\overline{C}^j_{n,d}(r') + \overline{C}^j_{n,d}(r-1)\right] \\ &= \frac{1}{r}\left[\sum_{r'<r-1}\overline{C}^j_{n,d}(r') + \overline{C}^j_{n,d}(r-1)\right] \\ &= \frac{1}{r}\sum_{r'<r}\overline{C}^j_{n,d}(r'), \end{aligned}$$

where the fourth step is by Equation (21). Since clearly the initial scores (16), (17) satisfy (21), by induction we conclude that the scores correspond to the average payoffs over all the previous iterations.

Remark 3.5. *Note that in (21) we are considering all the past costs obtained by player n for a fixed strategy, in other words, the payoffs obtained by the player if all the iterations was active or passive. In reality this not necessary happens so for those iterations r that we do not have a cost defined, because for example n was passive so he/she does not have an active cost at iteration r, it is enough to set this missed cost as the score of that strategy in the iteration $r-1$. This remark is purely a clarification to well define the computation just done.*

To conclude this section, we explain how the players compute their mixed strategy at each iteration based in their scores. For each player $n \in \mathcal{N}$ and iteration $r \in \mathbb{N}$, we define his/her mixed strategy as the Logit Probability

$$\mathbb{P}\left(n \in \mathcal{A}^r_d\right) := \; q^r_{n,d} \; := \frac{\exp\left(-\eta u^{\text{Act}}_{n,d}(r)\right)}{\exp\left(-\eta u^{\text{Act}}_{n,d}(r)\right) + \exp\left(-\eta u^{\text{Pass}}_{n,d}(r)\right)}, \tag{22}$$

$$\mathbb{P}\left(n \in \mathcal{P}^r_d\right) := \; 1 - q^r_{n,d} \; := \frac{\exp\left(-\eta u^{\text{Pass}}_{n,d}(r)\right)}{\exp\left(-\eta u^{\text{Act}}_{n,d}(r)\right) + \exp\left(-\eta u^{\text{Pass}}_{n,d}(r)\right)}, \tag{23}$$

where $\eta \geq 0$ and it is called the **Rationality coefficient**.

Remark 3.6. *Note that for $\eta = 0$ the Logit probability is uniform and therefore it is equiprobable being active or passive.*

This mechanism allows to the players to adapt their mixed strategies according to the payoffs observed during each iteration. Thanks to condition (20) over the sequence $(\gamma^d_r)_r$, we always have convergence of the scores and therefore, of the probabilities. After a sufficiently large number of iterations, the mixed strategy found for each player define a probability distribution over the set of strategies (being active or passive) and making a realization with this Nash Equilibrium independently between the users we can define the set of active players to then find the effective distribution of electricity and finally the cost of each user. Solving a day d with this algorithm we pass to the day $d+1$ and we continue until the end of the month. In the following algorithm we summarize the steps to find the monthly electricity scheduling.

Input: PV Generation $(g^d(t))_{t\in\mathcal{T}}^{d\in\mathcal{D}}$, Individual Consumption $(e_n^d(t))_{d\in\mathcal{D},t\in\mathcal{T}}^{n\in\mathcal{N}}$

for $d \in \mathcal{D}$ **do**
- $q_{n.d}^0 \leftarrow 1/2, u_{n,d}^{\text{Act}}(0), u_{n,d}^{\text{Pass}}(0) \leftarrow 0, \forall n \in \mathcal{N}$;
- **for** $r \in \mathbb{N}, r \leq r_{Max}$ **do**
  - **for** $n \in \mathcal{N}$ **do**
    - Compute the last daily cost $\overline{C}_{n,d}^{\text{Act}}(r-1)$ or $\overline{C}_{n,d}^{\text{Pass}}(r-1)$ using Equation (15) depending if the player was active or passive at iteration $r-1$ respectively;
    - Update the scores according to Equation (18) and (19) and compute $q_{n,d}^r$ using Equation (22);
    - Choose to be active or passive according to $(q_{n,d}^r)$;
    - Find the CES allocation $\hat{x}_d^r(t) := (\hat{x}_{n,d}^r(t))_{n\in\mathcal{A}_d^r}$, $\forall t \in \mathcal{T}$ with one of the allocation mechanisms of Section 3.1;
  - **end**
- **end**
- Solve the daily game using $q_{n,d}^{r_{\text{Max}}}, \forall n \in \mathcal{N}$ as their mixed strategies and one of the allocation mechanisms of Section 3.1. Find the daily electricity schedule $(x_n^d(t))_{n\in\mathcal{N},t\in\mathcal{T}}$ and compute the daily bills of the users;

end

Output: Monthly electricity scheduling $(x^d)^{d\in\mathcal{D}} := (x_n^d(t))_{n\in\mathcal{N},t\in\mathcal{T}}^{d\in\mathcal{D}}$

Algorithm 2: Monthly electricity scheduling algorithm. *Algorithm to compute a Nash Equilibrium in mixed strategies for each day and afterwards split the PV generation between the active users using a SAM, OtSAM, PAM or VSAM defined in Section 3.1.*

Remark 3.7. *Usually the most common stopping criteria used for this kind of algorithms are:*

$$\|u_{n,d}^j(r) - u_{n,d}^j(r-1)\| \leq \varepsilon \text{ or } \sum_{r'' \leq r' < r} \|u_{n,d}^j(r') - u_{n,d}^j(r'-1)\| \leq \varepsilon, \forall j \in \{Act, Pass\} \tag{24}$$

that is, the convergence of the scores to a particular point or the convergence of the tail of the series of the difference between two consecutive elements. Thanks to condition (20) we have that the sequence $(\gamma_r^d)_{r\geq 0}$ *converges to* 0 *and therefore the sequences of scores always satisfies the stopping criteria (24). It is because of this than it is enough with running the while loop a certain number of iteration* r_{Max}.

Remark 3.8. *The anonymity and lack of information of the users discussed in Remark 3.4 goes even further. For a given day, since the way that players update their scores, they do not even need to know their own individual consumption thanks that they only need to know their daily cost at the end of each iteration. This gives the option to the grid operator to take care of the individual consumption of the users. For this we mean that each day, knowing the total daily consumption of each users, the operator can distribute as he/she wants the electricity during the hours in order to give at the end of the day to each user the demand that they request, without respecting really the consumption than the users need at each hour, giving this way a more flexible nature to our electricity schedule.*

4 Numerical Study

In this section, we show and discuss the results obtained using the three allocation mechanisms of Section 3.1 plus VSAM that corresponds to a variant of the SAM considering the solar On-Peak/Off-Peak hours. We discuss their advantages to determine the best choice to find a scheduling of electricity in a longer time horizon.

The PV data can be found with open access on the *www.renewables.ninja* platform. We use the PV generation data of Texas during 2014. The individual consumption data was obtained from *www.pecanstreet.org* platform, which provides open access to students. The data used correspond to the individual consumption in Texas during the year 2016 and they represent the entire hourly consumption of the players, not discriminating by the different appliances that they can have.

4.1 Parameters, Self-Production and Self-Consumption

Besides the data obtained from the platforms, we consider the parameters given in Table 1.

Table 1: Summary of parameters used in the simulation.

α	β^+	β^-	Q_0	$Q_{\max}$	η
1	1	1	0 kW/h	10 kW/h	1

By considering $\alpha = \beta^+ = \beta^- = 1$ we are using a battery with perfect performance that stores all the electricity from the end of an hour to the beginning of the next one and it does not have loss of electricity by discharging or charging.

Before solving the problem, we have to determine the coefficients $\phi_t^d, \delta_t^d, \forall t \in \mathcal{T}, \forall d \in \mathcal{D}$ that determine the electricity price function of the active users. To determine them we follow the idea of [8]. Taking the aggregated consumption over the users during the considered month we define the **Maximal Aggregated Load** (MAL) and **Minimum Aggregated Load** (MIL) of consumption by

$$\text{MAL} := \max \left\{ \sum_{n \in \mathcal{N}} e_n^d(t) : \forall t \in \mathcal{T}, \forall d \in \mathcal{D} \right\},$$

$$\text{MIL} := \min \left\{ \sum_{n \in \mathcal{N}} e_n^d(t) : \forall t \in \mathcal{T}, \forall d \in \mathcal{D} \right\}.$$

Unlike [8], we only need to determine two coefficients for the unit price function and therefore it is enough with the maximum and the minimum aggregated load. Assigning these values to the On-Peak and Off-Peak prices given in [4] and considering a linear regression we can find the unit price function needed for the community. Table 2 shows different unit price function parameters using this calibration method.

