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> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([z, y, u[1], u[2]], [a, b, b1, c, d, e], M, verbose);
    The following coordinates have been protected:
          [z, y, u1, u2, a, b, b1, c, d, e]
    The following vector fields have been defined and protected:
          [D_z, D_y, D_u1, D_u2, D_a, D_b, D_b1, D_c, D_d, D_e]
    The following differential 1-forms have been defined and protected:
          [dz, dy, du1, du2, da, db, db1, dc, dd, de]
          frame name: M

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(1)

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> g := Matrix([[a^3, 0, 0, 0], [c, a^2, 0, 0], [d, b, a, 0], [e, b1, 0, a]]);
          g :=
          [
            a^3  0  0  0
             c  a^2  0  0
             d   b  a  0
             e  b1  0  a
          ]

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(2)

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M > h := MatrixInverse(g) :
M > A := map(evalDG, (ExteriorDerivative(g).h));
          A :=
          [
            3 da / a          0 dz          0 dz  0 dz
            - 2 c da / a^4 + dc / a^3          2 da / a          0 dz  0 dz
            - (d a^2 - b c) da / a^6 - c db / a^5 + dd / a^3          - b da / a^3 + db / a^2          da / a          0 dz
            - (e a^2 - b1 c) da / a^6 - c db1 / a^5 + de / a^3          - b1 da / a^3 + db1 / a^2          0 dz          da / a
          ]

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(3)

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M > t[1] := da / a :

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M > t[2] := - b da / a^3 + db / a^2 :

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M > t[3] := - 2 c da / a^4 + dc / a^3 :

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M > t[4] := - (d a^2 - b c) da / a^6 - c db / a^5 + dd / a^3 :

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M > t[5] := - (e a^2 - b1 c) da / a^6 - c db1 / a^5 + de / a^3 :

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M > t[6] := - b1 da / a^3 + db1 / a^2 :

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$$\mathbf{M} > t[8] := \frac{1}{4} I y^2 dz - \frac{1}{4} I z^2 dy + \left(-\frac{1}{2} z - \frac{1}{2} y \right) du_1 + \frac{1}{4} du_2 :$$

$$\mathbf{M} > t[9] := -\frac{1}{2} I y dz + \frac{1}{2} I z dy + \frac{1}{2} du_1 :$$

$$\mathbf{M} > V := \text{Vector}([t[8], t[9], dz, dy]) :$$

$$\mathbf{M} > W := g.V :$$

$$\mathbf{M} > FD := \text{FrameData}([t[1], t[2], t[3], t[4], t[5], t[6], W[1], W[2], W[3], W[4]], N) :$$

$$\mathbf{M} > \text{DGsetup}(FD, [E], [\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \text{alpha}[4], \text{alpha}[5], \alpha^\#[2], \text{sigma}, \text{rho}, \text{zeta}, \zeta^\#], \text{verbose});$$

The following coordinates have been protected:

$$[z, y, u_1, u_2, a, b, b1, c, d, e]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10]$$

The following differential 1-forms have been defined and protected:

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_2^\#, \sigma, \rho, \zeta, \zeta^\#]$$

frame name: N

(4)

$$\mathbf{N} > \text{ExteriorDerivative}(\text{sigma});$$

$$3 \alpha_1 \wedge \sigma + \frac{(d+e) \sigma \wedge \rho}{a^3} - \frac{c \sigma \wedge \zeta}{a^3} - \frac{c \sigma \wedge \zeta^\#}{a^3} + \rho \wedge \zeta + \rho \wedge \zeta^\#$$

(5)

$$\mathbf{N} > \text{ExteriorDerivative}(\text{rho});$$

$$2 \alpha_1 \wedge \rho + \alpha_3 \wedge \sigma - \frac{(-I b1 d a + I b e a - c d - c e) \sigma \wedge \rho}{a^6} \\ + \frac{I(I c^2 + e a^3 - b1 c a) \sigma \wedge \zeta}{a^6} + \frac{I(I c^2 - d a^3 + b c a) \sigma \wedge \zeta^\#}{a^6} + \frac{(c + I b1 a) \rho \wedge \zeta}{a^3} \\ - \frac{(-c + I b a) \rho \wedge \zeta^\#}{a^3} + I \zeta \wedge \zeta^\#$$

(6)

$$\mathbf{N} > \text{ExteriorDerivative}(\text{zeta});$$

$$\alpha_1 \wedge \zeta + \alpha_2 \wedge \rho + \alpha_4 \wedge \sigma - \frac{(-d e a - d^2 a + I b^2 e - I b1 d b) \sigma \wedge \rho}{a^7} \\ + \frac{I(I d a c + e a^2 b - b1 c b) \sigma \wedge \zeta}{a^7} + \frac{I(I d a c - d a^2 b + b^2 c) \sigma \wedge \zeta^\#}{a^7} \\ + \frac{(d a + I b1 b) \rho \wedge \zeta}{a^4} - \frac{(-d a + I b^2) \rho \wedge \zeta^\#}{a^4} + \frac{I b \zeta \wedge \zeta^\#}{a^2}$$

(7)

$$\mathbf{N} >$$