Titles and summaries of talks

M. Borovoi, Tel Aviv University (Israel)

Title: Fundamental group of a homogeneous space of a linear algebraic group

Abstract: Let X be a homogeneous space of a connected linear algebraic group G defined over the field of complex numbers $\mathbb{C}$. Let $x \in X(\mathbb{C})$. We denote by H the stabilizer of x in G. We show that the topological fundamental group $\pi_1(X(\mathbb{C}), x)$ can be defined algebraically. If $\mathrm{Pic}(G) = 0$ and H is connected or abelian, we compute $\pi_1(X(\mathbb{C}), x)$ in terms of the character groups of G and H. Furthermore, using ideas of Brion and Szamuely, when G and X are defined over an algebraically closed field of arbitrary characteristic p, we compute the prime-to-p étale fundamental group of X in terms of the character groups of G and H (if $\mathrm{Pic}(G) = 0$ and H is connected or abelian). This is a joint work with Cyril Demarche.

R. de la Bretèche, Université Paris 7 (France)

Title: Density of Châtelet surfaces failing the Hasse principle.

Abstract: Châtelet surfaces provide a rich source of geometrically rational surfaces which do not always satisfy the Hasse principle. I will present a joint work with Tim Browning where we investigate the frequency that such counter-exemples arise over the rationals.

F. Charles, CNRS, Université de Rennes I (France)

Title: Arithmetic aspects of the Noether-Lefschetz locus for families of K3 surfaces

Abstract: Given a family of smooth projective varieties, the Noether-Lefschetz locus is the set of points of the base corresponding to varieties containing strictly more divisors than the generic fiber. In the setting of complex algebraic geometry, Hodge theory has provided many tools leading to a thorough understanding of it. In this talk, we will describe some arithmetic aspects of the Noether-Lefschetz locus for families of K3 surfaces, first over finite fields, leading to a proof of the Tate conjecture for K3 surfaces, and then over number fields, where we expect it to shed light on the geometry of rational curves on these surfaces.

C. Demarche, Université Paris 6 (France)

Title: The unramified Brauer group of homogeneous spaces

Abstract: Let K be a field and let X be a homogeneous space of a connected linear algebraic group G over K. The unramified Brauer group of X is a cohomological invariant of X that plays an important role in the arithmetic of X (when K is a global field, via the Brauer-Manin obstruction to weak approximation) and in the geometry of X (especially to study the rationality of X over arbitrary fields K). In this talk, we will show a formula for the unramified Brauer group, under some connectedness assumption for the stabilizers of points in X. In particular, we will focus on the case of algebraically closed fields, global fields and finite fields. This is joint work with Mikhail Borovoi and David Harari.

P. Gille, C.N.R.S., École normale supérieure de Paris (France)

Title: Octonion algebras over rings and their norms

Abstract: Loos, Petersson and Racine defined octonion algebras over rings and relate them with étale torsors under the Chevalley group of type G_2. Over a field, an octonion algebra is determined by its norm which is a 3-Pfister form (Springer-Veldkamp). Using homogeneous spaces, we shall discuss the analogous question over rings.

Yong Hu, Université Paris-Sud 11 (France)

Title: Hasse principle for torsors over p-adic function fields

Abstract: Let K be the function field of an algebraic curve over a p-adic field and G a semisimple simply connected algebraic group over K. Colliot-Théène, Parimala and Suresh conjectured that for any G-torsor X over K, if for every discrete valuation v of K, X has points in the completion K_v of K at v, then X has a K-rational point. For most quasi-split groups, they have proved this conjecture using an injectivity property of the Rost invariant. In this talk, we will explain strategies for proving the conjecture for some classical groups in the non-quasi-split case. The method also applies to a more local situation and yields a parallel Hasse principle over the fraction field of a 2-dimensional henselian local domain with finite residue field.

Yongqi Liang, Université Paris-Sud 11 (France)

Title: Brauer-Manin obstruction for zero-cycles on rationally connected varieties defined over number fields.

