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> restart :
> with(LinearAlgebra) :
> with(DifferentialGeometry) :
> DGsetup([b, c, d, e, b1, c1, d1, e1, z[1], z[2], w[1], w[2], v], M2, verbose);

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The following coordinates have been protected:

$$[b, c, d, e, b1, c1, d1, e1, z_1, z_2, w_1, w_2, v]$$

The following vector fields have been defined and protected:

$$[D_b, D_c, D_d, D_e, D_{b1}, D_{c1}, D_{d1}, D_{e1}, D_{z_1}, D_{z_2}, D_{w_1}, D_{w_2}, D_v]$$

The following differential 1-forms have been defined and protected:

$$[db, dc, dd, de, db1, dc1, dd1, de1, dz_1, dz_2, dw_1, dw_2, dv]$$

frame name: M2

(1)

$$\begin{aligned}
 \mathbf{M2} > u[1] &:= \frac{dc}{c}; u[2] := -\frac{-db\ c + dc\ b}{c^2\ c1}; u[3] \\
 &:= \frac{dd\ c1\ c^2 - de\ b\ c1\ c - c1\ dc\ d\ c + c1\ dc\ e\ b + dc1\ c^2\ d - dc1\ c\ e\ b}{c1^2\ c^3}; u[4] := \\
 &-\frac{-de\ c\ c1 + e\ c1\ dc - e\ dc1\ c}{c1\ c^2}; u[5] := \frac{dc1}{c1}; u[7] := \\
 &-\frac{1}{c^2\ c1^3}(-dd1\ c\ c1^2 + de1\ b1\ c\ c1 - c1^2\ dc\ d1 + c1\ dc\ e1\ b1 + dc1\ c\ d1\ c1 \\
 &- dc1\ c\ e1\ b1); u[6] := -\frac{-db1\ c1 + dc1\ b1}{c1^2\ c}; u[8] \\
 &:= \frac{de1\ c\ c1 + e1\ c1\ dc - e1\ dc1\ c}{c\ c1^2};
 \end{aligned}$$

$$u_1 := \frac{dc}{c}$$

$$u_2 := -\frac{-db\ c + dc\ b}{c^2\ c1}$$

$$u_3 := \frac{dd\ c1\ c^2 - de\ b\ c1\ c - c1\ dc\ d\ c + c1\ dc\ e\ b + dc1\ c^2\ d - dc1\ c\ e\ b}{c1^2\ c^3}$$

$$u_4 := -\frac{-de\ c\ c1 + c1\ dc\ e - dc1\ c\ e}{c1\ c^2}$$

$$u_5 := \frac{dc1}{c1}$$

$$u_7 := -\frac{-dd1\ c\ c1^2 + de1\ b1\ c\ c1 - c1^2\ dc\ d1 + c1\ dc\ e1\ b1 + dc1\ c\ d1\ c1 - dc1\ c\ e1\ b1}{c^2\ c1^3}$$

$$u_6 := -\frac{-db1\ c1 + dc1\ b1}{c1^2\ c}$$

$$u_8 := \frac{de1\ c\ c1 + c1\ dc\ e1 - dc1\ c\ e1}{c\ c1^2}$$

(2)

