$$
\begin{aligned}
& >\text { restart: } \\
& >\text { with(DifferentialGeometry): } \\
& \text { - }>\text { with(Tools) : with(LinearAlgebra) : } \\
& >\operatorname{DGsetup}([z, y, u[1], u[2], u[3]],[a, b, b 1, c, d, e, f, g, h, k], M \text {, verbose); } \\
& \text { The following coordinates have been protected: } \\
& {\left[z, y, u_{1}, u_{2}, u_{3}, a, b, b 1, c, d, e, f, g, h, k\right]} \\
& \text { The following vector fields have been defined and protected: } \\
& {\left[D_{-} z, D_{-} y, D_{-} u_{1}, D_{-} u_{2}, D_{-} u_{3}, D_{-} a, D_{-} b, D_{-} b 1, D_{-} c, D_{-} d, D_{-} e, D \_f, D \_g, D \_h, D_{-} k\right]} \\
& \text { The following differential 1-forms have been defined and protected: } \\
& {\left[d z, d y, d u_{1}, d u_{2}, d u_{3}, d a, d b, d b 1, d c, d d, d e, d f, d g, d h, d k\right]} \\
& \text { frame name: } M \\
& >\operatorname{Ma}:=\operatorname{Matrix}\left(\left[\left[a^{4}, 0,0,0,0\right],\left[f, a^{3}, 0,0,0\right],\left[g, c, a^{2}, 0,0\right],[h, d, b, a, 0],[k, e, b 1,0, a]\right]\right) \text {; } \\
& M a:=\left[\begin{array}{lllll}
a^{4} & 0 & 0 & 0 & 0 \\
f & a^{3} & 0 & 0 & 0 \\
g & c & a^{2} & 0 & 0 \\
h & d & b & a & 0 \\
k & e & b 1 & 0 & a
\end{array}\right] \\
& {[\mathrm{M}>\operatorname{MaInv}:=\text { MatrixInverse }(M a):} \\
& \text { M }>A:=\operatorname{map}(\text { evalDG, (ExteriorDerivative(Ma).MaInv)); } \\
& A:=\left[\left[\frac{4 d a}{a}, 0 d z, 0 d z, 0 d z, 0 d z\right]\right. \text {, } \\
& {\left[-\frac{3 f d a}{a^{5}}+\frac{d f}{a^{4}}, \frac{3 d a}{a}, 0 d z, 0 d z, 0 d z\right] \text {, }} \\
& {\left[-\frac{2\left(g a^{3}-c f\right) d a}{a^{8}}-\frac{f d c}{a^{7}}+\frac{d g}{a^{4}},-\frac{2 c d a}{a^{4}}+\frac{d c}{a^{3}}, \frac{2 d a}{a}, 0 d z, 0 d z\right],} \\
& {\left[-\frac{\left(h a^{5}-d f a^{2}-b g a^{3}+b c f\right) d a}{a^{10}}-\frac{\left(g a^{3}-c f\right) d b}{a^{9}}-\frac{f d d}{a^{7}}+\frac{d h}{a^{4}},\right.} \\
& \left.-\frac{\left(d a^{2}-b c\right) d a}{a^{6}}-\frac{c d b}{a^{5}}+\frac{d d}{a^{3}},-\frac{b d a}{a^{3}}+\frac{d b}{a^{2}}, \frac{d a}{a}, 0 d z\right] \text {, } \\
& {\left[-\frac{\left(k a^{5}-e f a^{2}-b 1 g a^{3}+b 1 c f\right) d a}{a^{10}}-\frac{\left(g a^{3}-c f\right) d b 1}{a^{9}}-\frac{f d e}{a^{7}}+\frac{d k}{a^{4}},\right.} \\
& \left.\left.-\frac{\left(e a^{2}-b 1 c\right) d a}{a^{6}}-\frac{c d b 1}{a^{5}}+\frac{d e}{a^{3}},-\frac{b 1 d a}{a^{3}}+\frac{d b 1}{a^{2}}, 0 d z, \frac{d a}{a}\right]\right] \\
& {\left[>t[1]:=\frac{d a}{a}:\right.} \\
& {\left[\mathrm{M}>t[2]:=-\frac{b d a}{a^{3}}+\frac{d b}{a^{2}}:\right.}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{M}>t[3]:=-\frac{2 c d a}{a^{4}}+\frac{d c}{a^{3}}: \\
& \mathrm{M}>t[4]:=-\frac{\left(d a^{2}-b c\right) d a}{a^{6}}-\frac{c d b}{a^{5}}+\frac{d d}{a^{3}}: \\
& \mathrm{M}>t[5]:=-\frac{\left(e a^{2}-b 1 c\right) d a}{a^{6}}-\frac{c d b 1}{a^{5}}+\frac{d e}{a^{3}}: \\
& \mathrm{M}>t[6]:=-\frac{3 f d a}{a^{5}}+\frac{d f}{a^{4}} \text { : } \\
& {\left[\mathrm{M}>t[7]:=-\frac{2\left(g a^{3}-c f\right) d a}{a^{8}}-\frac{f d c}{a^{7}}+\frac{d g}{a^{4}}:\right.} \\
& \mathbf{M}>t[8]:=-\frac{\left(h a^{5}-d f a^{2}-b g a^{3}+b c f\right) d a}{a^{10}}-\frac{\left(g a^{3}-c f\right) d b}{a^{9}}-\frac{f d d}{a^{7}}+\frac{d h}{a^{4}}: \\
& \mathrm{M}>t[9]:=-\frac{\left(k a^{5}-e f a^{2}-b 1 g a^{3}+b 1 c f\right) d a}{a^{10}}-\frac{\left(g a^{3}-c f\right) d b 1}{a^{9}}-\frac{f d e}{a^{7}}+\frac{d k}{a^{4}}: \\
& \mathrm{M}>t[10]:=-\frac{b 1 d a}{a^{3}}+\frac{d b 1}{a^{2}}: \\
& \mathrm{M}>V:=\operatorname{Vector}\left(\left[-\frac{1}{12} \mathrm{I} y^{3} d z+\frac{1}{12} \mathrm{I} z^{3} d y+\left(\frac{1}{4} z^{2}+\frac{1}{2} z y+\frac{1}{4} y^{2}\right) d u_{1}+\left(-\frac{1}{4} z\right.\right.\right. \\
& \left.-\frac{1}{4} y\right) d u_{2}+\frac{1}{12} d u_{3}, \frac{1}{4} \mathrm{I} y^{2} d z-\frac{1}{4} \mathrm{I} z^{2} d y+\left(-\frac{1}{2} z-\frac{1}{2} y\right) d u_{1}+\frac{1}{4} d u_{2}, \\
& \left.\left.-\frac{1}{2} \mathrm{I} y d z+\frac{1}{2} \mathrm{I} z d y+\frac{1}{2} d u_{1}, d z, d y\right]\right):
\end{aligned}
$$