Table 2: Unit price function. *Unit price coefficients for different communities.*

Number of users	10	25	50	75	100
ϕ (c\$/kWh)	0.34	0.21	0.13	0.1	0.08
δ (c\$/kWh)	7.67	5.99	4.61	3.17	2.22

Note how the price coefficients decrease with the number of users due to the interpolation and the fact that with more users, larger are the MAL and the MIL. Obviously this is a consequence of the linear regression and the fact that for bigger communities the MAL increases. However, for the cases of big communities it is correct to have small unit price coefficients since considering large active grid loads the prices between active and passive players will be comparable. In other words, if the unit price coefficients are big for a large community, the price to pay for being active will be much more than by being passive, due to the individual consumption of each user have the same magnitude independently of the size of the community.

Among the objectives of this work we have mentioned the computation of the optimal size of the solar panel, however a criterion to evaluate this choice must be defined. Clearly a solar panel as big as possible is the best from the point of view of the users since that way they are able to cover most of their individual consumption with "free" electricity[9]. Since the point of view of the grid operator it may be enough if the players reach a certain level of the individual consumption from the solar panel.

To continue this discussion, note that the PV generation from the data corresponds to a solar panel of $1m^2$. Then, assuming that a solar panel of Mm^2 generates M times the energy of one of $1m^2$, the aim is to determine the optimal value of M. For this, we first define two key concepts in electricity models with PV generation. Fix $d \in \mathcal{D}$ and let $\mathcal{A}^d$ be the set of active players during the day d. Consider a profile of split electricity $x^d := (x_n^d(t))_{t\in\mathcal{T},n\in\mathcal{A}^d}$. We define the **Daily Self-Production** (DSP) by

$$\mathrm{DSP}\left(x^d\right) := \frac{\sum\limits_{t\in\mathcal{T}}\sum\limits_{n\in\mathcal{A}^d} x_n^d(t)}{\sum\limits_{t\in\mathcal{T}}\sum\limits_{n\in\mathcal{N}} e_n^d(t)}, \tag{25}$$

that is, the part of the aggregate consumption of all the users that is covered by the electricity obtained from the CES device. Note that constraint (1) implies that SP ≤ 1. A self-production equal to 1 means that users are getting all the electricity needed from the PV generation.

Remark 4.1. *Since the Daily Self-Production considers the PV generation consumed by the active users, it may be more logical to define this concept by just considering the active individual consumption, that is, just taking the sum over $\mathcal{A}^d$ and not over all $\mathcal{N}$ in the denominator. However since we want to use the Self-Production to determine the size of the solar panel for a given community, we opt by to consider the entire individual consumption.*

The second concept is the **Daily Self-Consumption** (DSC). Given the same setting than before we define

$$\mathrm{DSC}\left(x^d\right) := \frac{\sum\limits_{t\in\mathcal{T}}\sum\limits_{n\in\mathcal{A}^d} x_n^d(t)}{\sum\limits_{t\in\mathcal{T}} g^d(t)}, \tag{26}$$

that is, the proportion of electricity consumed from the PV generation. A daily self-consumption of 1 implies that the active users are consuming all the PV generation of the day d. Also, since the CES device is able to provide electricity to active players, the self-consumption may be bigger than 1 in some days.

The PV generation depends on the size of the solar panel that the community has. We assume that if g corresponds to the generation of a solar panel of $1m^2$, then Mm^2 generate M times g electricity. Under this assumption, we can compute the size of the solar panel M needed to achieve a certain level of Self-Production

[9] Note that when we say "'free" electricity we are not considering installation and transportation costs, the ones can be charged by a daily ticket.

desired. First, we need the monthly versions of Self-Production and Self-Consumption[10] defined by

$$\mathbf{SP}\left((x^d)_{d\in\mathcal{D}}\right) := \frac{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} \sum\limits_{n\in\mathcal{A}^d} x_n^d(t)}{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} e_n^d(t)}, \quad \mathbf{SC}\left((x^d)_{d\in\mathcal{D}}\right) := \frac{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} \sum\limits_{n\in\mathcal{A}^d} x_n^d(t)}{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} g^d(t)},$$

and let us suppose that

$$\sum_{t\in\mathcal{T}} g^d(t) \leq \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} e_n^d(t), \forall d \in \mathcal{D}, \tag{27}$$

i.e. that every day the daily PV generation is lower than the total electricity needed by the users. Since all the allocation mechanisms defined in Section 3.1 give as much electricity as possible to the users, by assumption (27) every day it is achieved a daily self-consumption equal to one. In particular, the monthly self-consumption is also equal to one (consider the aggregation of constraint (27) over the month) and then we can express the monthly self-production by:

$$\mathrm{SP} = \frac{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} g^d(t)}{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} e_n^d(t)}. \tag{28}$$

Considering a PV generation of $Mg^d(t)$ for a solar panel of size M, replacing this in (28), we obtain the monthly self-production:

$$\mathrm{SP} = M \cdot \frac{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} g^d(t)}{\sum\limits_{d\in\mathcal{D},t\in\mathcal{T}} \sum\limits_{n\in\mathcal{N}} e_n^d(t)} \tag{29}$$

Knowing the monthly unit generation of electricity and the individual consumption of the users, Equation (29) determines the optimal size of the solar panel to achieve a desired average level of daily self-production. Table 3 summarizes different sizes applying Equation (28) depending on the level of monthly self-production desired for a community of 25 users, considering 24 time slots by day.

Table 3: Solar panel size versus monthly Self-Production for a community of 25 users.

Self-production	0%	10%	25%	33%	50%	75%	90%	100%
Size (m^2)	0	50	126	166	252	378	453	504

Table 4 on the other hand shows the size of the solar panel needed to achieve a 25% of self-production depending on the number of users with 24 time slots by day.

Table 4: Solar panel size needed to achieve 25% of monthly self-production.

Number of users	10	25	50	75	100
Size (m^2)	47	126	253	378	504

[10] For the monthly versions of self-production and self-consumption we just denote them SP and SC. Note that the daily and monthly versions are evaluated in different arrays.

Remark 4.2. *The size of the solar panel does not affect only to the PV production, but also it affects directly to the energy received by the active users. Indeed, since the Available Energy (11) depends on the generation $g^d(t)$, the part of PV production that each active user receives is bigger and therefore, with a larger solar panel it is easier to satisfy the individual consumption of the users.*

Remark 4.3. *More than having the condition (27), the important part for this computation is to have a daily self-Consumption equal to one every day, or even more weakly, a monthly self-consumption equal to one. These assumptions are key for the formula (29) to provide a good estimation of the right PV panel size, since for a monthly self-consumption bigger than one the energy is not properly used. Note that if (27) is not satisfy then there is an excess of PV generation for the community and therefore active users can achieve bills almost zero. This last scenario does not have true interest since it is very rare in practice.*

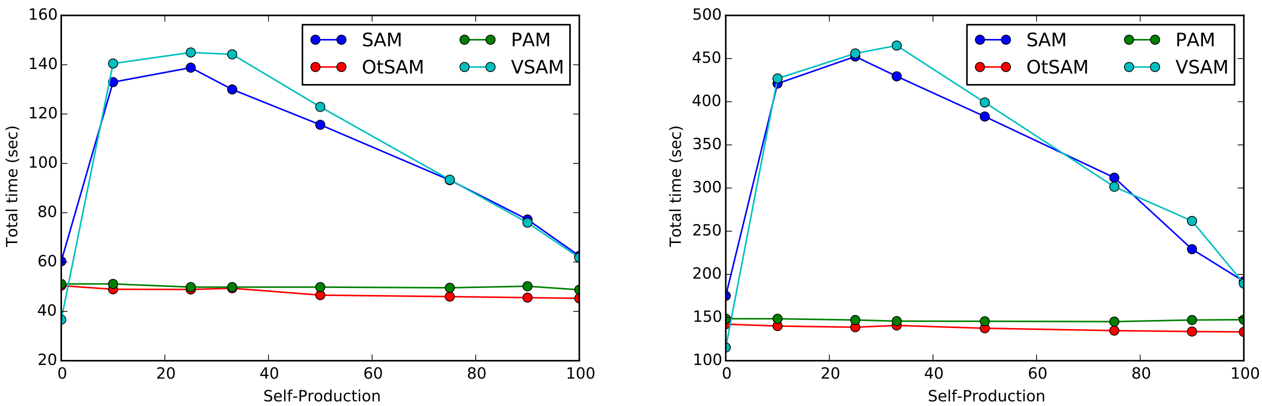
In the following section, we analyze the accuracy of (29) and the issues discussed in Remark 4.3.

4.2 Numerical Results

We compare four mechanisms defined in Section 3.1. For each of them, we have solved the electricity scheduling for January 2016, considering 24 time slots for each day and running $r_{\text{Max}} = 100$ times each daily game in Algorithm 2. We considered 5 different communities of 10, 25, 50, 75 and 100 users. Besides, for each of these communities, we consider the 8 levels of self-production showed in Table 3, that is, 0%, 10%, 25%, 33%, 50%, 75%, 90% and 100%.

