> restart :

> with(DifferentialGeometry) :

> with(Tools) : with(LinearAlgebra) :

> DGsetup([z, y, u[1], u[2]], [a, a1, b, b1, c, d, e], M, verbose);

The following coordinates have been protected:

$$[z, y, u_1, u_2, a, a1, b, b1, c, d, e]$$

The following vector fields have been defined and protected:

$$[D\_z, D\_y, D\_u_1, D\_u_2, D\_a, D\_a1, D\_b, D\_b1, D\_c, D\_d, D\_e]$$

The following differential 1-forms have been defined and protected:

$$[dz, dy, du_1, du_2, da, da1, db, db1, dc, dd, de]$$

frame name: M **(1)**

> $g := Matrix([[a^2 \cdot a1, 0, 0, 0], [c, a \cdot a1, 0, 0], [d, b, a, 0], [e, b1, 0, a1]]);$

$$g := \begin{bmatrix} a^2\,a1 & 0 & 0 & 0 \\ c & a\,a1 & 0 & 0 \\ d & b & a & 0 \\ e & b1 & 0 & a1 \end{bmatrix} \quad \textbf{(2)}$$

> h := MatrixInverse(g) :

> A := map(evalDG, (ExteriorDerivative(g).h));

$$A := \begin{bmatrix} \frac{2\,da}{a} + \frac{da1}{a1} & 0\,dz & 0\,dz & 0\,dz \\ -\frac{c\,da}{a1\,a^3} - \frac{c\,da1}{a1^2\,a^2} + \frac{dc}{a^2\,a1} & \frac{da}{a} + \frac{da1}{a1} & 0\,dz & 0\,dz \\ -\frac{(d\,a\,a1 - b\,c)\,da}{a^4\,a1^2} - \frac{c\,db}{a^3\,a1^2} + \frac{dd}{a^2\,a1} & -\frac{b\,da}{a^2\,a1} + \frac{db}{a\,a1} & \frac{da}{a} & 0\,dz \\ -\frac{(e\,a\,a1 - b1\,c)\,da1}{a^3\,a1^3} - \frac{c\,db1}{a^3\,a1^2} + \frac{de}{a^2\,a1} & -\frac{b1\,da1}{a\,a1^2} + \frac{db1}{a\,a1} & 0\,dz & \frac{da1}{a1} \end{bmatrix} \quad \textbf{(3)}$$

> $t[1] := \frac{da}{a}$:

M > $t[2] := -\frac{b\,da}{a^2\,a1} + \frac{db}{a\,a1}$:

M > $t[3] := -\frac{c\,da}{a1\,a^3} - \frac{c\,da1}{a1^2\,a^2} + \frac{dc}{a^2\,a1}$:

M > $t[4] := -\frac{(d\,a\,a1 - b\,c)\,da}{a^4\,a1^2} - \frac{c\,db}{a^3\,a1^2} + \frac{dd}{a^2\,a1}$:

M > $t[5] := -\frac{(e\,a\,a1 - b1\,c)\,da1}{a^3\,a1^3} - \frac{c\,db1}{a^3\,a1^2} + \frac{de}{a^2\,a1}$:

M > $t[6] := \frac{da1}{a1}$:

M > $t[7] := -\frac{b1\ da1}{a\ a1^2} + \frac{db1}{a\ a1}$:

M > $t[8] := \frac{1}{4}\, \mathrm{I} y^2\, dz - \frac{1}{4}\, \mathrm{I} z^2\, dy + \left(-\frac{1}{2}\, z - \frac{1}{2}\, y\right) du_1 + \frac{1}{4}\, du_2$:

M > $t[9] := -\frac{1}{2}\, \mathrm{I} y\, dz + \frac{1}{2}\, \mathrm{I} z\, dy + \frac{1}{2}\, du_1$:

M > $V := Vector([t[8], t[9], dz, dy])$:

M > $W := g.V$:

M > $FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], t[7], W[1], W[2], W[3], W[4]], N)$:

M > $DGsetup\left(FD, [E], \left[\mathrm{alpha}[1], \mathrm{alpha}[2], \mathrm{alpha}[3], \mathrm{alpha}[4], \mathrm{alpha}[5], \alpha^{\#}[1], \alpha^{\#}[2], \mathrm{sigma}, \mathrm{rho}, \mathrm{zeta}, \zeta^{\#}\right], verbose\right)$;

The following coordinates have been protected:

$$[z, y, u_1, u_2, a, a1, b, b1, c, d, e]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11]$$