M $>W$ : = Ma.V:
$\mathbf{M}>F D:=\operatorname{FrameData}([t[1], t[2], t[3], t[4], t[5], t[6], t[7], t[8], t[9], t[10], W[1]$,

$$
W[2], W[3], W[4], W[5]], N):
$$

$\mathbf{M}>\operatorname{DGsetup}(F D,[E],[\operatorname{alpha}[1]$, alpha[2], alpha[3], alpha[4], alpha[5], alpha[6], alpha[7], alpha[8], alpha[9], $\alpha^{\#}[2]$, tau, sigma, rho, zeta, $\left.\zeta^{\#}\right]$, verbose);

The following coordinates have been protected:

$$
\left[z, y, u_{1}, u_{2}, u_{3}, a, b, b 1, c, d, e, f, g, h, k\right]
$$

The following vector fields have been defined and protected:
[E1,E2,E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13, E14, E15]
The following differential 1-forms have been defined and protected:

$$
\begin{equation*}
\left[\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{6}, \alpha_{7}, \alpha_{8}, \alpha_{9}, \alpha_{2}^{\#}, \tau, \sigma, \rho, \zeta, \zeta^{\#}\right] \tag{4}
\end{equation*}
$$

frame name: $N$
$\mathrm{N}>$ ExteriorDerivative (tau);

$$
\begin{align*}
4 \alpha_{1} & \wedge \tau+\frac{\left(-g b l-g b+a^{2} h+a^{2} k\right) \tau \wedge \sigma}{a^{6}}+\frac{f(b+b 1) \tau \wedge \rho}{a^{6}}-\frac{f \tau \wedge \zeta}{a^{4}}-\frac{f \tau \wedge \zeta^{\#}}{a^{4}}  \tag{5}\\
& -\frac{(b+b 1) \sigma \wedge \rho}{a^{2}}+\sigma \wedge \zeta+\sigma \wedge \zeta^{\#}
\end{align*}
$$

