Erratum to Spiraling spectra of geodesic lines in negatively curved manifolds

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The correct statement of Proposition 1.4 of [PP] is the following one.

Proposition 1.4 *For the Golden Ratio* $\phi = \frac{1+\sqrt{5}}{2}$*, we have* $K_\phi = 3/\sqrt{5} - 1 \approx 0.34$*, and* K_ϕ *is not isolated in* Sp_ϕ.

The proof of Proposition 1.4 follows from the following corrected version of Proposition 4.11.

Proposition 4.11 *Let* Γ_0 *be the cyclic subgroup of* $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$ *generated by* $\gamma_1 = \pm\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$*, and let* $\mathscr{D} = (\mathbb{H}^2_{\mathbb{R}}, \Gamma, \Gamma_0, C_\infty)$*. Then* $K_{\mathscr{D}} = 3/\sqrt{5} - 1$*, and* $K_{\mathscr{D}}$ *is not isolated in the approximation spectrum* $\mathrm{Sp}(\mathscr{D})$.

Proof. The penultimate sentence of the proof of [PP, Prop. 4.11] is incorrect. For every $n \in \mathbb{N}$, let L_1, γ_n be as in the original version of the proof, and let A_n be the geodesic line from ∞ to the repelling fixed point γ_n^- of γ_n. In order to compute the (strictly increasing) limit, as n tends to $+\infty$, of the approximation constant $c(\gamma_n^-)$ of γ_n^- (using its expression given by Eq. (11) in [PP]), we need not only to consider the Γ-translates of L_1 intersecting A_n and to minimise $1 - \cos\theta$ where θ is its intersection angle, but also to consider the Γ-translates of L_1 not intersecting A_n and to minimise $\cosh \ell - 1$ where ℓ is its distance to A_n.

Consider the common perpendicular arc between the translation axis L_n of γ_n and a disjoint Γ-translate of L_1. By the symmetry at i and the computation (done in the original version of the proof) of the translation length of γ_n, we may restrict to the case when the endpoint on L_n of this common perpendicular arc lies between i and $i + n$. Let L be the translate by $z \mapsto z + 1$ of L_1, whose points at infinity are $\frac{3\pm\sqrt{5}}{2}$. Clearly (see in particular the picture in the original version of the proof), the common perpendicular arc δ_n between L_n and L realises the minimum distance between L_n and a Γ-translate of L_1 disjoint from L_n whose closest point on L_n lies between i and $i + n$. As $n \to \infty$, the segments δ_n converge (with strictly increasing lengths) to the common perpendicular arc δ_∞ between the positive imaginary axis and L. Since δ_∞ is contained in the Euclidean unit circle (which is the angle bisector through i of the equilateral geodesic triangle with vertices i, $1+i$, $\frac{1+i}{2}$), its hyperbolic length is $\operatorname{argcosh}\frac{3}{\sqrt{5}}$ by an easy computation. Since we analysed the contribution of the Γ-translates of L_1 that intersect L_n in the original version of the proof, and since $\frac{3}{\sqrt{5}} - 1 < 1 - \frac{1}{\sqrt{5}}$, the (strictly increasing) limit of $c(\gamma_n^-)$ is $\frac{3}{\sqrt{5}} - 1$.

To conclude, we also need to improve the last claim of the second paragraph of the proof of [PP, Prop. 4.11]. Let T be a triangle as in this second paragraph. The distance from a geodesic line γ meeting T to the geodesic line containing the side of T which is not cut by γ is maximal when γ goes through its opposite vertex and is perpendicular to the angle bisector of T at this vertex. This distance is equal to $\operatorname{argcosh}\frac{3}{\sqrt{5}}$ by the above computation. Since we analysed the contribution of the sides of T intersecting γ in the original version of the proof, and since $\frac{3}{\sqrt{5}} - 1 < 1 - \frac{1}{\sqrt{5}}$, we have $c(\xi) \leq \frac{3}{\sqrt{5}} - 1$ for every $\xi \in \mathbb{R} - \mathbb{Q}$. The result follows. □

We are grateful to Yann Bugeaud for pointing out the mistake. See [Bug] for an arithmetic proof of the above result.

References

[Bug] Y. Bugeaud. On the quadratic Lagrange spectrum. Preprint 2012.

[PP] J. Parkkonen and F. Paulin. Spiraling spectra of geodesic lines in negatively curved manifolds. *Math. Z.* **268** (2011) 101–142.

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