4.2.1 Execution time

We start by showing the execution time of each mechanism. This corresponds to the time taken to compute the equilibrium for each day and then the final electricity scheduling. Figure 2 shows the execution time (in seconds) taken by the four mechanisms in finding the monthly electricity schedule, for the community of 25 users (Figure 2a) and 75 users (Figure 2b), for the 8 levels of monthly self-production considered, i.e. the self-production as in Equation (29). Besides these two cases, for the other three communities the plots have the same shape and can be seen in the Appendix.



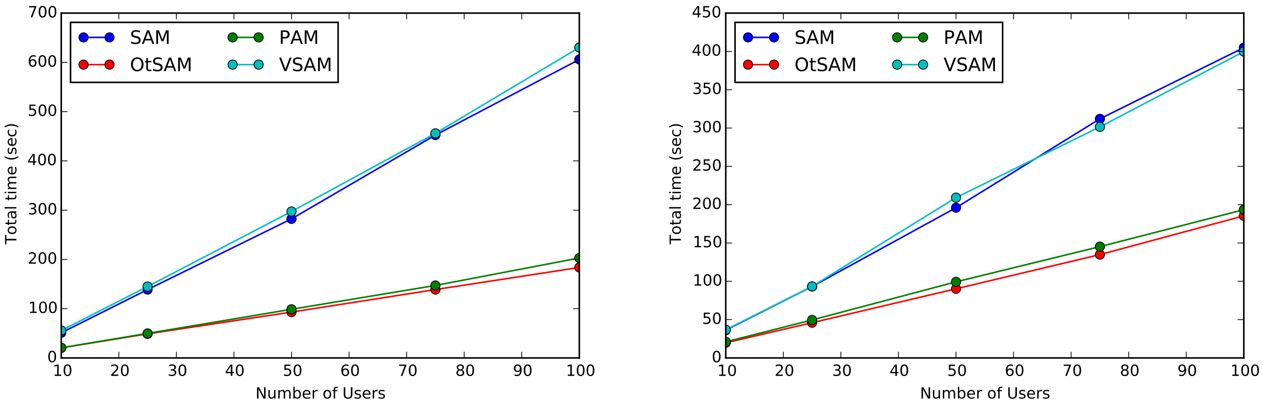
(a) Community of 25 users

(b) Community of 75 users

Figure 2: Execution time versus self-production, January 2016, 24 time slots. *For both communities, the execution time of PAM and OtSAM is practically constant, while for the other two allocation mechanisms it depends strongly on the level of self-production desired, being considerably higher for small levels.*

We can observe two principal behaviors: On the one hand the execution times of OtSAM and PAM are practically constant independently of the size of the solar panel used, for both figures. On the other hand, SAM and VSAM execution times are initially increasing then decreasing with the solar panel size. This difference is due to the re-splitting that SAM and VSAM do: indeed for high levels of self-production desired, since there is a large PV generation, the re-splitting is not necessary due to the mechanisms are able to cover all the individual consumption in few iterations. For a small solar panel able to just achieve 10%-33% of monthly SP, the re-splitting is done several times until all the PV generation of each hour is given to the users, needing more iterations to finish. As for OtSAM and PAM the allocation is done in a unique iteration; this explains why they take the same time to solve the problem independently of the PV generation available.

To complement these figures, Figure 3 shows the execution time (in seconds) of the four mechanisms versus the number of users for two different levels of monthly self-production desired: 25% and 75%.



(a) 25% of Self-production (b) 75% of Self-production

Figure 3: Execution time versus number of players, January 2016, 24 time slots. *For both communities, the execution time is linearly increasing with the number of users. Due to the re-splitting, SAM and VSAM have a higher slope.*

Once again, we observe how OtSAM and PAM take considerably less time in solving the problem than SAM and VSAM, and the reason is the same than before, the re-splitting presented in the first two mechanism. This difference in time resolution is smaller in Figure 3b than in Figure 3a since a higher PV production available makes the number of re-splits needed to decrease.

It is also interesting to observe the linear behaviour of the execution time versus the number of players. Due to for bigger communities the iteration over the users are costlier in time, it takes more time to assign the energy at each instant.

It is important to have in mind that when we talk about a fixed level of monthly self-production, we are using different sizes of solar panels for different communities, as Table 4 shows. Due to this, at each instance we are increasing the amount of electricity available at each hour and the number of players that are going to receive this energy, in a proportional way. This proportional increase results in the linear shape that we observe in Figure 3, being a higher slope in the case of SAM and VSAM because of the re-splitting.

4.2.2 Monthly Bills

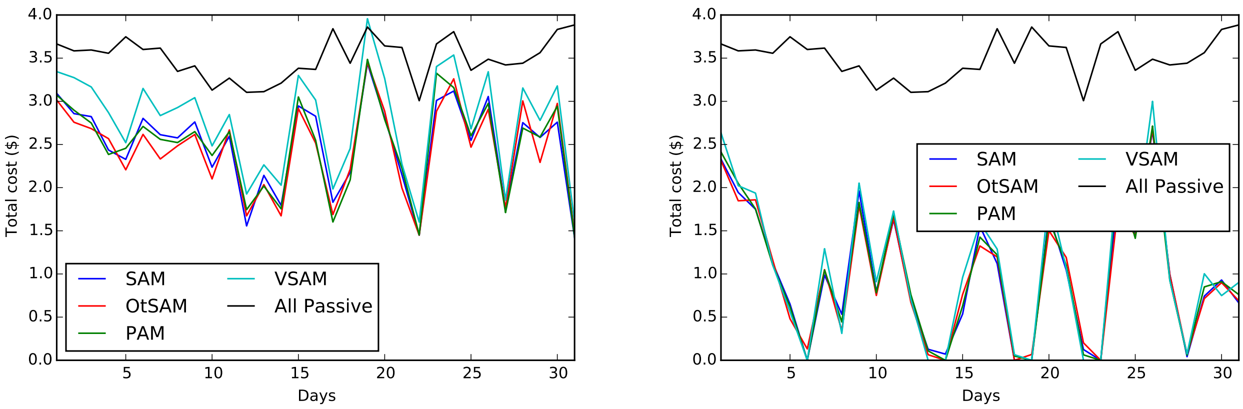
The second criterion to compare the mechanisms is the monthly bills paid by the users. Similarly to (12), we define two concepts: The *Total Cost* of all the users on day $d \in \mathcal{D}$ by:

$$\mathrm{TC}^d := \sum_{n \in \mathcal{N}} \overline{C}_n(d) = \frac{1}{|\mathcal{T}|} \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} C_n^d(t), \tag{30}$$

and the *Total Bill* of all the users on day $d \in \mathcal{D}$ by:

$$\mathrm{TB}^d := |\mathcal{T}| \cdot \sum_{n \in \mathcal{N}} \overline{C}_n(d) = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} C_n^d(t) = |\mathcal{T}| \cdot \mathrm{TC}^d, \tag{31}$$

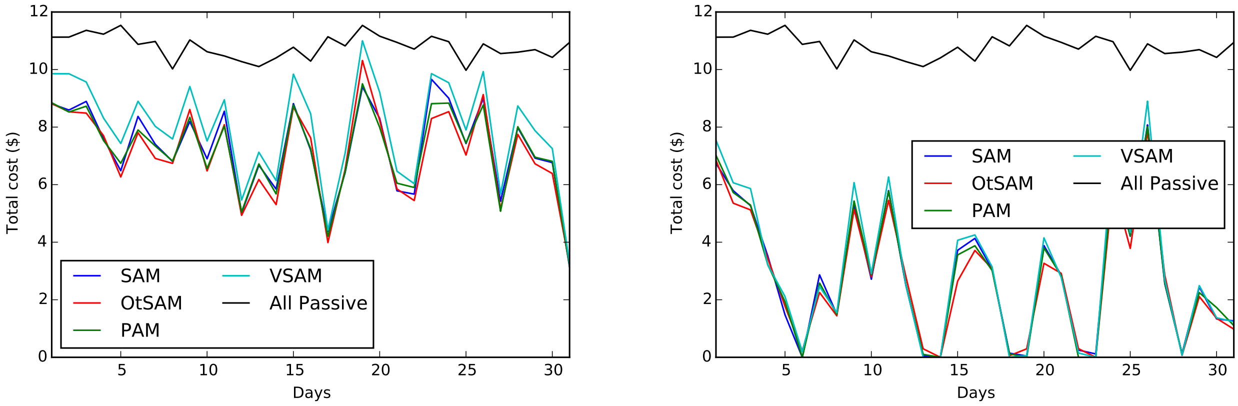
For each $d \in \mathcal{D}$, the Total Cost corresponds to the aggregated costs (15) of all the users during the day, while the Total Bill corresponds to the aggregated bills of the users during the day. In practice, these two concepts just differ by a factor of $|\mathcal{T}|$. Both depend strongly on the electricity scheduling made by the different allocation mechanisms. We compare the Total Cost achieved by the four mechanisms at equilibrium. We consider the same cases than before, two communities of 25 and 75 consumers both with monthly self-production levels of 25% and 75%. More plots are provided in the Appendix. Besides the Total Cost of each mechanism, we show the cost obtained if all the players decide to be passive. Figure 4 corresponds to the small community and it shows how the four mechanisms achieve better daily costs than if all the players were passive, with clearly better results for the bigger solar panel Figure 4b, since the community is able to cover a higher percentage of the consumption with solar energy and therefore, to buy less from the grid.