$\lceil\mathrm{N}>$ ExteriorDerivative( sigma);

$$
\left[\begin{array}{l}
3 \alpha_{1} \wedge \sigma+\alpha_{6} \wedge \tau  \tag{6}\\
\\
\quad-\frac{\left(f b 1 g+f b g+c a^{3} h-e a^{3} g+c a^{3} k-d a^{3} g-f a^{2} k-f a^{2} h\right) \tau \wedge \sigma}{a^{10}} \\
\quad+\frac{\left(b 1 f^{2}+b f^{2}-e a^{3} f-d f a^{3}+k a^{6}+h a^{6}\right) \tau \wedge \rho}{a^{10}}-\frac{\left(g a^{4}-a c f+f^{2}\right) \tau \wedge \zeta}{a^{8}} \\
\\
\quad-\frac{\left(g a^{4}-a c f+f^{2}\right) \tau \wedge \zeta^{\#}}{a^{8}}+\frac{\left(a^{3} e+d a^{3}-b f-b 1 f\right) \sigma \wedge \rho}{a^{6}}-\frac{(a c-f) \sigma \wedge \zeta}{a^{4}} \\
\end{array}\right.
$$

$\mathrm{N}>$ ExteriorDerivative (rho);
$2 \alpha_{1} \wedge \rho+\alpha_{3} \wedge \sigma+\alpha_{7} \wedge \tau-\frac{1}{a^{10}}((\mathrm{I} e a b g+\mathrm{I} b 1 c a h-\mathrm{I} d a b 1 g-\mathrm{I} b c a k-e g c$

$$
\begin{equation*}
\left.\left.-d g c+\mathrm{I} d a^{3} k-\mathrm{I} e a^{3} h+c^{2} k+c^{2} h-g a^{2} k-g a^{2} h+g^{2} b l+g^{2} b\right) \tau \wedge \sigma\right) \tag{7}
\end{equation*}
$$

$$
-\frac{1}{a^{10}}\left(\left(e c f-c a^{3} k-c a^{3} h+d c f-f b 1 g-f b g+\mathrm{I} b a^{4} k-\mathrm{I} b 1 a^{4} h+\mathrm{I} b 1 d a f\right.\right.
$$

$$
-\mathrm{I} b e a f) \tau \wedge \rho)
$$

$$
+\frac{\left(-\mathrm{I} e a^{3} f-\mathrm{I} a^{4} b 1 g+\mathrm{I} b 1 c a f+\mathrm{I} a^{6} k-g a^{3} c+c^{2} f-g a^{2} f\right) \tau \wedge \zeta}{a^{10}}
$$

$$
-\frac{\left(\mathrm{I} a^{6} h+\mathrm{I} b c a f-\mathrm{I} a^{4} b g-\mathrm{I} d a^{3} f+g a^{3} c-c^{2} f+g a^{2} f\right) \tau \wedge \zeta^{\#}}{a^{10}}
$$

$$
-\frac{(g b+g b 1-c e-c d-\mathrm{I} b 1 d a+\mathrm{I} b e a) \sigma \wedge \rho}{a^{6}}
$$

$$
+\frac{\left(-\mathrm{I} b 1 c a+\mathrm{I} e a^{3}+g a^{2}-c^{2}\right) \sigma \wedge \zeta}{a^{6}}-\frac{\left(\mathrm{I} d a^{3}-g a^{2}-\mathrm{I} b c a+c^{2}\right) \sigma \wedge \zeta^{\#}}{a^{6}}
$$

$$
+\frac{(c+\mathrm{I} b 1 a) \rho \wedge \zeta}{a^{3}}-\frac{(-c+\mathrm{I} b a) \rho \wedge \zeta^{\#}}{a^{3}}+\mathrm{I} \zeta \wedge \zeta^{\#}
$$