$$\begin{aligned}
\mathbf{M2} > \text{Lambda}[1] := & -\frac{1}{2} \text{Iccl} (w_1 + w_2 z_1) dz_1 + \frac{\frac{1}{4} \text{Iccl} (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} \\
& + \frac{1}{2} \text{Iccl} (z_1 + w_1 z_2) dw_1 - \frac{\frac{1}{4} \text{Iccl} (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} \\
& + \frac{1}{2} \text{ccl} (-1 + z_2 w_2) dv :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > \text{Lambda}[2] := & \left(c - \frac{1}{2} \text{Ib} w_1 - \frac{1}{2} \text{Ib} w_2 z_1 \right) dz_1 \\
& + \frac{1}{4} \frac{(-4 c w_1 - 4 c w_2 z_1 + \text{Ib} w_1^2 + 2 \text{Ib} w_2 z_1 w_1 + \text{Ib} z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} + \frac{1}{2} \text{Ib} (z_1 \\
& + w_1 z_2) dw_1 - \frac{\frac{1}{4} \text{Ib} (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} + \frac{1}{2} b (-1 + z_2 w_2) dv :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > \text{Lambda}[3] := & \left(e - \frac{1}{2} \text{Id} w_1 - \frac{1}{2} \text{Id} w_2 z_1 \right) dz_1 \\
& + \frac{1}{4} \frac{1}{\text{cl} (-1 + z_2 w_2)} \left((\text{Id} z_1^2 w_2^2 \text{cl} - 4 e w_2 z_1 \text{cl} + 2 \text{Id} w_2 z_1 w_1 \text{cl} + 4 c \right. \\
& \left. - 4 e w_1 \text{cl} + \text{Id} w_1^2 \text{cl}) dz_2 \right) + \frac{1}{2} \text{Id} (z_1 + w_1 z_2) dw_1 \\
& - \frac{\frac{1}{4} \text{Id} (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} + \frac{1}{2} d (-1 + z_2 w_2) dv :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > \text{Lambda}[4] := & -\frac{1}{2} \text{Ibl} (w_1 + w_2 z_1) dz_1 + \frac{\frac{1}{4} \text{Ibl} (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} \\
& + \left(\text{cl} + \frac{1}{2} \text{Ibl} z_1 + \frac{1}{2} \text{Ibl} w_1 z_2 \right) dw_1 \\
& - \frac{1}{4} \frac{(4 \text{cl} z_1 + 4 \text{cl} w_1 z_2 + \text{Ibl} z_1^2 + 2 \text{Ibl} z_2 z_1 w_1 + \text{Ibl} w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} \\
& + \frac{1}{2} \text{bl} (-1 + z_2 w_2) dv :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > \text{Lambda}[5] := & -\frac{1}{2} \text{Idl} (w_1 + w_2 z_1) dz_1 + \frac{\frac{1}{4} \text{Idl} (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} \\
& + \left(e l + \frac{1}{2} \text{Idl} z_1 + \frac{1}{2} \text{Idl} w_1 z_2 \right) dw_1 \\
& - \frac{1}{4} \frac{1}{c (-1 + z_2 w_2)} \left((\text{Idl} w_1^2 z_2^2 c + 4 e l w_1 z_2 c + 2 \text{Idl} z_2 z_1 w_1 c - 4 \text{cl} \right. \\
& \left. + 4 e l z_1 c + \text{Idl} z_1^2 c) dw_2 \right) + \frac{1}{2} dl (-1 + z_2 w_2) dv :
\end{aligned}$$

$$\mathbf{M2} > \text{Fr3} := \text{FrameData}([u[1], u[2], u[3], u[4], u[5], u[6], u[7], u[8], \text{Lambda}[1],$$

Lambda[2], Lambda[3], Lambda[4], Lambda[5]], P2) :

M2 > $DGsetup(Fr3, [E], [\text{beta}[1], \text{beta}[2], \text{beta}[3], \text{beta}[4], \text{beta}[5], \text{beta}[6], \text{beta}[7], \text{beta}[8], \text{rho}, \text{kappa}, \text{zeta}, \kappa^\#, \zeta^\#]);$
frame name: P2 (3)

P2 > $visualisation := \text{proc}(l); \text{subs}(\{\text{beta}[5] = \text{conjugate}(\text{beta}[1]), \text{beta}[6] = \text{conjugate}(\text{beta}[2]), \text{beta}[7] = \text{conjugate}(\text{beta}[3]), \text{beta}[8] = \text{conjugate}(\text{beta}[4]), w[1] = \text{conjugate}(z[1]), w[2] = \text{conjugate}(z[2]), b1 = \text{conjugate}(b), c1 = \text{conjugate}(c), d1 = \text{conjugate}(d), e1 = \text{conjugate}(e), \kappa^\# = \text{conjugate}(\text{kappa}), \zeta^\# = \text{conjugate}(\text{zeta})\}, l); \text{end proc};$

P2 > $visualisation(\text{ExteriorDerivative}(\text{rho}));$

$$\beta_1 \wedge \rho + \beta_5 \wedge \rho - \frac{I(Ie\bar{c}^2\bar{z}_2 - c\bar{b})\rho \wedge \kappa}{\bar{c}c^2} - \frac{\bar{z}_2\bar{c}\rho \wedge \zeta}{c} - \frac{I(I\bar{e}c^2z_2 + \bar{c}b)\rho \wedge \kappa^\#}{c\bar{c}^2} - \frac{z_2c\rho \wedge \zeta^\#}{c} + I\kappa \wedge \kappa^\# \quad (4)$$