Abstract: In this talk, we consider rationally connected varieties defined over number fields. We will state a general relation between the local-global principle for rational points and for zero-cycles. As an application, we prove the exactness of a sequence of local-global type for homogeneous spaces of linear algebraic groups.

M. Lieblich, University of Washington (USA)

Title: Rational curves in the moduli space of supersingular K3 surfaces

Abstract: I will describe a way to make rational curves through general points of the moduli space of supersingular K3 surfaces by exploiting the bizarre properties of the second cohomology of a K3 surface with coefficients in the sheaf of p-th roots of unity.

L. Matthiesen, University of Bristol (UK)

Title: Rational points on conic bundle surfaces via additive combinatorics

Abstract: Methods of Green and Tao can be used to prove the Hasse principle and weak approximation for some special intersections of quadrics defined over $\mathbb{Q}$. This implies that the Brauer-Manin obstruction controls weak approximation on conic bundles with an arbitrary number of degenerate fibres, all defined over $\mathbb{Q}$. This is joint work with Tim Browning and Alexei Skorobogatov.

L. Moret-Bailly, Université de Rennes I (France)

Title: Topological properties of torsors and homogeneous spaces over valued fields

Abstract: This is a report on work in progress with Philippe Gille. Let K be the fraction field of a henselian valuation ring R of positive characteristic p. Let Y be a K-variety, H an algebraic group over K, and $f : X \to Y$ an H-torsor over Y. We consider the induced map $X(K) \to Y(K)$, which is continuous for the topologies deduced from the valuation. If Z denotes the image of this map, we investigate the following questions: (a) Is Z locally closed (resp. closed) in $Y(K)$? (b) Is the continuous bijection $X(K)/H(K) \to Z$ a homeomorphism? We give positive answers to (a) and (b) in two cases: (1) R is an excellent discrete valuation ring with finite residue field. (2) $X = G$ is an algebraic group containing H, and $Y = G/H$; moreover R is a complete discrete valuation ring with countable residue field, and K has finite p-rank. In case (1) we use a recent compactification theorem due to Gabber. In case (2), the main tool is (a generalization of) a constructibility theorem due to Bernstein. When K is a local field, case (2) is essentially due to Bernstein and Zelevinsky (1976).

R. Parimala, Emory University (USA)

Title: Splitting ramification of division algebras over arithmetic surfaces

Abstract: There are open questions concerning bounding the index of division algebras over finitely generated fields in terms of the exponent. We shall explain the significance of this question with respect to bounding the u-invariant of function fields of curves over totally imaginary number fields. This leads to the following: the u-invariant of the function field of curves over such number fields is bounded provided a conjecture of Colliot-Thélène on the existence of 0-cycles of degree one on smooth projective varieties over number fields with trivial Brauer-Manin obstruction holds. (Joint work with Lieblich and Suresh)

E. Peyre, Université de Grenoble (France)

Title: Asymptotics for curves:

Abstract: Inspired by the analogy with the program of Batyrev and Manin for rational points, it is natural to ask whether the moduli space of very free morphisms from a given curve to an almost Fano variety converges, after renormalisation. This talk shall give some evidence for this question.

T. Schlank, Hebrew University of Jerusalem (Israel)

Title: Étale Homotopical Obstructions For Homogeneous Spaces

Abstract: An important tool in studying rational points on algebraic varieties over number fields are the various obstructions for the local global principle (e.g the Brauer-Manin obstruction, the descent obstruction...). Lately Y. Harpaz and myself found a way the describe these obstructions in a unified way using étale homotopy. This approach allows one to easily generalize these obstructions to work over fields of higher cohomological dimension.

In the case where the variety studied is a homogeneous space under the action of connected linear algebraic group, M. Borovoi gave a description for the Brauer-Manin obstruction using the stabilizer of the action and Galois cohomology. In this talk I will explain how to connect these two points of view. This will allow us to generalize Borovoi's construction to homogeneous spaces over fields of higher cohomological dimension.