N $\mathrm{N}>$ ExteriorDerivative(zeta);
$\alpha_{1} \wedge \zeta+\alpha_{2} \wedge \rho+\alpha_{4} \wedge \sigma+\alpha_{8} \wedge \tau-\frac{1}{a^{11}}((\operatorname{acdk}+a c d h-e a g d+h a b 1 g+h a b g$
$+\mathrm{I} e b^{2} g+\mathrm{I} d a^{2} k b-\mathrm{I} e a^{2} h b-\mathrm{I} b^{2} c k+\mathrm{I} b 1 c h b-\mathrm{I} d b 1 g b-h^{2} a^{3}-d^{2} a g$
$\left.\left.-h a^{3} k\right) \tau \wedge \sigma\right)-\frac{1}{a^{11}}\left(\left(a d^{2} f+a d e f-b 1 h a f-b h a f+\mathrm{I} k a^{3} b^{2}-a^{4} d k\right.\right.$
$\left.\left.-a^{4} d h+\mathrm{I} b 1 d f b-\mathrm{I} b 1 h a^{3} b-\mathrm{I} e f b^{2}\right) \tau \wedge \rho\right)$
$+\frac{\left(a c f d+\mathrm{I} k a^{5} b-g a^{4} d-h a^{3} f-\mathrm{I} e f a^{2} b-\mathrm{I} b 1 g a^{3} b+\mathrm{I} b 1 c f b\right) \tau \wedge \zeta}{a^{11}}$

$$
\begin{aligned}
& -\frac{\left(-a c f d+g a^{4} d+h a^{3} f-\mathrm{I} d f a^{2} b+\mathrm{I} h a^{5} b-\mathrm{I} b^{2} g a^{3}+\mathrm{I} b^{2} c f\right) \tau \wedge \zeta^{\#}}{a^{11}} \\
& -\frac{\left(b 1 h a+\mathrm{I} e b^{2}-a d^{2}-\mathrm{I} b 1 d b-a d e+b h a\right) \sigma \wedge \rho}{a^{7}} \\
& +\frac{\left(h a^{3}+\mathrm{I} e a^{2} b-\mathrm{I} b 1 c b-a c d\right) \sigma \wedge \zeta}{a^{7}} \\
& -\frac{\left(-h a^{3}+\mathrm{I} d a^{2} b-\mathrm{I} b^{2} c+a c d\right) \sigma \wedge \zeta^{\#}}{a^{7}}+\frac{(a d+\mathrm{I} b 1 b) \rho \wedge \zeta}{a^{4}} \\
& -\frac{\left(-a d+\mathrm{I} b^{2}\right) \rho \wedge \zeta^{\#}}{a^{4}}+\frac{\mathrm{I} b \zeta \wedge \zeta^{\#}}{a^{2}}
\end{aligned}
$$

$\mathrm{N}>$ ExteriorDerivative $\left(\zeta^{\#}\right)$;

$$
\alpha_{1} \wedge \zeta^{\#}+\alpha_{5} \wedge \sigma+\alpha_{9} \wedge \tau+\alpha_{2}^{\#} \wedge \rho-\frac{1}{a^{11}}\left(\left(\mathrm{I} e b 1 g b-\mathrm{I} b c k b 1-e a g d-e^{2} a g\right.\right.
$$

$$
-k^{2} a^{3}+a c e k+a c e h+k a b 1 g+k a b g+\mathrm{I} b 1^{2} c h+\mathrm{I} d a^{2} k b 1-\mathrm{I} e a^{2} h b 1
$$

$$
\left.\left.-\mathrm{I} d b 1^{2} g-h a^{3} k\right) \tau \wedge \sigma\right)-\frac{1}{a^{11}}\left(\left(a d e f-a^{4} e k+a e^{2} f-\mathrm{I} b e f b 1-a^{4} e h\right.\right.
$$

$$
\left.\left.-b 1 k a f-b k a f-\mathrm{I} h a^{3} b 1^{2}+\mathrm{I} d f b 1^{2}+\mathrm{I} b k a^{3} b 1\right) \tau \wedge \rho\right)
$$