(a) 25% of Self-production (b) 75% of Self-production

Figure 4: Total daily cost achieved for the four allocation mechanisms, compared with the case of all the users being passive, during January 2016, $N = 25$ users. *SAM, OtSAM and PAM obtain similar costs, while VSAM has higher bills especially for a small solar panel.*

As Figure 5 shows, for the community of 75 users we obtain similar results. There are two important aspects to note. First the fact than the total costs are proportional to the size of the community, that is, the costs obtained for the community of 75 users are of the magnitude of three times the costs obtained by the small community, what it means than independently of the number of users, if the size of the solar panel is sufficient, the average daily costs of each user will be similar.



(a) 25% of Self-production (b) 75% of Self-production

Figure 5: Total daily cost achieved for the four allocation mechanisms, compared with the case of all the users being passive, during January 2016, $N = 75$ users. *SAM, OtSAM and PAM obtain similar costs, while VSAM has higher bills especially for a small solar panel.*

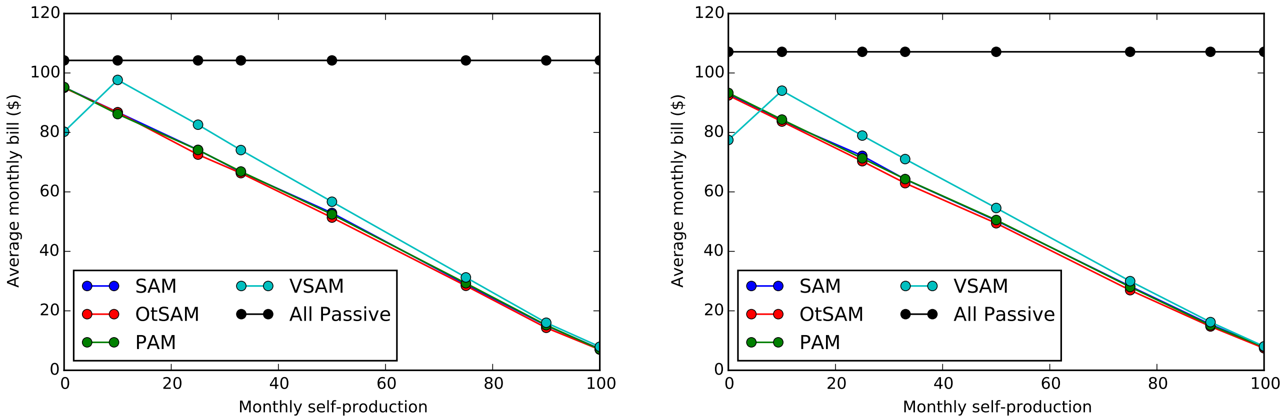
The second important aspect to mention is related to the performance taking total costs as a metric of each mechanism. SAM, OtSAM and PAM achieve really similar performances, that considering in addition the execution time gives an advatage to OtSAM and PAM. As for VSAM, it is always located between the other mechanisms and the total cost of all the users being passive. In addition to the execution time, it discards VSAM from being a useful mechanism to solve the electricity scheduling problem in this context.

The reason why VSAM achieves higher daily costs is not because it is not using optimally the PV generation, as discussed in the following section, but the strategy of changing the On-Peak/Off-Peak hours according to the PV generation is useless if we do not give the faculty to the users to have flexible demands, and then to have a higher consumption during the hours with more PV generation. If we only change the On-Peak/Off-Peak hours and we use the same demands than in the normal case, we are just charging the users higher prices exactly in the moments that they usually consume more, which unluckily are often also the moments with lowest PV generation.

Besides the total cost we present the average monthly bill that a user achieve with each mechanism, and we compare this with the case when all players are passive. We continue with the same instances: Figure 6a shows a community of $N = 25$ users while Figure 6b a community of $N = 75$ users.

Note how in both figures the users get the same monthly bills independently of the number of users of the community. Since for each community we use a solar panel sufficiently big to achieve the level of self-production desired, without matter the size of the community each player will get the same average bill at the end of the month. Figure 7 shows the same result than before but now versus the number of players in the community and fixing the level of self-production desired.

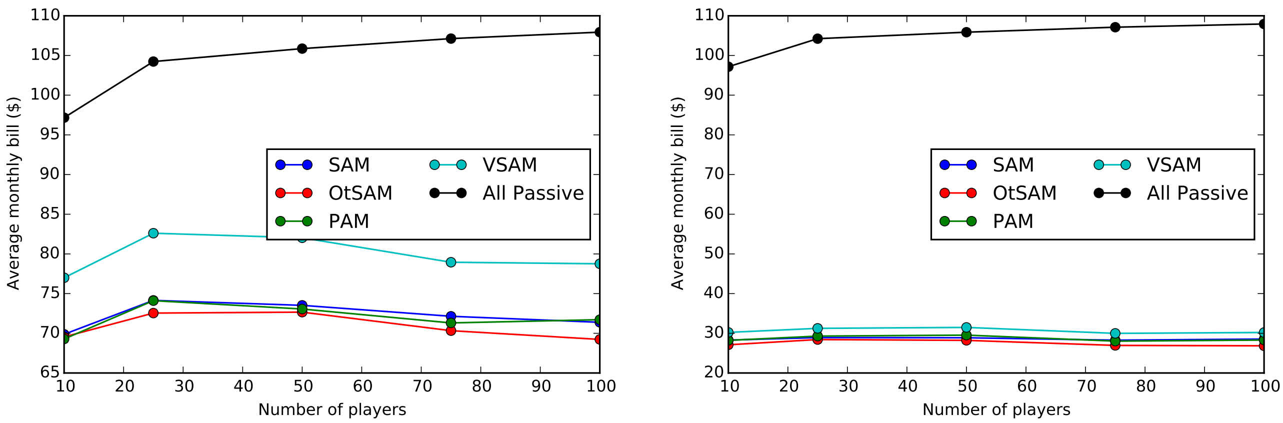
We observe how the monthly bill than in average a user obtains is very similar independently of the size of the community, just as we mentioned before, being this bill clearly lower for a bigger self-production. It is interesting to note that using the proper solar panel for a given community, similar final bills are achieved. This allow to study small communities to compute final prices before of implementing cases with larger amount of users.



(a) $N = 25$ users

(b) $N = 75$ users

Figure 6: Average daily bill of a user versus the size of the solar panel during January 2016. *Each player obtains a decreasing monthly bill with the size of the solar panel thanks to the bigger PV generation available. Independently of the size of the community in average the players obtain the same monthly bill.*



(a) 25% of self-production

(b) 75% of self-production

Figure 7: Average daily bill of a user versus the size of the community during January 2016. *The player obtain a decreasing monthly bill with the size of the solar panel thanks to the bigger PV generation available.*

Finally, we compare the mechanisms under a last criterion, the level of monthly self-production reached and the Peak Grid Load.

4.2.3 Self-Production and Peak Grid Load

The last two criteria to compare the mechanisms are the monthly self-production effectively achieved and the Peak Grid Load. Recall that we study these results always at equilibrium obtained for that with Algorithm 2. As discussed in Remark 2.2, it is generally preferable for the grid load profile as much constant as possible, and in order to analyze this, the metric of Peak to Average Ratio (PAR) is usually considered. PAR is defined by:

$$\text{PAR}^d := \frac{\max_{t\in\mathcal{T}} L^d(t)}{\overline{L}^d(t)}, \text{ where } \overline{L}^d(t) := \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} L^d(t) \text{ is the average grid load.} \tag{32}$$

Remark 4.4. *Note that the definition of PAR that we use consider the total aggregated load, that is, the Active load and the Passive load. Considering passive users in the formula may increase the values of the numerator and denominator of (32).*

Unfortunately, there are two disadvantages of the PAR. First, for high levels of PV generation it is not a good metric. Since there are days with enough PV production to cover almost all the demands of the active users, the Average Grid Load and the Peak Grid Load may differ considerably even if both correspond to small values, obtaining finally a large PAR.

