MOMENTS, POSITIVE POLYNOMIALS & OPTIMIZATION

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In this advanced short course we consider the *global* optimization problem $\mathbf{P}$: $f^* = \inf\{f(\mathbf{x}) : \mathbf{x} \in \mathbf{K}\}$, with $\mathbf{K} = \{\mathbf{x} : g_j(\mathbf{x}) \geq 0,\ j = 1, \ldots, m\}$, and where the data $(f, g_1, \ldots, g_m)$ are polynomials (extension to semi-algebraic functions is also possible). In particular, $\mathbf{P}$ includes the important class of non convex quadratic as well as discrete 0/1 optimization problems.

In fact, Polynomial Optimization can be viewed as a particular instance (and even the simplest instance) of the *Generalized Moment Problem* (GMP) and the methodology described in this short course also applies for solving the GMP whose list of applications is endless and is concerned with many different fields, including algebraic geometry, optimization, signal & image processing, probability & statistics, computational geometry, control & optimal control, etc.

To address problem $\mathbf{P}$ (and more generally to address the GMP) we invoke powerful results from Real Algebraic Geometry (RAG) and their dual counterparts in Functional Analysis concerned with the so-called $\mathbf{K}$-moment problem. In this short course we will describe:

- Main results on Positive Polynomials, and in particular *(positivity) certificates* for a polynomial to be positive on a compact semi-algebraic set $\mathbf{K} \subset \mathbb{R}^n$.
- The dual counterparts of positivity certificates, the $\mathbf{K}$-moment problem.
- How to use such results for solving the polynomial optimization problem $\mathbf{P}$ via solving a *hierarchy* of *linear* or *semidefinite programs* of increasing size.

If time permits we will also describe how to use those positivity certificates and the above methodology in some applications outside optimization, viewed as instances of the GMP. For example: In computational geometry for computing the volume of a compact semi-algebraic set, in probability for providing bounds on measures with (finitely many) prescribed moments, in optimal control.

Basic knowledge of Linear Programming (LP) and Semidefinite Programming (SDP) is desirable but not mandatory.

References

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[2] **J.B. Lasserre**. *An Introduction to Polynomial & Semi-Algebraic Optimization*, Cambridge University Press, Cambridge, UK, 2015.

[3] **M. Laurent**. Sums of squares, moment matrices and optimization over polynomials. In *Emerging Applications of Algebraic Geometry*, Vol. 149 of IMA Volumes in Mathematics and its Applications, M. Putinar and S. Sullivant (eds.), Springer, pages 157–270, 2009. `http://homepages.cwi.nl/~monique/files/moment-ima-update-new.pdf`