$$
+\frac{\left(-g a^{4} e-k a^{3} f-\mathrm{I} e f a^{2} b 1+a c f e-\mathrm{I} b 1^{2} g a^{3}+\mathrm{I} b 1^{2} c f+\mathrm{I} k a^{5} b 1\right) \tau \wedge \zeta}{a^{11}}
$$

$$
-\frac{\left(g a^{4} e+k a^{3} f-\mathrm{I} d f a^{2} b 1-a c f e+\mathrm{I} h a^{5} b 1-\mathrm{I} b 1 g a^{3} b+\mathrm{I} b 1 c f b\right) \tau \wedge \zeta^{\#}}{a^{11}}
$$

$$
-\frac{\left(b k a-\mathrm{I} d b 1^{2}-a e^{2}+\mathrm{I} b e b 1+b 1 k a-a d e\right) \sigma \wedge \rho}{a^{7}}
$$

$$
+\frac{\left(k a^{3}-a c e+\mathrm{I} e a^{2} b 1-\mathrm{I} b 1^{2} c\right) \sigma \wedge \zeta}{a^{7}}
$$

$$
-\frac{\left(-k a^{3}+\mathrm{I} d a^{2} b 1+a c e-\mathrm{I} b 1 c b\right) \sigma \wedge \zeta^{\#}}{a^{7}}+\frac{\left(a e+\mathrm{I} b 1^{2}\right) \rho \wedge \zeta}{a^{4}}
$$

$$
-\frac{(-a e+\mathrm{I} b 1 b) \rho \wedge \zeta^{\#}}{a^{4}}+\frac{\mathrm{I} b 1 \zeta \wedge \zeta^{\#}}{a^{2}}
$$

frame2 $>$ List $:=$ GenerateForms ([alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], alpha[6], alpha[7], alpha[8], alpha[9], $\alpha^{\#}[2]$, tau, sigma, rho, zeta, $\left.\left.\zeta^{\#}\right], 2\right)$ :
frame2 $>$ Torsion $:=\operatorname{proc}(S, i, j)$ local $k, X ; k:=15 \cdot(i-1)-\frac{i \cdot(i-1)}{2}+j-i ; X$ $:=G e t C o m p o n e n t s(S, L i s t) ; \operatorname{expand}(X[k])$; end proc:
frame2 $>$ result $:=\operatorname{proc}(l) \operatorname{local} k, t, X ; X:=0: t:=\operatorname{expand}($ GetComponents $(l$,
$\mathbf{N}>\operatorname{result}($ ExteriorDerivative $($ tau $)$ );

$$
\begin{align*}
4 \alpha_{1} & \wedge \tau+\left(-\frac{g b 1}{a^{6}}-\frac{g b}{a^{6}}+\frac{h}{a^{4}}+\frac{k}{a^{4}}\right) \tau \wedge \sigma+\left(\frac{f b}{a^{6}}+\frac{f b 1}{a^{6}}\right) \tau \wedge \rho-\frac{f \tau \wedge \zeta}{a^{4}}  \tag{10}\\
& -\frac{f \tau \wedge \zeta^{\#}}{a^{4}}+\left(-\frac{b}{a^{2}}-\frac{b 1}{a^{2}}\right) \sigma \wedge \rho+\sigma \wedge \zeta+\sigma \wedge \zeta^{\#}
\end{align*}
$$

$\mathrm{N}>\operatorname{result}($ ExteriorDerivative(sigma));

