```
> restart :
> with(DifferentialGeometry) :
> with(Tools) :
> DGsetup( [z, y, u[1], u[2], u[3]], M, verbose);
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3]$$

The following vector fields have been defined and protected:

$$[D\_z, D\_y, D\_u_1, D\_u_2, D\_u_3]$$

The following differential 1-forms have been defined and protected:

$$[dz, dy, du_1, du_2, du_3]$$

frame name: M **(1)**

```
> L := evalDG(D_z + I·y· D_u[1] + I·(2·z·y + y^2)·D_u_2 + I·(3·z^2·y + 3·z·y^2 + y^3)·D_u_3);
```

$$L := D\_z + \mathrm{I}\, y\, D\_u_1 + \mathrm{I}\, y\, (2\, z + y)\, D\_u_2 + \mathrm{I}\, y\, \left(3\, z^2 + 3\, z\, y + y^2\right) D\_u_3 \quad \textbf{(2)}$$

```
M > L^# := evalDG(D_y − I·z· D_u[1]−I·(2·z·y + z^2)·D_u_2−I·(3·z^2·y + 3·z·y^2 + z^3)
        ·D_u_3);
```

$$L^{\#} := D\_y - \mathrm{I}\, z\, D\_u_1 - \mathrm{I}\, z\, (2\, y + z)\, D\_u_2 - \mathrm{I}\, z\, \left(3\, z\, y + 3\, y^2 + z^2\right) D\_u_3 \quad \textbf{(3)}$$

```
M > T := evalDG(I·LieBracket(L, L^#));
```

$$T := 2\, D\_u_1 + (4\, z + 4\, y)\, D\_u_2 + \left(6\, z^2 + 12\, z\, y + 6\, y^2\right) D\_u_3 \quad \textbf{(4)}$$

```
M > S := LieBracket(L, T);
```

$$S := 4\, D\_u_2 + (12\, z + 12\, y)\, D\_u_3 \quad \textbf{(5)}$$

```
M > R := LieBracket(L, S);
```

$$R := 12\, D\_u_3 \quad \textbf{(6)}$$

```
M > Fr := FrameData([R, S, T, L, L^#], N) :
M > DGsetup(Fr, [E], [tau[0], sigma[0], rho[0], zeta[0], ζ^#[0]], verbose) :
```

The following coordinates have been protected:

$$[z, y, u_1, u_2, u_3]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5]$$

The following differential 1-forms have been defined and protected:

$$\left[\tau_0, \sigma_0, \rho_0, \zeta_0, \zeta^{\#}{}_0\right] \quad \textbf{(7)}$$

```
N > ExteriorDerivative(tau[0]);
```

$$\sigma_0 \wedge \zeta_0 + \sigma_0 \wedge \zeta^{\#}{}_0 \quad \textbf{(8)}$$

```
N > ExteriorDerivative(sigma[0]);
```

$$\rho_0 \wedge \zeta_0 + \rho_0 \wedge \zeta^{\#}{}_0 \quad \textbf{(9)}$$

```
N > ExteriorDerivative(zeta[0]);
```

$$0\, \tau_0 \wedge \sigma_0 \quad \textbf{(10)}$$

N > *ExteriorDerivative*$(\zeta^{\#}[0])$;

$$0\,\tau_0 \wedge \sigma_0 \tag{11}$$

N > *ExteriorDerivative*(rho[0]);

$$\mathrm{I}\,\zeta_0 \wedge \zeta^{\#}{}_0 \tag{12}$$

N > *DualBasis*$([R, S, T, L, L^{\#}])$;

$$\left[-\frac{1}{12}\,\mathrm{I}\,y^3\,dz + \frac{1}{12}\,\mathrm{I}\,z^3\,dy + \left(\frac{1}{4}\,z^2 + \frac{1}{2}\,z\,y + \frac{1}{4}\,y^2\right) du_1 + \left(-\frac{1}{4}\,z - \frac{1}{4}\,y\right) du_2 + \frac{1}{12}\,du_3, \frac{1}{4}\,\mathrm{I}\,y^2\,dz - \frac{1}{4}\,\mathrm{I}\,z^2\,dy + \left(-\frac{1}{2}\,z - \frac{1}{2}\,y\right) du_1 + \frac{1}{4}\,du_2, -\frac{1}{2}\,\mathrm{I}\,y\,dz + \frac{1}{2}\,\mathrm{I}\,z\,dy + \frac{1}{2}\,du_1, dz, dy\right] \tag{13}$$

M >