```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup( [z, y, u[1], u[2], u[3]], [a, a1, b, b1, c, d, e, f, g, h, k], M, verbose);
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3, a, a1, b, b1, c, d, e, f, g, h, k]$$

The following vector fields have been defined and protected:

$$[D\_z, D\_y, D\_u_1, D\_u_2, D\_u_3, D\_a, D\_a1, D\_b, D\_b1, D\_c, D\_d, D\_e, D\_f, D\_g, D\_h, D\_k]$$

The following differential 1-forms have been defined and protected:

$$[dz, dy, du_1, du_2, du_3, da, da1, db, db1, dc, dd, de, df, dg, dh, dk]$$

frame name: M **(1)**

```
> Ma := Matrix( [ [a^3·a1, 0, 0, 0, 0], [f, a^2·a1, 0, 0, 0], [g, c, a·a1, 0, 0], [h, d, b, a, 0], [k, e,
      b1, 0, a1 ] ]);
```

$$Ma := \begin{bmatrix} a^3\,a1 & 0 & 0 & 0 & 0 \\ f & a^2\,a1 & 0 & 0 & 0 \\ g & c & a\,a1 & 0 & 0 \\ h & d & b & a & 0 \\ k & e & b1 & 0 & a1 \end{bmatrix} \quad \textbf{(2)}$$