$$
\begin{align*}
3 \alpha_{1} & \wedge \sigma+\alpha_{6} \wedge \tau+\left(-\frac{f b 1 g}{a^{10}}-\frac{f b g}{a^{10}}-\frac{c h}{a^{7}}+\frac{e g}{a^{7}}-\frac{c k}{a^{7}}+\frac{d g}{a^{7}}+\frac{f k}{a^{8}}+\frac{f h}{a^{8}}\right) \tau \wedge \sigma  \tag{11}\\
& +\left(\frac{b 1 f^{2}}{a^{10}}+\frac{b f^{2}}{a^{10}}-\frac{e f}{a^{7}}-\frac{d f}{a^{7}}+\frac{k}{a^{4}}+\frac{h}{a^{4}}\right) \tau \wedge \rho+\left(-\frac{g}{a^{4}}+\frac{c f}{a^{7}}-\frac{f^{2}}{a^{8}}\right) \tau \wedge \zeta \\
& +\left(-\frac{g}{a^{4}}+\frac{c f}{a^{7}}-\frac{f^{2}}{a^{8}}\right) \tau \wedge \zeta^{\#}+\left(\frac{e}{a^{3}}+\frac{d}{a^{3}}-\frac{f b}{a^{6}}-\frac{f b 1}{a^{6}}\right) \sigma \wedge \rho+\left(-\frac{c}{a^{3}}\right. \\
& \left.+\frac{f}{a^{4}}\right) \sigma \wedge \zeta+\left(-\frac{c}{a^{3}}+\frac{f}{a^{4}}\right) \sigma \wedge \zeta^{\#}+\rho \wedge \zeta+\rho \wedge \zeta^{7}
\end{align*}
$$

$\mathrm{N}>\operatorname{result}($ ExteriorDerivative (rho) );
$2 \alpha_{1} \wedge \rho+\alpha_{3} \wedge \sigma+\alpha_{7} \wedge \tau+\left(-\frac{\mathrm{I} e b g}{a^{9}}-\frac{\mathrm{I} b 1 c h}{a^{9}}+\frac{\mathrm{I} d b 1 g}{a^{9}}+\frac{\mathrm{I} b c k}{a^{9}}+\frac{e g c}{a^{10}}\right.$

$$
\begin{align*}
& \left.+\frac{d g c}{a^{10}}-\frac{\mathrm{I} d k}{a^{7}}+\frac{\mathrm{I} e h}{a^{7}}-\frac{c^{2} k}{a^{10}}-\frac{c^{2} h}{a^{10}}+\frac{g k}{a^{8}}+\frac{g h}{a^{8}}-\frac{g^{2} b 1}{a^{10}}-\frac{g^{2} b}{a^{10}}\right) \tau \wedge \sigma+  \tag{12}\\
& -\frac{e c f}{a^{10}}+\frac{c k}{a^{7}}+\frac{c h}{a^{7}}-\frac{d c f}{a^{10}}+\frac{f b 1 g}{a^{10}}+\frac{f b g}{a^{10}}-\frac{\mathrm{I} b k}{a^{6}}+\frac{\mathrm{I} b 1 h}{a^{6}}-\frac{\mathrm{I} b 1 d f}{a^{9}} \\
& \left.+\frac{\mathrm{I} b e f}{a^{9}}\right) \tau \wedge \rho+\left(-\frac{\mathrm{I} e f}{a^{7}}-\frac{\mathrm{I} b 1 g}{a^{6}}+\frac{\mathrm{I} b 1 c f}{a^{9}}+\frac{\mathrm{I} k}{a^{4}}-\frac{g c}{a^{7}}+\frac{c^{2} f}{a^{10}}-\frac{g f}{a^{8}}\right) \tau \wedge \zeta \\
& +\left(-\frac{\mathrm{I} h}{a^{4}}-\frac{\mathrm{I} b c f}{a^{9}}+\frac{\mathrm{I} b g}{a^{6}}+\frac{\mathrm{I} d f}{a^{7}}-\frac{g c}{a^{7}}+\frac{c^{2} f}{a^{10}}-\frac{g f}{a^{8}}\right) \tau \wedge \zeta^{\#}+\left(-\frac{g b}{a^{6}}-\frac{g b 1}{a^{6}}\right. \\
& \left.+\frac{c e}{a^{6}}+\frac{c d}{a^{6}}+\frac{\mathrm{I} b 1 d}{a^{5}}-\frac{\mathrm{I} b e}{a^{5}}\right) \sigma \wedge \rho+\left(-\frac{\mathrm{I} b 1 c}{a^{5}}+\frac{\mathrm{I} e}{a^{3}}+\frac{g}{a^{4}}-\frac{c^{2}}{a^{6}}\right) \sigma \wedge \zeta+( \\
& \left.-\frac{\mathrm{I} d}{a^{3}}+\frac{g}{a^{4}}+\frac{\mathrm{I} b c}{a^{5}}-\frac{c^{2}}{a^{6}}\right) \sigma \wedge \zeta^{\#}+\left(\frac{c}{a^{3}}+\frac{\mathrm{I} b 1}{a^{2}}\right) \rho \wedge \zeta+\left(\frac{c}{a^{3}}-\frac{\mathrm{I} b}{a^{2}}\right) \rho \wedge \zeta^{\#} \\
& +\mathrm{I} \zeta \wedge \zeta^{\#}
\end{align*}
$$