The second disadvantage is the one showed in Figure 8. In black, we observe the PAR reached without PV generation, while the other curves correspond to the different allocation mechanisms. For each curve there is also a dashed lines corresponding to the average value.

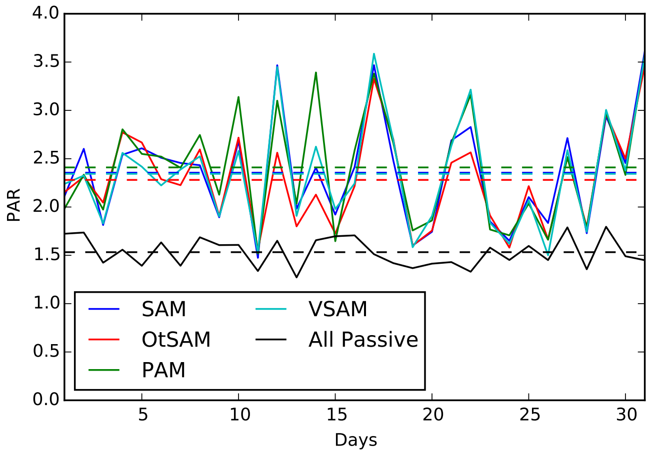
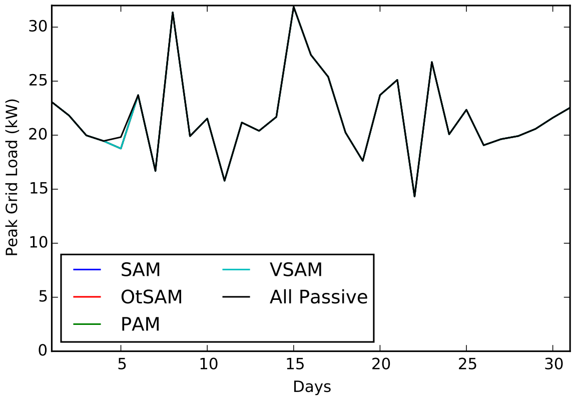


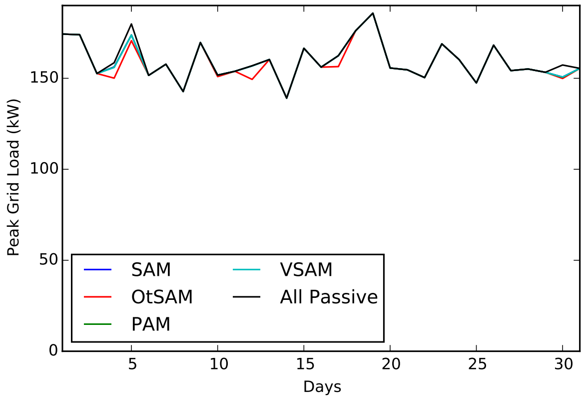
Figure 8: Example of PAR as a defective metric. $N = 50$ users, 33% of monthly self-production. *Although the Peak Grid Load and the Average Grid Load decrease with PV generation, the PAR is bigger than the case without solar energy.*

Clearly the presence of solar energy reduces the *Peak Grid Load* (numerator of (32)) and the *Average Grid Load* (denominator of (32)) since the users can cover part of their demands with PV "free" electricity. However, reducing both at the same time may increase the ratio between them, as we observe in Figure 8. To complement this analysis we study the *Peak Grid Load* reached by each mechanism. For that we show four different instances:

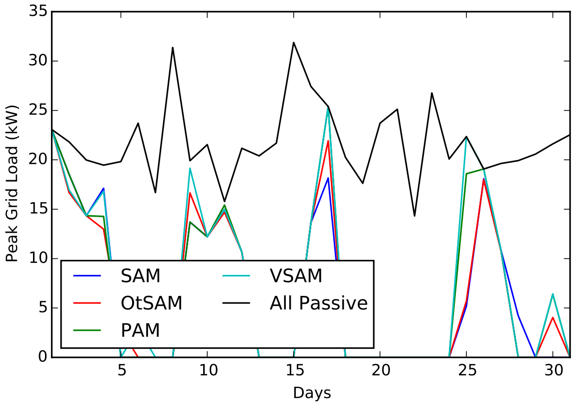
1. $N = 10$ users with 10% of monthly self-production in Figure 9a,
2. $N = 100$ users with 10% of monthly self-production in Figure 9b,
3. $N = 10$ users with 90% of monthly self-production in Figure 9c,
4. $N = 100$ users with 90% of monthly self-production in Figure 9d.



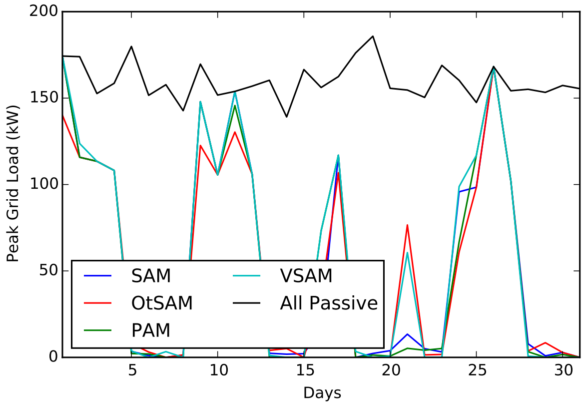
(a) $N = 10$ Users, 10% of monthly Self-production



(b) $N = 100$ Users, 10% of monthly Self-production



(c) $N = 10$ Users, 90% of monthly Self-production



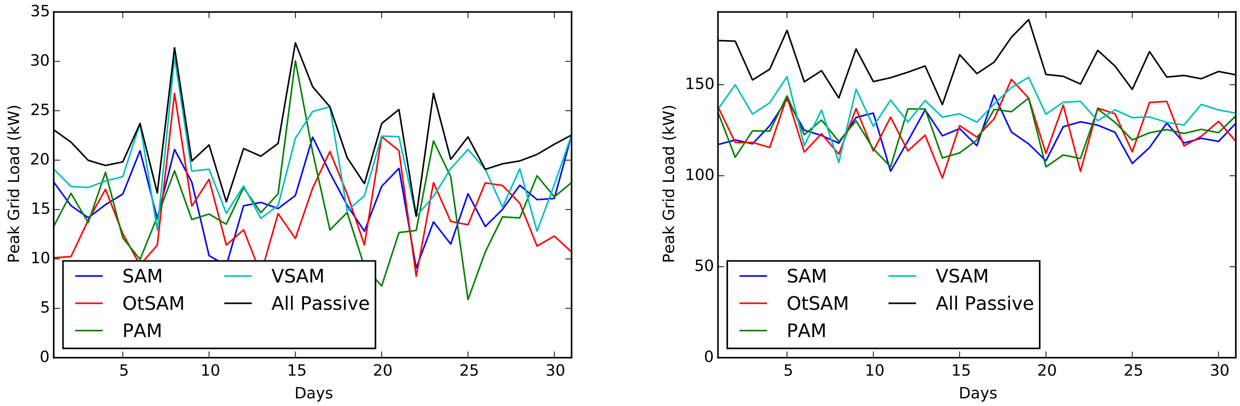
(d) $N = 100$ Users, 90% of monthly Self-production

Figure 9: Peak Grid Load (numerator of (32)) achieved by the four allocation mechanisms compared with the case without PV generation. *The allocation mechanisms have similar results between them. For some days the mechanisms coincide with the case without PV generation. For a small solar panel the PGL is almost equal to when all the players are passive.*

In Figure 9 we observe the Peak Grid Load (PGL) achieved each day by the four mechanisms, plus the one if all the users were passive. We observe how even with a huge PV generation able to cover 90% of the demand (Figure 9c and Figure 9d), there exist days with a PGL as big as the case without generation. There are two reasons for this phenomenon. First, since players decide if being active or passive through mixed strategies, it may happen that the user who attain the PGL exactly the same day decided to be passive, making to coincide the PGL with the case without PV generation. The second reason to observe a PGL of the mechanisms equal to the one without PV generation is the form of the data. During the month exist some days with a low PV production so that, even for a big solar panel, the PV generation is not enough to reduce de PGL.

For communities with a small solar panel as Figure 9a and Figure 9b, the PGL can be equal to the one without PV generation. One important observation of why this happens is the presence of passive players as

Remark 4.4 mentions. These players do not get energy from the solar panel and therefore their individual grid loads are equal to their individual demands. Figure 10 shows the PGL achieved just by the active users in the same setting than Figure 9a and Figure 9b. We observe now a decrease of the PGL in comparison with the case without PV generation.



