```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([z, y, u[1], u[2], u[3]], [a, d, e, g, h, k], M, verbose);
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3, a, d, e, g, h, k]$$

The following vector fields have been defined and protected:

$$[D\_z, D\_y, D\_u_1, D\_u_2, D\_u_3, D\_a, D\_d, D\_e, D\_g, D\_h, D\_k]$$

The following differential 1-forms have been defined and protected:

$$[dz, dy, du_1, du_2, du_3, da, dd, de, dg, dh, dk]$$

frame name: M **(1)**

```
> Ma := Matrix([[a^4, 0, 0, 0, 0], [0, a^3, 0, 0, 0], [g, 0, a^2, 0, 0], [h, d, 0, a, 0], [k, e, 0, 0, a]]);
```

$$Ma := \begin{bmatrix} a^4 & 0 & 0 & 0 & 0 \\ 0 & a^3 & 0 & 0 & 0 \\ g & 0 & a^2 & 0 & 0 \\ h & d & 0 & a & 0 \\ k & e & 0 & 0 & a \end{bmatrix} \quad \textbf{(2)}$$

```
M > MaInv := MatrixInverse(Ma) :
M > A := map(evalDG, (ExteriorDerivative(Ma).MaInv)) ;
```

$$A := \begin{bmatrix} \frac{4\,da}{a} & 0\,dz & 0\,dz & 0\,dz & 0\,dz \\ 0\,dz & \frac{3\,da}{a} & 0\,dz & 0\,dz & 0\,dz \\ -\frac{2\,g\,da}{a^5} + \frac{dg}{a^4} & 0\,dz & \frac{2\,da}{a} & 0\,dz & 0\,dz \\ -\frac{h\,da}{a^5} + \frac{dh}{a^4} & -\frac{d\,da}{a^4} + \frac{dd}{a^3} & 0\,dz & \frac{da}{a} & 0\,dz \\ -\frac{k\,da}{a^5} + \frac{dk}{a^4} & -\frac{e\,da}{a^4} + \frac{de}{a^3} & 0\,dz & 0\,dz & \frac{da}{a} \end{bmatrix} \quad \textbf{(3)}$$

```
M > t[1] := da/a :
M > t[2] := -d*da/a^4 + dd/a^3 :
M > t[3] := -e*da/a^4 + de/a^3 :
M > t[4] := -2*g*da/a^5 + dg/a^4 :
```

```
> t[5] := -h*da/a^5 + dh/a^4 :
> t[6] := -k*da/a^5 + dk/a^4 :
> V := Vector([ -1/12 I y^3 dz + 1/12 I z^3 dy + (1/4 z^2 + 1/2 z y + 1/4 y^2) du1 + (-1/4 z
      - 1/4 y) du2 + 1/12 du3, 1/4 I y^2 dz - 1/4 I z^2 dy + (-1/2 z - 1/2 y) du1 + 1/4 du2,
      -1/2 I y dz + 1/2 I z dy + 1/2 du1, dz, dy]) :
> W := Ma.V :
> FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], W[1], W[2], W[3], W[4], W[5]],
      N) :
> DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], alpha[6], tau,
      sigma, rho, zeta, ζ#], verbose);
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3, a, d, e, g, h, k]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11]$$

The following differential 1-forms have been defined and protected:

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \tau, \sigma, \rho, \zeta, \zeta^{\#}]$$

frame name: N **(4)**

```
> ExteriorDerivative(tau);
```

$$4\,\alpha_1 \wedge \tau + \frac{(h+k)\,\tau \wedge \sigma}{a^4} + \sigma \wedge \zeta + \sigma \wedge \zeta^{\#} \quad \textbf{(5)}$$

```
> ExteriorDerivative(sigma);
```

$$3\,\alpha_1 \wedge \sigma + \frac{g\,(e+d)\,\tau \wedge \sigma}{a^7} + \frac{(h+k)\,\tau \wedge \rho}{a^4} - \frac{g\,\tau \wedge \zeta}{a^4} - \frac{g\,\tau \wedge \zeta^{\#}}{a^4} + \frac{(e+d)\,\sigma \wedge \rho}{a^3} + \rho \wedge \zeta + \rho \wedge \zeta^{\#} \quad \textbf{(6)}$$

```
> ExteriorDerivative(rho);
```

$$2\,\alpha_1 \wedge \rho + \alpha_4 \wedge \tau - \frac{(-h\,g - k\,g - \mathrm{I}\,e\,h\,a + \mathrm{I}\,d\,k\,a)\,\tau \wedge \sigma}{a^8} + \frac{\mathrm{I}\,k\,\tau \wedge \zeta}{a^4} - \frac{\mathrm{I}\,h\,\tau \wedge \zeta^{\#}}{a^4} + \frac{(g + \mathrm{I}\,e\,a)\,\sigma \wedge \zeta}{a^4} - \frac{(-g + \mathrm{I}\,d\,a)\,\sigma \wedge \zeta^{\#}}{a^4} + \mathrm{I}\,\zeta \wedge \zeta^{\#} \quad \textbf{(7)}$$

```
> ExteriorDerivative(zeta);
```

$$\alpha_1 \wedge \zeta + \alpha_2 \wedge \sigma + \alpha_5 \wedge \tau + \frac{\left(a^2\,h\,k + e\,d\,g + a^2\,h^2 + d^2\,g\right)\,\tau \wedge \sigma}{a^{10}} + \frac{(h+k)\,d\,\tau \wedge \rho}{a^7} - \frac{d\,g\,\tau \wedge \zeta}{a^7} - \frac{d\,g\,\tau \wedge \zeta^{\#}}{a^7} + \frac{(e+d)\,d\,\sigma \wedge \rho}{a^6} + \frac{h\,\sigma \wedge \zeta}{a^4} + \frac{h\,\sigma \wedge \zeta^{\#}}{a^4} + \frac{d\,\rho \wedge \zeta}{a^3} \quad \textbf{(8)}$$

$$+\frac{d\,\rho\wedge\zeta^{\#}}{a^3}$$

```
> ExteriorDerivative(ζ#);
```

$$\alpha_1\wedge\zeta^{\#}+\alpha_3\wedge\sigma+\alpha_6\wedge\tau+\frac{\left(a^2 h k+e d g+a^2 k^2+e^2 g\right)\tau\wedge\sigma}{a^{10}}+\frac{(h+k)\,e\,\tau\wedge\rho}{a^7}$$
$$-\frac{e\,g\,\tau\wedge\zeta}{a^7}-\frac{e\,g\,\tau\wedge\zeta^{\#}}{a^7}+\frac{(e+d)\,e\,\sigma\wedge\rho}{a^6}+\frac{k\,\sigma\wedge\zeta}{a^4}+\frac{k\,\sigma\wedge\zeta^{\#}}{a^4}+\frac{e\,\rho\wedge\zeta}{a^3}$$
$$+\frac{e\,\rho\wedge\zeta^{\#}}{a^3} \qquad (9)$$

```
>
```