$\mathrm{N}>\operatorname{result}($ ExteriorDerivative(zeta));

$$
\begin{align*}
\alpha_{1} & \wedge \zeta+\alpha_{2} \wedge \rho+\alpha_{4} \wedge \sigma+\alpha_{8} \wedge \tau+\left(-\frac{c d k}{a^{10}}-\frac{c d h}{a^{10}}+\frac{e g d}{a^{10}}-\frac{h b 1 g}{a^{10}}-\frac{h b g}{a^{10}}\right.  \tag{13}\\
& -\frac{\mathrm{I} e b^{2} g}{a^{11}}-\frac{\mathrm{I} d k b}{a^{9}}+\frac{\mathrm{I} e h b}{a^{9}}+\frac{\mathrm{I} b^{2} c k}{a^{11}}-\frac{\mathrm{I} b 1 c h b}{a^{11}}+\frac{\mathrm{I} d b 1 g b}{a^{11}}+\frac{h^{2}}{a^{8}}+\frac{d^{2} g}{a^{10}}
\end{align*}
$$

$$
\begin{aligned}
& \left.+\frac{h k}{a^{8}}\right) \tau \wedge \sigma+\left(-\frac{d^{2} f}{a^{10}}-\frac{d e f}{a^{10}}+\frac{b 1 h f}{a^{10}}+\frac{b h f}{a^{10}}-\frac{\mathrm{I} k b^{2}}{a^{8}}+\frac{d k}{a^{7}}+\frac{d h}{a^{7}}\right. \\
& \left.-\frac{\mathrm{I} b 1 d f b}{a^{11}}+\frac{\mathrm{I} b l h b}{a^{8}}+\frac{\mathrm{I} e f b^{2}}{a^{11}}\right) \tau \wedge \rho+\left(\frac{d c f}{a^{10}}+\frac{\mathrm{I} b k}{a^{6}}-\frac{d g}{a^{7}}-\frac{f h}{a^{8}}-\frac{\mathrm{I} b e f}{a^{9}}\right. \\
& \left.-\frac{\mathrm{I} b 1 g b}{a^{8}}+\frac{\mathrm{I} b l c f b}{a^{11}}\right) \tau \wedge \zeta+\left(\frac{d c f}{a^{10}}-\frac{d g}{a^{7}}-\frac{f h}{a^{8}}+\frac{\mathrm{I} d f b}{a^{9}}-\frac{\mathrm{I} h b}{a^{6}}+\frac{\mathrm{I} b^{2} g}{a^{8}}\right. \\
& \left.-\frac{\mathrm{I} b^{2} c f}{a^{11}}\right) \tau \wedge \zeta^{\#}+\left(-\frac{b 1 h}{a^{6}}-\frac{\mathrm{I} e b^{2}}{a^{7}}+\frac{d^{2}}{a^{6}}+\frac{\mathrm{I} b 1 d b}{a^{7}}+\frac{d e}{a^{6}}-\frac{h b}{a^{6}}\right) \sigma \wedge \rho \\
& +\left(\frac{h}{a^{4}}+\frac{\mathrm{I} b e}{a^{5}}-\frac{\mathrm{I} b 1 c b}{a^{7}}-\frac{c d}{a^{6}}\right) \sigma \wedge \zeta+\left(\frac{h}{a^{4}}-\frac{\mathrm{I} d b}{a^{5}}+\frac{\mathrm{I} b^{2} c}{a^{7}}-\frac{c d}{a^{6}}\right) \sigma \wedge \zeta^{\#} \\
& +\left(\frac{d}{a^{3}}+\frac{\mathrm{I} b 1 b}{a^{4}}\right) \rho \wedge \zeta+\left(\frac{d}{a^{3}}-\frac{\mathrm{I} b^{2}}{a^{4}}\right) \rho \wedge \zeta^{\#}+\frac{\mathrm{I} b \zeta \wedge \zeta^{\#}}{a^{2}}
\end{aligned}
$$