```
M > MaInv := MatrixInverse(Ma) :
M > A := map(evalDG, (ExteriorDerivative(Ma).MaInv) );
```

$$A := \left[\left[\frac{3\,da}{a} + \frac{da1}{a1}, 0\,dz, 0\,dz, 0\,dz, 0\,dz\right], \right. \quad \textbf{(3)}$$

$$\left[-\frac{2\,f\,da}{a1\,a^4} - \frac{f\,da1}{a1^2\,a^3} + \frac{df}{a^3\,a1}, \frac{2\,da}{a} + \frac{da1}{a1}, 0\,dz, 0\,dz, 0\,dz\right],$$

$$\left[-\frac{(g\,a^2\,a1 - c\,f)\,da}{a1^2\,a^6} - \frac{(g\,a^2\,a1 - c\,f)\,da1}{a1^3\,a^5} - \frac{f\,dc}{a^5\,a1^2} + \frac{dg}{a^3\,a1}, -\frac{c\,da}{a1\,a^3} - \frac{c\,da1}{a1^2\,a^2}\right.$$

$$\left. + \frac{dc}{a^2\,a1}, \frac{da}{a} + \frac{da1}{a1}, 0\,dz, 0\,dz\right],$$

$$\left[-\frac{(h\,a^3\,a1^2 - d\,f\,a\,a1 - b\,g\,a^2\,a1 + b\,c\,f)\,da}{a^7\,a1^3} - \frac{(g\,a^2\,a1 - c\,f)\,db}{a^6\,a1^3} - \frac{f\,dd}{a^5\,a1^2}\right.$$

$$+ \frac{dh}{a^3\,a1}, -\frac{(d\,a\,a1 - b\,c)\,da}{a^4\,a1^2} - \frac{c\,db}{a^3\,a1^2} + \frac{dd}{a^2\,a1}, -\frac{b\,da}{a^2\,a1} + \frac{db}{a\,a1}, \frac{da}{a}, 0\,dz\Big],$$

$$\left[-\frac{(k\,a^3\,a1^2 - e\,f\,a\,a1 - b1\,g\,a^2\,a1 + b1\,c\,f)\,da1}{a^6\,a1^4} - \frac{(g\,a^2\,a1 - c\,f)\,db1}{a^6\,a1^3} - \frac{f\,de}{a^5\,a1^2}\right.$$

$$\left. + \frac{dk}{a^3\,a1}, -\frac{(e\,a\,a1 - b1\,c)\,da1}{a^3\,a1^3} - \frac{c\,db1}{a^3\,a1^2} + \frac{de}{a^2\,a1}, -\frac{b1\,da1}{a\,a1^2} + \frac{db1}{a\,a1}, 0\,dz, \frac{da1}{a1}\right]\Big]$$

$$t[1] := \frac{da}{a} :$$

$$t[2] := -\frac{b\,da}{a^2\,a1} + \frac{db}{a\,a1} :$$

$$t[3] := -\frac{c\,da}{a1\,a^3} - \frac{c\,da1}{a1^2\,a^2} + \frac{dc}{a^2\,a1} :$$

$$t[4] := -\frac{(d\,a\,a1 - b\,c)\,da}{a^4\,a1^2} - \frac{c\,db}{a^3\,a1^2} + \frac{dd}{a^2\,a1} :$$

$$t[5] := -\frac{(e\,a\,a1 - b1\,c)\,da1}{a^3\,a1^3} - \frac{c\,db1}{a^3\,a1^2} + \frac{de}{a^2\,a1} :$$

$$t[6] := -\frac{2\,f\,da}{a1\,a^4} - \frac{f\,da1}{a1^2\,a^3} + \frac{df}{a^3\,a1} :$$

$$t[7] := -\frac{(g\,a^2\,a1 - c\,f)\,da}{a1^2\,a^6} - \frac{(g\,a^2\,a1 - c\,f)\,da1}{a1^3\,a^5} - \frac{f\,dc}{a^5\,a1^2} + \frac{dg}{a^3\,a1} :$$

$$t[8] := -\frac{(h\,a^3\,a1^2 - d\,f\,a\,a1 - b\,g\,a^2\,a1 + b\,c\,f)\,da}{a^7\,a1^3} - \frac{(g\,a^2\,a1 - c\,f)\,db}{a^6\,a1^3} - \frac{f\,dd}{a^5\,a1^2} + \frac{dh}{a^3\,a1} :$$

$$t[9] := -\frac{(k\,a^3\,a1^2 - e\,f\,a\,a1 - b1\,g\,a^2\,a1 + b1\,c\,f)\,da1}{a^6\,a1^4} - \frac{(g\,a^2\,a1 - c\,f)\,db1}{a^6\,a1^3} - \frac{f\,de}{a^5\,a1^2} + \frac{dk}{a^3\,a1} :$$

$$t[10] := \frac{da1}{a1} :$$

$$t[11] := -\frac{b1\,da1}{a\,a1^2} + \frac{db1}{a\,a1} :$$

$$form[1] := -\frac{1}{12}\,\mathrm{I}y^3\,dz + \frac{1}{12}\,\mathrm{I}z^3\,dy + \left(\frac{1}{4}\,z^2 + \frac{1}{2}\,z\,y + \frac{1}{4}\,y^2\right)du_1 + \left(-\frac{1}{4}\,z - \frac{1}{4}\,y\right)du_2 + \frac{1}{12}\,du_3 :$$

$$form[2] := \frac{1}{4}\,\mathrm{I}y^2\,dz - \frac{1}{4}\,\mathrm{I}z^2\,dy + \left(-\frac{1}{2}\,z - \frac{1}{2}\,y\right)du_1 + \frac{1}{4}\,du_2 :$$

$$form[3] := -\frac{1}{2}\,\mathrm{I}y\,dz + \frac{1}{2}\,\mathrm{I}z\,dy + \frac{1}{2}\,du_1 :$$

$$V := Vector\left(\left[-\frac{1}{12}\,\mathrm{I}y^3\,dz + \frac{1}{12}\,\mathrm{I}z^3\,dy + \left(\frac{1}{4}\,z^2 + \frac{1}{2}\,z\,y + \frac{1}{4}\,y^2\right)du_1 + \left(-\frac{1}{4}\,z - \frac{1}{4}\,y\right)du_2 + \frac{1}{12}\,du_3,\ \frac{1}{4}\,\mathrm{I}y^2\,dz - \frac{1}{4}\,\mathrm{I}z^2\,dy + \left(-\frac{1}{2}\,z - \frac{1}{2}\,y\right)du_1 + \frac{1}{4}\,du_2,\ -\frac{1}{2}\,\mathrm{I}y\,dz + \frac{1}{2}\,\mathrm{I}z\,dy + \frac{1}{2}\,du_1,\ dz,\ dy\right]\right) :$$

$$W := Ma.V :$$

$$FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], t[7], t[8], t[9], t[10], t[11], W[1], W[2], W[3], W[4], W[5]], N) :$$