R. Schulze-Pillot, Universität des Saarlandes (Germany)

Title: Recent progress in integral quadratic forms

Abstract: We will discuss recent work in the arithmetic theory of quadratic forms, mainly focussing on questions of representations of integers and of integral quadratic forms of lower rank by a given form.

T. Szamuely, Alfred Rényi Institute for Mathematics (Hungary)

Title: On fundamental groups and Brauer groups of algebraic groups and their homogeneous spaces

A. Várilly-Alvarado, Rice University (USA)

Title: Brauer classes on del Pezzo surfaces of degree 4 are vertical

Abstract: This is a report on joint work with Bianca Viray. Let X be a locally soluble del Pezzo surface of degree 4 over a number field k. I will explain how to construct, for every non-trivial element A of $\mathrm{Br}X/\mathrm{Br}k$, a rational genus-one fibration $X \to \mathbf{P}^1$ such that A is vertical for this map. This implies, for example, that if there is a Brauer-Manin obstruction to the Hasse principle on X arising from a single Brauer

class, then there is a genus-one fibration $X \to \mathbf{P}^1$ where none of the fibers are locally soluble, giving a concrete, geometric way of "seeing" the Brauer-Manin obstruction.

F. Voloch, University of Texas (USA)

Title: Anabelian geometry and descent obstructions on moduli spaces

Abstract: We will discuss the section conjecture of anabelian geometry and the sufficiency of the finite descent obstruction to the Hasse principle for the moduli spaces of principally polarized abelian varieties and of curves over number fields. For the former we show that both the section conjecture and the finite descent obstruction fail in a very controlled way. For the latter, we prove some partial results that suggest that the finite descent obstruction suffices. We also show how this sufficiency implies the same for all hyperbolic curves.

Dasheng Wei, Academy of Mathematics and System Science, CAS (China)

Title: The unramified Brauer group of norm one tori

Abstract: It is well known that the Brauer-Manin obstruction to the Hasse principle and weak approximation for rational points is the only one on homogeneous spaces of tori. To compute the Brauer-Manin obstruction, one need to construct the Brauer group. Recently, Colliot-Thélène gave an explicit construction for a multi-norm torus of dimension 5. However, for general tori, it is still open, even for the norm one torus $R^1_{K/\mathbb{Q}}(\mathbb{G}_m)$, where $K/\mathbb{Q}$ biquadratic. Let k be a number field and K/k Galois. We transform the construction of the unramified Brauer group of the norm one torus $R^1_{K/k}(\mathbb{G}_m)$ into the construction of a special abelian extension over K. If $k = \mathbb{Q}$ and $K/\mathbb{Q}$ biquadratic, we explicitly construct the unramified Brauer group of $R^1_{K/\mathbb{Q}}(\mathbb{G}_m)$.

O. Wittenberg, C.N.R.S., École normale supérieure de Paris (France)

Title: Index of varieties over Henselian fields and Euler characteristic of coherent sheaves

Abstract : Let X be a smooth proper variety over the quotient field of a Henselian discrete valuation ring with algebraically closed residue field of characteristic p. We show that if $p = 0$ or $p > \dim X + 1$, the index of X divides the Euler characteristic of any coherent sheaf on X. The same assertion holds in general for the prime-to-p part of the index of X. We prove a similar divisibility for the cobordism class of X. When $p = 0$, such statements have implications for the possible multiplicities of singular fibers in degenerations of complex projective varieties. (Joint work with H. Esnault and M. Levine.)

Yi Zhu, University of Utah (USA)

Title: de Jong's principle over function fields

Abstract: A basic question in arithmetic geometry is whether a given variety defined over a non-closed field admits a rational point. Let K be either a function field of a complex algebraic surface or a global function field. Johan de Jong formulated the following principle: a "rationally simply connected" (RSC) K-variety admits a rational point if and only if the elementary obstruction vanshies. In this talk, I will discuss how the principle works. I will also survey recent results for low degree hypersurfaces and projective homogeneous spaces under this principle.