$\mathrm{N}>\operatorname{result}\left(\right.$ ExteriorDerivative $\left.\left(\zeta^{\#}\right)\right)$;

$$
\begin{align*}
\alpha_{1} & \wedge \zeta^{\#}+\alpha_{5} \wedge \sigma+\alpha_{9} \wedge \tau+\alpha_{2}^{\#} \wedge \rho+\left(-\frac{\mathrm{I} e b 1 g b}{a^{11}}+\frac{\mathrm{I} b c k b 1}{a^{11}}+\frac{e g d}{a^{10}}+\frac{e^{2} g}{a^{10}}+\frac{k^{2}}{a^{8}}\right.  \tag{14}\\
& -\frac{c e k}{a^{10}}-\frac{c e h}{a^{10}}-\frac{k b 1 g}{a^{10}}-\frac{k b g}{a^{10}}-\frac{\mathrm{I} b 1^{2} c h}{a^{11}}-\frac{\mathrm{I} d k b 1}{a^{9}}+\frac{\mathrm{I} e h b 1}{a^{9}}+\frac{\mathrm{I} d b 1^{2} g}{a^{11}} \\
& \left.+\frac{h k}{a^{8}}\right) \tau \wedge \sigma+\left(-\frac{d e f}{a^{10}}+\frac{e k}{a^{7}}-\frac{e^{2} f}{a^{10}}+\frac{\mathrm{I} b e f b 1}{a^{11}}+\frac{e h}{a^{7}}+\frac{b 1 k f}{a^{10}}+\frac{b k f}{a^{10}}\right. \\
& \left.+\frac{\mathrm{I} h b 1^{2}}{a^{8}}-\frac{\mathrm{I} d f b 1^{2}}{a^{11}}-\frac{\mathrm{I} b k b 1}{a^{8}}\right) \tau \wedge \rho+\left(-\frac{e g}{a^{7}}-\frac{f k}{a^{8}}-\frac{\mathrm{I} e f b 1}{a^{9}}+\frac{e c f}{a^{10}}\right. \\
& \left.-\frac{\mathrm{I} b 1^{2} g}{a^{8}}+\frac{\mathrm{I} b 1^{2} c f}{a^{11}}+\frac{\mathrm{I} k b 1}{a^{6}}\right) \tau \wedge \zeta+\left(-\frac{e g}{a^{7}}-\frac{f k}{a^{8}}+\frac{\mathrm{I} b 1 d f}{a^{9}}+\frac{e c f}{a^{10}}-\frac{\mathrm{I} b 1 h}{a^{6}}\right. \\
& \left.+\frac{\mathrm{I} b 1 g b}{a^{8}}-\frac{\mathrm{I} b 1 c f b}{a^{11}}\right) \tau \wedge \zeta^{\#}+\left(-\frac{b k}{a^{6}}+\frac{\mathrm{I} d b 1^{2}}{a^{7}}+\frac{e^{2}}{a^{6}}-\frac{\mathrm{I} b e b 1}{a^{7}}-\frac{k b 1}{a^{6}}\right. \\
& \left.+\frac{d e}{a^{6}}\right) \sigma \wedge \rho+\left(\frac{k}{a^{4}}-\frac{c e}{a^{6}}+\frac{\mathrm{I} e b 1}{a^{5}}-\frac{\mathrm{I} b 1^{2} c}{a^{7}}\right) \sigma \wedge \zeta+\left(\frac{k}{a^{4}}-\frac{\mathrm{I} b 1 d}{a^{5}}-\frac{c e}{a^{6}}\right. \\
& \left.+\frac{\mathrm{I} b 1 c b}{a^{7}}\right) \sigma \wedge \zeta^{\#}+\left(\frac{e}{a^{3}}+\frac{\mathrm{I} b 1^{2}}{a^{4}}\right) \rho \wedge \zeta+\left(\frac{e}{a^{3}}-\frac{\mathrm{I} b 1 b}{a^{4}}\right) \rho \wedge \zeta^{\#}+\frac{\mathrm{I} b 1 \zeta \wedge \zeta^{\#}}{a^{2}}
\end{align*}
$$

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