P2 > $visualisation(\text{ExteriorDerivative}(\text{kappa}));$

$$\beta_1 \wedge \kappa + \beta_2 \wedge \rho + \frac{(Ibc\bar{b} + \bar{c}^2\bar{z}_2be - e\bar{c}cb - \bar{c}^2\bar{z}_2dc)\rho \wedge \kappa}{\bar{c}^2c^3} + \frac{\bar{b}\rho \wedge \zeta}{c\bar{c}} - \frac{(-\bar{e}c^2z_2b - \bar{c}^2be + Ib^2\bar{c} + \bar{c}^2dc)\rho \wedge \kappa^\#}{c^2\bar{c}^3} - \frac{z_2b\rho \wedge \zeta^\#}{\bar{c}^2} - \frac{\bar{z}_2\bar{c}\kappa \wedge \zeta}{c} + \frac{(-e\bar{c} + Ib)\kappa \wedge \kappa^\#}{\bar{c}c} + \zeta \wedge \kappa^\# \quad (5)$$

P2 > $visualisation(\text{ExteriorDerivative}(\text{zeta}));$

$$\beta_1 \wedge \zeta + \beta_3 \wedge \rho + \beta_4 \wedge \kappa - \beta_5 \wedge \zeta + \frac{(-\bar{c}^2\bar{b}e^2 + ec^2\bar{b}\bar{e}z_2 - ec^2\bar{d}\bar{c}z_2 + Idc\bar{c}\bar{b})\rho \wedge \kappa}{c^3\bar{c}^3} + \frac{(\bar{z}_2\bar{c}^3be - \bar{z}_2\bar{c}^3cd - z_2c^3\bar{b}\bar{e} + z_2c^3\bar{d}\bar{c} + \bar{c}^2c\bar{b}e)\rho \wedge \zeta}{c^3\bar{c}^3} - \frac{(\bar{c}^2cde + \bar{e}c^2bez_2 - 2\bar{e}c^3dz_2 + Id\bar{c}cb - \bar{c}^2be^2)\rho \wedge \kappa^\#}{\bar{c}^3c^3} + \frac{z_2(-2cd + be)\rho \wedge \zeta^\#}{\bar{c}^2c} - \frac{\bar{c}ez_2\kappa \wedge \zeta}{c^2} + \frac{(\bar{e}c^2ez_2 - e^2\bar{c}^2 + I\bar{c}cd)\kappa \wedge \kappa^\#}{c^2\bar{c}^2} - \frac{ez_2\kappa \wedge \zeta^\#}{c} - \frac{(\bar{e}c^2z_2 - e\bar{c}^2)\zeta \wedge \kappa^\#}{\bar{c}^2c} + \frac{z_2c\zeta \wedge \zeta^\#}{c} \quad (6)$$

P2 > $\text{new} := \text{proc}(l); \text{subs}(\{b = -Ie \cdot c1, b1 = Ie1 \cdot c\}, l); \text{end proc};$

> **for** i **from** 3 **to** 8 **do** $p[i] := \text{new}(u[i]);$ **od;**

$$p_3 := \frac{dd\ c\ l\ c^2 + I\ d\ e\ e\ c\ l^2\ c - c\ l\ d\ c\ d\ c - I\ c\ l^2\ d\ c\ e^2 + d\ c\ l\ c^2\ d + I\ d\ c\ l\ c\ e^2\ c\ l}{c\ l^2\ c^3}$$

$$p_4 := -\frac{-d\ e\ c\ c\ l + c\ l\ d\ c\ e - d\ c\ l\ c\ e}{c\ l\ c^2}$$

$$p_5 := \frac{d\ c\ l}{c\ l}$$

$$p_6 := -\frac{-d\ b\ l\ c\ l + I\ d\ c\ l\ e\ l\ c}{c\ l^2\ c}$$

$$p_7 := -\frac{-d\ d\ l\ c\ c\ l^2 + I\ d\ e\ l\ e\ l\ c^2\ c\ l - c\ l^2\ d\ c\ d\ l + I\ c\ l\ d\ c\ e\ l^2\ c + d\ c\ l\ c\ d\ l\ c\ l - I\ d\ c\ l\ c^2\ e\ l^2}{c^2\ c\ l^3}$$

$$p_8 := \frac{d\ e\ l\ c\ c\ l + c\ l\ d\ c\ e\ l - d\ c\ l\ c\ e\ l}{c\ c\ l^2}$$

(7)