```
> DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], alpha[6],
    alpha[7], alpha[8], alpha[9], α#[1], α#[2], tau, sigma, rho, zeta, ζ#], verbose);
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3, a, a1, b, b1, c, d, e, f, g, h, k]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13, E14, E15, E16]$$

The following differential 1-forms have been defined and protected:

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha^{\#}_1, \alpha^{\#}_2, \tau, \sigma, \rho, \zeta, \zeta^{\#}]$$

frame name: N **(4)**

```
> ExteriorDerivative(tau);
```

$$3\,\alpha_1 \wedge \tau + \alpha^{\#}_1 \wedge \tau + \frac{(k\,a^2\,a1 + a\,a1^2\,h - a\,b1\,g - a1\,b\,g)\,\tau \wedge \sigma}{a^4\,a1^3} + \frac{f\,(b1\,a + b\,a1)\,\tau \wedge \rho}{a^4\,a1^3} \quad \textbf{(5)}$$

$$- \frac{f\tau \wedge \zeta}{a^3\,a1} - \frac{f\tau \wedge \zeta^{\#}}{a^2\,a1^2} - \frac{(b1\,a + b\,a1)\,\sigma \wedge \rho}{a\,a1^2} + \sigma \wedge \zeta + \frac{a\,\sigma \wedge \zeta^{\#}}{a1}$$

```
> ExteriorDerivative(sigma);
```

$$2\,\alpha_1 \wedge \sigma + \alpha_6 \wedge \tau + \alpha^{\#}_1 \wedge \sigma - \frac{1}{a^7\,a1^4}\left(\left(a^3\,a1\,c\,k + a^2\,a1^2\,c\,h - e\,a^3\,a1\,g - d\,a^2\,a1^2\,g\right.\right. \quad \textbf{(6)}$$

$$\left.\left. - f\,a^2\,a1\,k - f\,a\,a1^2\,h + f\,a\,b1\,g + f\,a1\,b\,g\right)\,\tau \wedge \sigma\right)$$

$$+ \frac{(-d\,a^2\,a1^2\,f - a^3\,a1\,e\,f + b1\,a\,f^2 + b\,a1\,f^2 + a^5\,a1^2\,k + a^4\,a1^3\,h)\,\tau \wedge \rho}{a^7\,a1^4}$$

$$- \frac{(f^2 + a^3\,a1\,g - a\,c\,f)\,\tau \wedge \zeta}{a^6\,a1^2} - \frac{(f^2 + a^3\,a1\,g - a\,c\,f)\,\tau \wedge \zeta^{\#}}{a^5\,a1^3}$$

$$+ \frac{(a^3\,a1\,e + a^2\,a1^2\,d - b1\,a\,f - b\,a1\,f)\,\sigma \wedge \rho}{a^4\,a1^3} - \frac{(-f + a\,c)\,\sigma \wedge \zeta}{a^3\,a1}$$

$$- \frac{(-f + a\,c)\,\sigma \wedge \zeta^{\#}}{a^2\,a1^2} + \rho \wedge \zeta + \frac{a\,\rho \wedge \zeta^{\#}}{a1}$$

```
> ExteriorDerivative(rho);
```

$$\alpha_1 \wedge \rho + \alpha_3 \wedge \sigma + \alpha_7 \wedge \tau + \alpha^{\#}_1 \wedge \rho - \frac{1}{a^7\,a1^4}\left(\left(-e\,a\,g\,c - d\,a1\,g\,c + \mathrm{I}\,d\,a^2\,a1^2\,k\right.\right. \quad \textbf{(7)}$$

$$\left.\left. - \mathrm{I}\,e\,a^2\,a1^2\,h + \mathrm{I}\,e\,a\,a1\,b\,g + \mathrm{I}\,b1\,c\,a1\,a\,h - \mathrm{I}\,d\,a\,a1\,b1\,g - \mathrm{I}\,b\,c\,a\,k\,a1 + a\,c^2\,k + a1\,c^2\,h\right.\right.$$

