Title of the course: Approximate independence in II_1 factors
Instructor: Sorin Popa
Meetings: to be determined

The course is built around the following theme: Given a II_1 factor M, a subalgebra $B \subset M$, a finite set of "special" elements $F \subset M$, construct elements $u \in \mathcal{U}(B)$ that are "as independent as possible" with respect to F, i.e., words with alternating letters in $\{u^k \mid k \in \mathbb{Z}, k \neq 0\}$ and in F, should have moments close to zero, $\tau(x_0 u^{k_1} x_1 u^{k_2} ...) \approx 0$, $x_i \in F$, $k_i \neq 0$. But the fact that x_i are "constraint" to be in F and u in B may force that only part of this is possible, resulting in a set $\mathcal{R}$ of "achievable goals" (e.g., "all letters x_i from F within a given word must be distinct", or "all k_i must be positive", or "length of words must be ≤ 6") This can often be viewed as an "approximate embedding" ("simulation") of an algebra Q, known by generators and relations, into $B \subset M$. As it turns out, the solution to some special cases of this innocent-looking problem has important applications to various areas of von Neumann algebras, such as cohomology theory, subfactor theory, orbit equivalence, or paving (Kadison-Singer) type problems.

Our scope is to develop a technique to solve such problems, called *incremental patching*, and to discuss applications.

Here are some particular cases that we will cover:

• **Free semigroup independence.** Given any finite set of unitaries $F \subset M$, any diffuse abelian von Neumann algebra $A \subset M$, any n and $\varepsilon > 0$, one can find Haar unitaries $u \in A$ such that $|\tau(u^{i_0} v_1 u^{i_2} v_2 ... v_{2k-1} u^{i_{2k}})| < \varepsilon$, $\forall 1 \leq k \leq n$, $i_0 \geq 0$, $i_1, ..., i_k > 0$. We will use this to derive vanishing cohomology results for II_1 factors.

• **Bernoulli independence.** Given any group Γ, one can "simulate" the Bernoulli action $\Gamma \curvearrowright \mathbb{T}^\Gamma$ inside any free ergodic p.m.p. action $\Gamma \curvearrowright X$. We will relate this to various problems in orbit equivalence ergodic theory.

• **Independence in MASAs**. Given any MASA $A \subset M$, any $F \subset M \ominus A$ and any $\varepsilon > 0$, there exist Haar unitaries $u \in A$ such that $|\tau(u^{i_1} x_1 .... u^{i_k} x_k)| \leq \varepsilon$, $\forall k \leq 3$, $0 < |i_j| \leq \varepsilon^{-1}$, $x_j \in F$. Moreover, if A is a singular MASA, then this holds true $\forall k$. We will relate this to Kadison-Singer type paving problems.

• **Independence in subfactors**. Given a subfactor $N \subset M$ and a finite set $F \subset M \ominus (N' \cap M)$, construct diffuse subalgebras $Q \subset N$ that are free independent to F. We'll show how this leads to the axiomatization of standard invariants in Jones theory of subfactors, through amalgamated free product *reconstruction*, as well as to the fact that free product of "hyperfine" standard invariants is hyperfine.

The course will be rather elementary and self-contained, but basic knowledge in operator algebra and II_1 factors is assumed.