(a) $N = 10$ Users, 10% of monthly Self-production

(b) $N = 100$ Users, 10% of monthly Self-production

Figure 10: Active PGL achieved by two communities with 10% of monthly self-production. *Considering just active load the mechanisms reduce the PGL compared with the aggregated case.*

Although this may be an argument in favor of the allocation mechanisms, from the point of view of the grid operator the entire grid load and not just the active one is important and therefore, in terms of Peak Grid Load none of them is particularly better then the others.

The final result that we show in this section is the self-production effectively achieved by each mechanism. Table 5 presents the results obtained by SAM different instances. Since the four allocation mechanisms achieve almost the same results we only present here the numbers for SAN and we leave the other three tables to the Appendix.

Table 5: Self-Production achieved by SAM. *For monthly self-production (Equation (29)) bigger than* 50% *the accuracy between effective monthly SP values - inside of the table - and the monthly SP desired - top row of the table - decreases.*

SAM		Level of monthly Self-Production desired							
		0%	10%	25%	33%	50%	75%	90%	100%
Number of users (N)	10	0	10.3	25.4	32.9	48.3	70.9	84.1	92.5
	25	0	9.9	25.1	32.7	48.7	71.3	84.6	92.7
	50	0	10.1	25.1	32.2	48.1	70.9	83.9	91.6
	75	0	10.1	25.1	32.5	48.2	70.9	84.3	91.9
	100	0	10.1	24.9	32.3	47.9	70.7	83.9	91.5

We can test the accuracy of Equation (29) and continue the discussion of Remark 4.3. For a small solar panel able to achieve no more than a 50% monthly self-production, Equation (29) has good results and the four mechanisms achieve, in average, the desired self-production. However, for bigger sizes, the difference between monthly SP used for choosing the PV panel and effective monthly SP values achieved increases due to assumption (27) is not always satisfied, having almost a 9% of error in the biggest case.

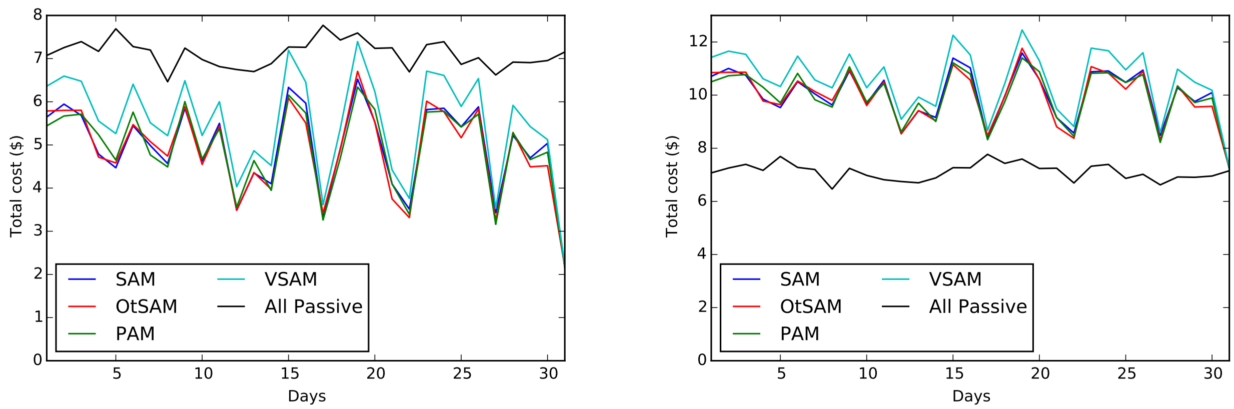
4.3 Daily Ticket

Clearly, from the point of view of the users, the best for the community is to have the biggest solar panel possible in order to reduce as much as possible their bills. However, this is only true if the investment cost (and installation) is zero for them. Until now these costs have been considered as if the PV panel was already installed in the building or house. In reality, the grid operator may charge a small tariff to the users for using the solar panel, which in our game model is represented by an extra cost to be paid by players to become active at the beginning of each day. Determining the value of this *daily ticket* is fundamental for the grid operator since a too high price may induce all the players deciding more often being passive, and a too small value may induce not enough financial resources to cover the investment costs.

To determine the daily ticket, we consider a solar panel *SW 250 Poly*[11] of $1.61m^2$ at 290\$, from the German brand Solar World[12]. Considering a life of 25 years we can compute[13] the daily price of $1m^2$ of this solar panel obtaining a daily ticket of $2c\$$ per square meter. This daily ticket is shared uniformly between all the active users, therefore at the beginning of each day $d \in \mathcal{D}$ all of them have to pay the extra cost:

$$\text{Daily Active Ticket} := \frac{M}{|\mathcal{A}^d|} \cdot 2c\$,$$

with M the size of the solar panel. Let us consider a community of $N = 50$ users with a solar panel of $M = 253m^2$, enough to achieve a 25% of self-production. Figure 11 shows the total cost obtained with the four mechanisms considering and not considering the daily ticket, at equilibrium.



(a) Without daily ticket

(b) With daily ticket

Figure 11: Monthly costs $N = 50$ Users, 25% of monthly Self-production (29). *The daily ticket results to be too costly compared with the monthly bills achieved thanks to the solar panel. Charging this daily ticket without a subvention will provoke all the users to be passive all the days.*

Because of the high price of the square meter of solar panel, the total cost of the users is significantly increased if we consider the daily ticket, making at the end more convenient for them to be passive every day

[11] www.siliconsolar.com/documents/solarworld-sw250-250-watt-poly-solar-panel-datasheet.pdf.
[12] www.solarworld.de/en/home/.
[13] For this computation we do not consider an interest rate reducing the future value of the solar panel.

and not participate in the community connected to the solar panel. This phenomenon was exactly the mentioned before when the grid's operator imposes a daily ticket too expensive. To effectively charge a daily ticket to the users first the operator must to reduce it somehow, for example, paying her a part of the solar panel.

It is important to have in mind that these issues must be considered before to installing the solar panel and designing the community rules. If an electricity company wants to propose to a community the installation of such a solar panel and then the service like the ones modeled here, it must offer to the users to pay part of the daily ticket in order to make this service attractive and useful.

5 Conclusion and further work

As mentioned at the beginning of this report, the study of smart grids has become more and more relevant in the last years thanks to the multiple advantages that it may imply. In our case, we have seen how the presence of solar energy in a community of consumers can be used to cover individual or aggregated demands and thanks to that, to reduce the bills of the users.

To utilize a CES device in a community, several issues must be addressed, as the sizing of the solar panel, the mechanism to distribute the PV generation between the users and from the point of view of the electricity company the definition of a good tariff to the users for using this solar energy or give it them for free, among others.

We have studied how to determine these three objectives in a Game Theory setting. We have proposed a model to analyze these three objectives, modeling mathematically the situation of a community where each user can decide, at the beginning of each day, if being connected or not to the CES device during that day, a device that works as a storage of the PV production obtained from a solar panel. Besides this model, the same section presents an *Electricity Cost Model* where the cost functions of connected users (active users) and non connected users (passive users) are defined based on the decisions of all the users. On the one hand the passive users confront a non-flexible cost model that charge them a fixed unit price for the electricity. On the other hand, the active users have a flexible cost model in which the unit price is fixed depending on the total net load consumed by these active players from the grid.

Section 3 defines four allocation mechanisms for the CES operator to split the energy between the active users connected to the device. Then considering each day separately it defines a *finite game* for each of them where all the users have only two strategies: being "active" or "passive". Finally, this section proposes an algorithm based in the work of Cominetti, Melo, Sorin in [3] and Cominetti, Dumett in [5] to compute a Nash Equilibrium in mixed strategies for each daily game.

To determine the size of the solar panel for a given community two key concepts are *Self-Consumption* and *Self-Production*. The first one expresses the portion of PV generation effectively consumed by the users, and the second one shows the portion of demand covered by this PV generation consumed. Having a self-consumption close to one allows us to compute the size of the solar panel needed to achieve a given level of monthly self-production and this way, to the grid operator to determine how big has to be the solar panel for a given community depending on how much energy want to be covered by PV generation.

Numerically we have compared the allocation mechanisms showing their Execution Times for different communities with several sizes of solar panel, besides the aggregated costs achieved by each of them as well as the Peak Grid Load (PGL). We also studied the option of charging a daily ticket to the active users for connecting to the CES device each day. It is important to state that all these results are under the assumption that players play on equilibrium. The comparison of allocation mechanisms as well as the computation of optimal sizes of solar panel out of equilibrium are out of the scope of this work. A sensibility analysis of players acting out of the Nash Equilibrium can be done as an extension of this work.