$$\left.\left. - g\,a\,a1^2\,h + g^2\,a\,b1 + g^2\,a1\,b - g\,a^2\,a1\,k\right)\,\tau \wedge \sigma\right) - \frac{1}{a^7\,a1^4}\left(\left(-a^3\,a1\,c\,k\right.\right.$$

$$\left.\left. - a^2\,a1^2\,c\,h + \mathrm{I}\,b1\,a\,a1\,d\,f - \mathrm{I}\,b\,a1\,a\,e\,f + a\,c\,e\,f + a1\,c\,d\,f - f\,a\,b1\,g - f\,a1\,b\,g\right.\right.$$

$$\left.\left. + \mathrm{I}\,b\,a1^2\,a^3\,k - \mathrm{I}\,b1\,a^3\,a1^2\,h\right)\,\tau \wedge \rho\right)$$

$$+ \frac{(\mathrm{I}\,a\,b1\,c\,f - a^2\,a1\,g\,c + c^2\,f + \mathrm{I}\,a^4\,k\,a1^2 - g\,a\,a1\,f - \mathrm{I}\,a^2\,e\,f\,a1 - \mathrm{I}\,a^3\,b1\,g\,a1)\,\tau \wedge \zeta}{a^7\,a1^3}$$

$$-\frac{(\mathrm{I}\,al\,b\,c\,f+a^2\,al\,g\,c-c^2\,f+g\,a\,al\,f+\mathrm{I}\,al^3\,a^3\,h-\mathrm{I}\,al^2\,d\,a\,f-\mathrm{I}\,al^2\,b\,g\,a^2)\,\tau\wedge\zeta^{\#}}{a^6\,al^4}$$

$$-\frac{(-\mathrm{I}\,bl\,a\,al\,d+\mathrm{I}\,b\,al\,a\,e+a\,bl\,g-a\,c\,e-al\,c\,d+al\,b\,g)\,\sigma\wedge\rho}{a^4\,al^3}$$

$$+\frac{(-c^2+\mathrm{I}\,a^2\,e\,al+g\,a\,al-\mathrm{I}\,a\,bl\,c)\,\sigma\wedge\zeta}{a^4\,al^2}$$

$$-\frac{(c^2-\mathrm{I}\,al\,b\,c-g\,a\,al+\mathrm{I}\,al^2\,a\,d)\,\sigma\wedge\zeta^{\#}}{a^3\,al^3}+\frac{(c+\mathrm{I}\,bl\,a)\,\rho\wedge\zeta}{a^2\,al}$$

$$-\frac{(-c+\mathrm{I}\,b\,al)\,\rho\wedge\zeta^{\#}}{a\,al^2}+\mathrm{I}\,\zeta\wedge\zeta^{\#}$$

```
N > ExteriorDerivative(zeta);
```

$$\alpha_1\wedge\zeta+\alpha_2\wedge\rho+\alpha_4\wedge\sigma+\alpha_8\wedge\tau-\frac{1}{a^7\,al^4}((-d^2\,al\,g-h\,a^2\,al\,k+\mathrm{I}\,bl\,c\,h\,b-\mathrm{I}\,d\,bl\,g\,b \qquad \textbf{(8)}$$

$$+\mathrm{I}\,e\,b^2\,g+\mathrm{I}\,d\,a\,al\,b\,k-\mathrm{I}\,e\,a\,al\,h\,b-\mathrm{I}\,b^2\,c\,k-h^2\,a\,al^2+a\,c\,d\,k+al\,c\,d\,h-e\,a\,g\,d$$

$$+h\,a\,bl\,g+h\,al\,b\,g)\,\tau\wedge\sigma)-\frac{1}{a^7\,al^4}((-d\,a^3\,al\,k-d\,a^2\,al^2\,h+d^2\,al\,f+d\,a\,e\,f$$