The following differential 1-forms have been defined and protected:

$$\left[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha^{\#}{}_1, \alpha^{\#}{}_2, \sigma, \rho, \zeta, \zeta^{\#}\right]$$

frame name: N **(4)**

N > $ExteriorDerivative(\mathrm{sigma})$;

$$2\,\alpha_1 \wedge \sigma + \alpha^{\#}{}_1 \wedge \sigma + \frac{(e\,a + d\,a1)\,\sigma \wedge \rho}{a^2\,a1^2} - \frac{c\,\sigma \wedge \zeta}{a^2\,a1} - \frac{c\,\sigma \wedge \zeta^{\#}}{a\,a1^2} + \rho \wedge \zeta + \frac{a\,\rho \wedge \zeta^{\#}}{a1} \tag{5}$$

N > $ExteriorDerivative(\mathrm{rho})$;

$$\alpha_1 \wedge \rho + \alpha_3 \wedge \sigma + \alpha^{\#}{}_1 \wedge \rho - \frac{(-c\,a\,e - \mathrm{I}\,b1\,a\,a1\,d + \mathrm{I}\,a1\,b\,a\,e - c\,a1\,d)\,\sigma \wedge \rho}{a^4\,a1^3} + \frac{\mathrm{I}\,(\mathrm{I}\,c^2 + a^2\,a1\,e - a\,b1\,c)\,\sigma \wedge \zeta}{a^4\,a1^2} + \frac{\mathrm{I}\,(\mathrm{I}\,c^2 - d\,a\,a1^2 + a1\,b\,c)\,\sigma \wedge \zeta^{\#}}{a^3\,a1^3} + \frac{(c + \mathrm{I}\,b1\,a)\,\rho \wedge \zeta}{a^2\,a1} - \frac{(-c + \mathrm{I}\,a1\,b)\,\rho \wedge \zeta^{\#}}{a\,a1^2} + \mathrm{I}\,\zeta \wedge \zeta^{\#} \tag{6}$$

N > $ExteriorDerivative(\mathrm{zeta})$;

$$\alpha_1 \wedge \zeta + \alpha_2 \wedge \rho + \alpha_4 \wedge \sigma - \frac{(\mathrm{I}\,b^2\,e - \mathrm{I}\,b1\,d\,b - d\,a\,e - a1\,d^2)\,\sigma \wedge \rho}{a^4\,a1^3} + \frac{\mathrm{I}\,(\mathrm{I}\,d\,a1\,c + a\,a1\,e\,b - b\,b1\,c)\,\sigma \wedge \zeta}{a^4\,a1^3} + \frac{\mathrm{I}\,(\mathrm{I}\,d\,a\,c - d\,a\,a1\,b + b^2\,c)\,\sigma \wedge \zeta^{\#}}{a1^3\,a^4} + \frac{(d\,a1 + \mathrm{I}\,b1\,b)\,\rho \wedge \zeta}{a^2\,a1^2} - \frac{(-d\,a + \mathrm{I}\,b^2)\,\rho \wedge \zeta^{\#}}{a1^2\,a^2} + \frac{\mathrm{I}\,b\,\zeta \wedge \zeta^{\#}}{a\,a1} \tag{7}$$

N > $ExteriorDerivative\left(\zeta^{\#}\right)$;

(8)

$$\alpha_5 \wedge \sigma + \alpha^{\#}_1 \wedge \zeta^{\#} + \alpha^{\#}_2 \wedge \rho - \frac{\left(-e\, a1\, d - a\, e^2 - \mathrm{I}\, b1^2\, d + \mathrm{I}\, b\, e\, b1\right) \sigma \wedge \rho}{a^4\, a1^3} + \frac{\mathrm{I}\left(\mathrm{I}\, e\, a1\, c + e\, a\, a1\, b1 - b1^2\, c\right) \sigma \wedge \zeta}{a^4\, a1^3} + \frac{\mathrm{I}\left(\mathrm{I}\, e\, a\, c - a\, a1\, d\, b1 + b\, b1\, c\right) \sigma \wedge \zeta^{\#}}{a1^3\, a^4} + \frac{\left(e\, a1 + \mathrm{I}\, b1^2\right) \rho \wedge \zeta}{a^2\, a1^2} - \frac{\left(-e\, a + \mathrm{I}\, b1\, b\right) \rho \wedge \zeta^{\#}}{a1^2\, a^2} + \frac{\mathrm{I}\, b1\, \zeta \wedge \zeta^{\#}}{a\, a1} \quad \textbf{(8)}$$

N >