All the results show a clear advantage from OtSAM and PAM over SAM and VSAM. The first two mechanisms, unlike SAM and VSAM, does not have re-splitting of electricity which reduces considerably the execution time for all the instances tested. This is not the only advantage, since in spite of the intuition that SAM should provide smaller bills thanks to the re-splitting, the results got by SAM, OtSAM and PAM are almost the same. The PGL values obtained are comparable with the ones in a case without PV panel. The PGL does not change the results obtained until this point.

As a summary of these numerical tests, the more useful allocation mechanisms for the CES operator seems to be OtSAM and PAM. There is only one important issue to consider about PAM. Since the players obtain a portion of PV generation proportional to their individual consumption, they have incentives to announce higher demands than their real needs in order to reduce their bills. Because of this, to use such a proportional allocation mechanism it is important to design it carefully in order to avoid this kind of untruthful behaviour from the users.

The implementation of a daily ticket is more delicate. Adding a daily ticket proportional to the number of active players and the investment cost in the PV panel may make all the users decide to be passive due to this additional cost. In order to implement this idea, the CES operator has to finance part of the daily ticket in order to make attractive to the users to become active.

There are several ways to continue this work. As mentioned before, a sensibility analysis can be done to study the results of the mechanisms out of equilibrium. Besides, other ways to allocate the PV generation can be proposed as well as other algorithms to compute the Nash Equilibria. The way that users update their scores each day to compute their strategies was chosen for the speed of convergence that has the algorithm and the good results that Cominetti, Melo, Sorin in [3] and Cominetti, Dumett in [5] have proved. However, there is no use of information to update the scores. For example, a fully informed version can be designed where users update both scores at each iteration by knowing the active users at each iteration and computing their possible costs if choosing the other option. Partially informed algorithms also can be also designed, where users get information only from their neighbours, giving relevance to the topology of the network and obtaining results depending of it.

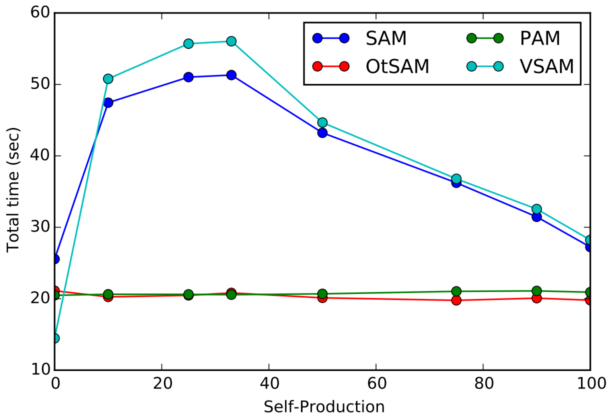
Besides the options mentioned, it is important to implement a forecast for the PV generation. As discussed in Remark 2.3 this is an essential but difficult task in this kind of work. An inaccurate forecast may mean a decreased performance of the scheduling decisions taken in the model and therefore it is important to carefully design this part. A good idea to try could be the one proposed in [7].

6 Appendix

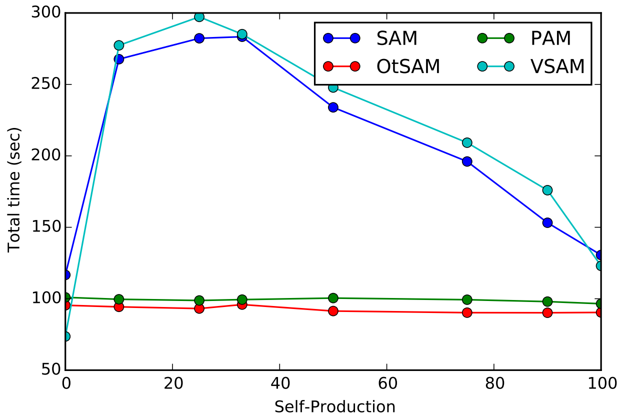
In this Appendix we show complementary figures and tables for Section 4.

6.1 Execution Time

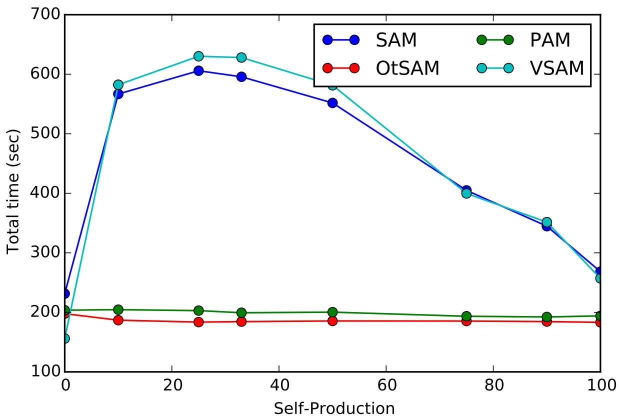
We start showing the Execution Time figures for the communities with 10, 50 and 100 users.



(a) Community of 10 users



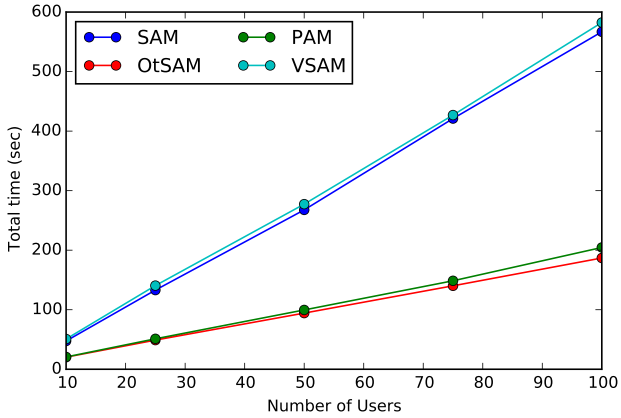
(b) Community of 50 users



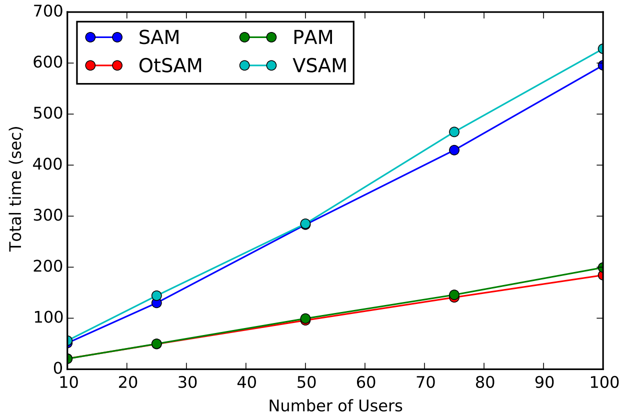
(c) Community of 100 users

Figure 12: Execution time versus self-production, January 2016, 24 time slots

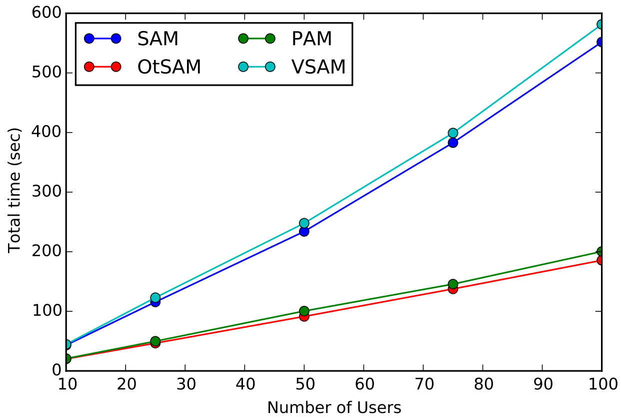
We observe how the mechanisms have the same behavior independently of the size of the community. SAM and VSAM takes considerable more time than OtSAM and PAM for low levels of self-production due to the re-splitting of PV generation. This difference decreases with bigger levels of self-production. We also obtain the same behaviour in the figures of Execution Time versus number of users. We show the cases with 10%, 33%, 50%, 90% and 100% self-production.