$$-bl\,a\,h\,f-b\,al\,h\,f+\mathrm{I}\,b^2\,al\,a^2\,k+\mathrm{I}\,bl\,d\,f\,b-\mathrm{I}\,bl\,a^2\,al\,h\,b-\mathrm{I}\,b^2\,e\,f)\,\tau\wedge\rho)$$

$$+\frac{1}{a^7\,al^4}((-d\,a^2\,al^2\,g-f\,a\,al^2\,h+al\,c\,d\,f+\mathrm{I}\,b\,al^2\,a^3\,k-\mathrm{I}\,b\,al\,a\,e\,f-\mathrm{I}\,a^2\,bl\,g\,al\,b$$

$$+\mathrm{I}\,bl\,c\,f\,b)\,\tau\wedge\zeta)$$

$$-\frac{1}{al^4\,a^7}((a^3\,al\,g\,d+h\,a^2\,al\,f-a\,c\,f\,d+\mathrm{I}\,al^2\,h\,a^3\,b-\mathrm{I}\,al\,b^2\,g\,a^2+\mathrm{I}\,b^2\,c\,f$$

$$-\mathrm{I}\,al\,d\,f\,a\,b)\,\tau\wedge\zeta^{\#})-\frac{(-d^2\,al+\mathrm{I}\,b^2\,e-\mathrm{I}\,bl\,d\,b-d\,a\,e+b\,al\,h+bl\,h\,a)\,\sigma\wedge\rho}{a^4\,al^3}$$

$$+\frac{(-al\,c\,d+\mathrm{I}\,b\,al\,a\,e-\mathrm{I}\,bl\,c\,b+a\,al^2\,h)\,\sigma\wedge\zeta}{a^4\,al^3}$$

$$-\frac{(a\,c\,d-h\,a^2\,al-\mathrm{I}\,b^2\,c+\mathrm{I}\,al\,d\,a\,b)\,\sigma\wedge\zeta^{\#}}{al^3\,a^4}+\frac{(d\,al+\mathrm{I}\,bl\,b)\,\rho\wedge\zeta}{a^2\,al^2}$$

$$-\frac{(-d\,a+\mathrm{I}\,b^2)\,\rho\wedge\zeta^{\#}}{al^2\,a^2}+\frac{\mathrm{I}\,b\,\zeta\wedge\zeta^{\#}}{a\,al}$$

```
N > ExteriorDerivative(zeta#):
frame2 > List := GenerateForms([alpha[1], alpha[2], alpha[3], alpha[4],
                 alpha[5], alpha[6], alpha[7], alpha[8], alpha[9], alpha#[1], alpha#[2], tau,
                 sigma, rho, zeta, zeta#], 2):
frame2 > Torsion := proc(S, i, j) local k, X; k := 16·(i-1) - (i·(i-1))/2 + j - i; X
```

```
:= GetComponents(S, List); X[k]; end proc:
frame2 > result :=proc(l) local k, t, X; X := 0 : t := expand(GetComponents(l,
         List)) : for k from 1 to 120 do X := X + t[k]·List[k] od; X; end
         proc:
N > result(ExteriorDerivative(tau));
```

$$3\,\alpha_1 \wedge \tau + \overset{\#}{\alpha}_1 \wedge \tau + \left(\frac{k}{a^2\,a1^2} + \frac{h}{a^3\,a1} - \frac{b1\,g}{a^3\,a1^3} - \frac{b\,g}{a^4\,a1^2} \right) \tau \wedge \sigma + \left(\frac{f\,b1}{a^3\,a1^3} + \frac{f\,b}{a^4\,a1^2} \right) \tau \wedge \rho - \frac{f\,\tau \wedge \zeta}{a^3\,a1} - \frac{f\,\tau \wedge \overset{\#}{\zeta}}{a^2\,a1^2} + \left(-\frac{b1}{a1^2} - \frac{b}{a\,a1} \right) \sigma \wedge \rho + \sigma \wedge \zeta + \frac{a\,\sigma \wedge \overset{\#}{\zeta}}{a1} \tag{9}$$

```
N >
```