> for i from 1 to 5 do T[i] := new(Lambda[i]) od ;

$$T_1 := -\frac{1}{2} I\ c\ c\ l\ (w_1 + w_2\ z_1)\ dz_1 + \frac{\frac{1}{4} I\ c\ c\ l\ (w_1^2 + 2\ w_2\ z_1\ w_1 + z_1^2\ w_2^2)\ dz_2}{-1 + z_2\ w_2} + \frac{1}{2} I\ c\ c\ l\ (z_1$$

$$+ w_1\ z_2)\ dw_1 - \frac{\frac{1}{4} I\ c\ c\ l\ (z_1^2 + 2\ z_2\ z_1\ w_1 + w_1^2\ z_2^2)\ dw_2}{-1 + z_2\ w_2} + \frac{1}{2} c\ c\ l\ (-1 + z_2\ w_2)\ dv$$

$$T_2 := \left(c - \frac{1}{2} e\ w_1\ c\ l - \frac{1}{2} e\ w_2\ z_1\ c\ l \right) dz_1$$

$$+ \frac{1}{4} \frac{(-4\ c\ w_1 - 4\ c\ w_2\ z_1 + e\ w_1^2\ c\ l + 2\ w_1\ e\ w_2\ z_1\ c\ l + w_2^2\ z_1^2\ e\ c\ l)\ dz_2}{-1 + z_2\ w_2} + \frac{1}{2} e\ c\ l\ (z_1$$

$$+ w_1\ z_2)\ dw_1 - \frac{1}{4} \frac{e\ c\ l\ (z_1^2 + 2\ z_2\ z_1\ w_1 + w_1^2\ z_2^2)\ dw_2}{-1 + z_2\ w_2} - \frac{1}{2} I\ e\ c\ l\ (-1 + z_2\ w_2)\ dv$$

$$T_3 := \left(e - \frac{1}{2} I\ d\ w_1 - \frac{1}{2} I\ d\ w_2\ z_1 \right) dz_1$$

$$+ \frac{1}{4} \frac{(I\ d\ z_1^2\ w_2^2\ c\ l - 4\ e\ w_2\ z_1\ c\ l + 2\ I\ d\ w_2\ z_1\ w_1\ c\ l + 4\ c - 4\ e\ w_1\ c\ l + I\ d\ w_1^2\ c\ l)\ dz_2}{c\ l\ (-1 + z_2\ w_2)}$$

$$+ \frac{1}{2} I\ d\ (z_1 + w_1\ z_2)\ dw_1 - \frac{\frac{1}{4} I\ d\ (z_1^2 + 2\ z_2\ z_1\ w_1 + w_1^2\ z_2^2)\ dw_2}{-1 + z_2\ w_2} + \frac{1}{2} d\ (-1 + z_2\ w_2)\ dv$$

$$T_4 := \frac{1}{2} e\ l\ c\ (w_1 + w_2\ z_1)\ dz_1 - \frac{1}{4} \frac{e\ l\ c\ (w_1^2 + 2\ w_2\ z_1\ w_1 + z_1^2\ w_2^2)\ dz_2}{-1 + z_2\ w_2} + \left(c\ l - \frac{1}{2} e\ l\ z_1\ c \right.$$

$$\left. - \frac{1}{2} e\ l\ w_1\ z_2\ c \right) dw_1$$

$$- \frac{1}{4} \frac{(4\ c\ l\ z_1 + 4\ c\ l\ w_1\ z_2 - e\ l\ z_1^2\ c - 2\ e\ l\ w_1\ z_2\ c\ z_1 - e\ l\ w_1^2\ z_2^2\ c)\ dw_2}{-1 + z_2\ w_2} + \frac{1}{2} I\ e\ l\ c\ (-1$$

$$\begin{aligned}
& + z_2 w_2) dv \\
T_5 := & -\frac{1}{2} Idl (w_1 + w_2 z_1) dz_1 + \frac{\frac{1}{4} Idl (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} + \left(el + \frac{1}{2} Idl z_1 \right. \\
& \left. + \frac{1}{2} Idl w_1 z_2 \right) dw_1 \\
& - \frac{1}{4} \frac{(Idl w_1^2 z_2^2 c + 4 el w_1 z_2 c + 2 Idl z_2 z_1 w_1 c - 4 cl + 4 el z_1 c + Idl z_1^2 c) dw_2}{c (-1 + z_2 w_2)} \\
& + \frac{1}{2} dl (-1 + z_2 w_2) dv
\end{aligned} \tag{8}$$

M2 >