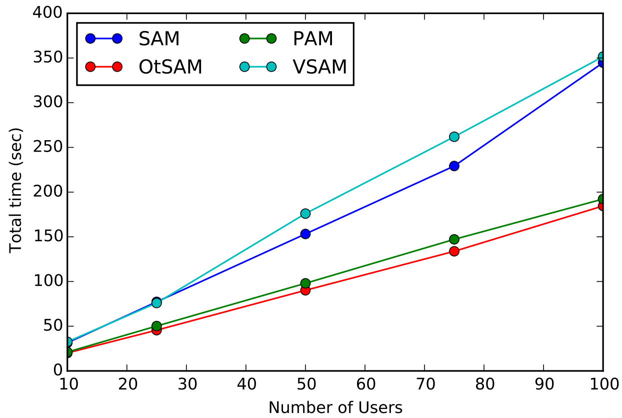
(a) 10% of Self-production



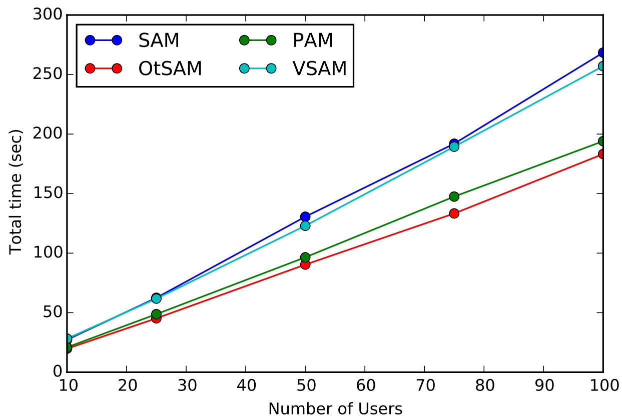
(b) 33% of Self-production



(c) 50% of Self-production



(d) 90% of Self-production

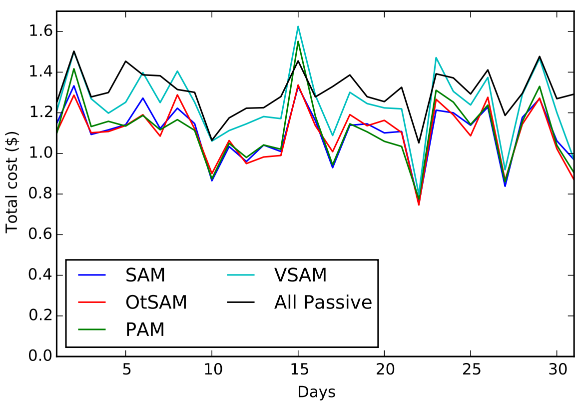


(e) 100% of Self-production

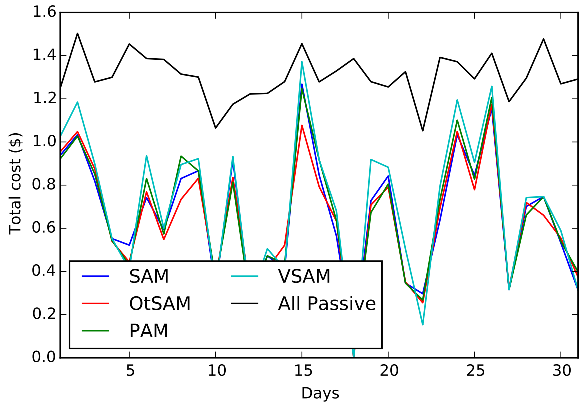
Figure 13: Execution time versus number of players, January 2016, 24 time slots

6.2 Monthly Bills

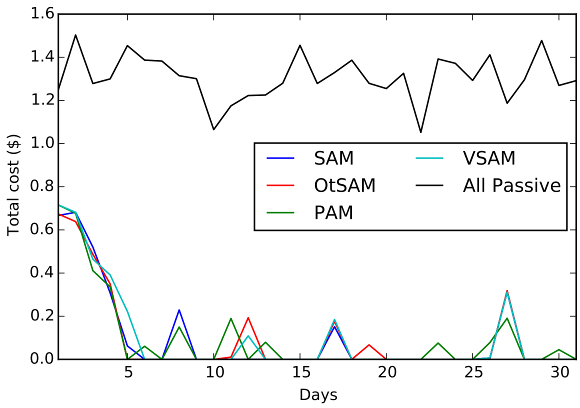
Here can be found the monthly bills for the communities of 10 and 50 users, each on them with 10%, 50% and 100% of self-production.



(a) 10% of Self-production



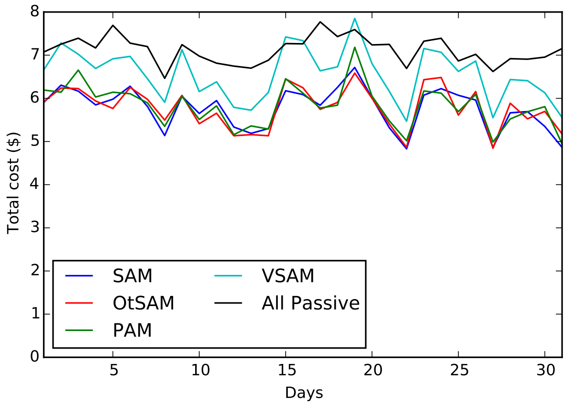
(b) 50% of Self-production



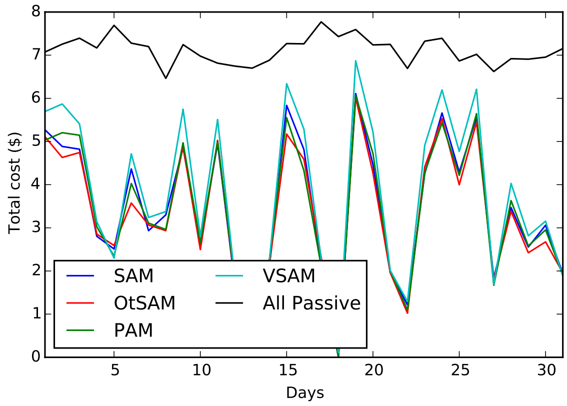
(c) 100% of Self-production

Figure 14: Total cost during January 2016, 10 users

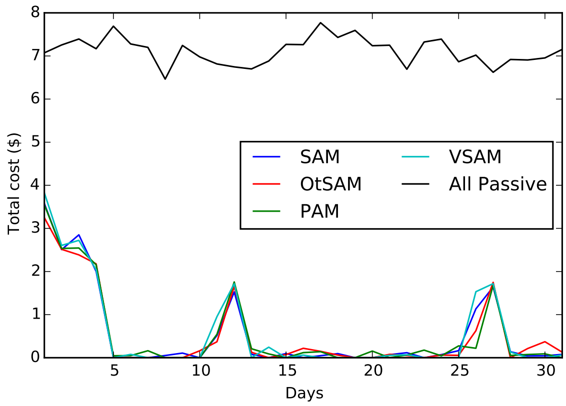
Once again we obtain the results showed for the communities of 25 and 75 users. The four mechanisms achieve similar results, being VSAM the one with worst performance. Except for particular days and for very small levels of self-production, the four mechanisms attain a better bill than the case where all the users are passive.



(a) 10% of Self-production



(b) 50% of Self-production



(c) 100% of Self-production

Figure 15: Total cost during January 2016, 50 users

6.3 Self-Production

Finally we show the tables corresponding to the self-production achieved by OtSAM, PAM and VSAM.

Table 6: Self-Production achieved by OtSAM, all the instances.

OtSAM		Level of Self-Production							
		0%	10%	25%	33%	50%	75%	90%	100%
Number of Users	10	0	10.3	25.3	32.5	48.1	70.7	84.1	92.2
	25	0	9.9	25.1	32.8	48.6	71.2	84.6	92.9
	50	0	10.0	25.1	32.5	48.9	70.9	83.7	91.4
	75	0	10.0	25.0	32.5	48.2	71.0	84.1	91.9
	100	0	10.0	25.0	32.3	47.9	70.7	83.8	91.7

We can see how the four mechanisms have similar results. As discussed in Section 4.2.3 our way to compute the solar panel size to achieve a given self-production level works independently of the allocation mechanism, and its accuracy depends only in having a self-consumption equal to one.

Table 7: Self-Production achieved by PAM, all the instances.

PAM		Level of Self-Production							
		0%	10%	25%	33%	50%	75%	90%	100%
Number of Users	10	0	10.3	25.3	32.9	48.0	70.9	84.4	92.3
	25	0	9.9	25.1	32.7	48.7	71.3	84.6	92.7
	50	0	10.0	25.1	32.4	48.1	70.9	83.9	91.6
	75	0	10.0	25.1	32.4	48.2	70.9	84.1	91.9
	100	0	10.0	25.0	32.3	47.9	70.7	83.9	91.6

Table 8: Self-Production achieved by VSAM, all the instances.

VSAM		Level of Self-Production							
		0%	10%	25%	33%	50%	75%	90%	100%
Number of Users	10	0	10.3	25.4	32.9	48.4	70.9	84.4	92.5
	25	0	10.0	25.1	32.9	48.7	71.2	84.6	92.8
	50	0	10.0	25.1	32.5	48.1	70.9	84.0	91.7
	75	0	10.1	25.1	32.6	48.2	71.0	84.3	92.1
	100	0	10.0	25.0	32.4	47.9	70.7	84.1	91.7

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