> restart :

> with(LinearAlgebra) :

> with(DifferentialGeometry) : with(Tools) :

> DGsetup([b, c, d, e, f, b1, c1, d1, e1, f1, z[1], z[2], w[1], w[2], v], M, verbose);

The following coordinates have been protected:

$$[b, c, d, e, f, b1, c1, d1, e1, f1, z_1, z_2, w_1, w_2, v]$$

The following vector fields have been defined and protected:

$$[D\_b, D\_c, D\_d, D\_e, D\_f, D\_b1, D\_c1, D\_d1, D\_e1, D\_f1, D\_z_1, D\_z_2, D\_w_1, D\_w_2, D\_v]$$

The following differential 1-forms have been defined and protected:

$$[db, dc, dd, de, df, db1, dc1, dd1, de1, df1, dz_1, dz_2, dw_1, dw_2, dv]$$

frame name: M **(1)**

M > $t[1] := \frac{dc}{c}$:

M > $t[2] := \frac{db}{c\,c1} - \frac{b\,dc}{c^2\,c1}$:

M > $t[3] := \frac{dd}{c\,c1} - \frac{b\,de}{c^2\,c1} + \frac{(-d\,c + e\,b)\,df}{c^2\,c1\,f}$:

M > $t[4] := \frac{de}{c} - \frac{e\,df}{c\,f}$:

M > $t[5] := \frac{df}{f}$:

M > $s[1] := \frac{dc1}{c1}$:

M > $s[2] := \frac{db1}{c\,c1} - \frac{b1\,dc1}{c\,c1^2}$:

M > $s[3] := \frac{dd1}{c\,c1} - \frac{b1\,de1}{c\,c1^2} + \frac{(-d1\,c1 + e1\,b1)\,df1}{c1^2\,c\,f1}$:

M > $s[4] := \frac{de1}{c1} - \frac{e1\,df1}{c1\,f1}$:

M > $s[5] := \frac{df1}{f1}$:

M > $\Omega[1] := evalDG\left(-\frac{1}{2}\,I\,(w_1 + z_1\,w_2)\,dz_1 + \frac{\frac{1}{4}\,I\,(w_1^2 + 2\,w_2\,z_1\,w_1 + z_1^2\,w_2^2)\,dz_2}{-1 + z_2\,w_2} + \frac{1}{2}\,I\,(z_1 + w_1\,z_2)\,dw_1 - \frac{\frac{1}{4}\,I\,(z_1^2 + 2\,z_2\,z_1\,w_1 + w_1^2\,z_2^2)\,dw_2}{-1 + z_2\,w_2} + \left(-\frac{1}{2} + \frac{1}{2}\,z_2\,w_2\right)\,dv\right)$:

M > $\Omega[2] := evalDG\left(dz[1] - \frac{(w_1 + z_1\,w_2)\,dz_2}{-1 + z_2\,w_2}\right)$:

M > Omega[3] := dz_2 :

M > Omega[4] := $dw_1 - \frac{(z_1 + w_1 z_2)\, dw_2}{-1 + z_2 w_2}$:

M > Omega[5] := $dw[2]$:

M > Theta[1] := *evalDG*(*c*·*c1*·Omega[1]) :

M > Theta[2] := *evalDG*(*b*·Omega[1] + *c*· Omega[2]) :

M > Theta[3] := *evalDG*(*d* ·Omega[1] + e·Omega[2] + *f*· Omega[3]) :

M > Theta[4] := *evalDG*(*b1*·Omega[1] + *c1* Omega[4]) :

M > Theta[5] := *evalDG*(*d1*·Omega[1] + *e1*·Omega[4] + *f1*·Omega[5]) :

M >

M > *Fr2* := *FrameData*([*t*[1], *t*[2], *t*[3], *t*[4], *t*[5], *s*[1], *s*[2], *s*[3], *s*[4], *s*[5], Theta[1], Theta[2], Theta[3], Theta[4], Theta[5]], *P*) :

M > *DGsetup*(*Fr2*, [*E*], [alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], $\alpha^{\#}$[1], $\alpha^{\#}$[2], $\alpha^{\#}$[3], $\alpha^{\#}$[4], $\alpha^{\#}$[5], rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$]);

frame name: P **(2)**

List := *GenerateForms*([alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], $\alpha^{\#}$[1], $\alpha^{\#}$[2], $\alpha^{\#}$[3], $\alpha^{\#}$[4], $\alpha^{\#}$[5], rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$], 2) :

Torsion :=**proc**(*S*, *i*, *j*) **local** *k*, *X*; $k := 15\cdot(i-1) - \frac{i\cdot(i-1)}{2} + j - i$; *X* := *GetComponents*(*S*, *List*); *X*[*k*]; **end proc**:

result :=**proc**(*l*) **local** *k*, *t*, *X*; *X* := 0 : *t* := (*expand*(*simplify*(*GetComponents*(*l*, *List*)))) : **for** *k* **from** 1 **to** 105 **do** *X* := *X* + *t*[*k*]·*List*[*k*] **od**; *X*; **end proc**:

P > *visualisation* :=**proc** (*l*); *subs*({$\alpha^{\#}$[2] = *conjugate*(alpha[2]), $\alpha^{\#}$[3] = *conjugate*(alpha[3]), $\alpha^{\#}$[4] = *conjugate*(alpha[4]), $\alpha^{\#}$[5] = *conjugate*(alpha[5]), $\alpha^{\#}$[1] = *conjugate*(alpha[1]), *w*[1] = *conjugate*(*z*[1]), *w*[2] = *conjugate*(*z*[2]), *b1* = *conjugate*(*b*), *c1* = *conjugate*(*c*), *d1* = *conjugate*(*d*), *e1* = *conjugate*(*e*), *f1* = *conjugate*(*f*), $\kappa^{\#}$ = *conjugate*(kappa), $\zeta^{\#}$ = *conjugate*(zeta)}, *l*); **end proc**:

P > *result*(*ExteriorDerivative*(rho));

$$\alpha_1 \wedge \rho + \alpha^{\#}_1 \wedge \rho + \left(\frac{e\, w_2}{f c\, (-1 + z_2 w_2)} - \frac{\mathrm{I}\, b1}{c\, c1\, (-1 + z_2 w_2)} + \frac{\mathrm{I}\, w_2\, b1\, z_2}{c\, c1\, (-1 + z_2 w_2)}\right) \rho \wedge \kappa - \frac{w_2\, \rho \wedge \zeta}{f\, (-1 + z_2 w_2)} + \left(\frac{z_2\, e1}{f1\, c1\, (-1 + z_2 w_2)} + \frac{\mathrm{I}\, b}{c1\, c\, (-1 + z_2 w_2)} - \frac{\mathrm{I}\, z_2 w_2\, b}{c1\, c\, (-1 + z_2 w_2)}\right) \rho \wedge \kappa^{\#} - \frac{z_2\, \rho \wedge \zeta^{\#}}{f1\, (-1 + z_2 w_2)} + \mathrm{I}\, \kappa \wedge \kappa^{\#} \tag{3}$$

P > *result*(*ExteriorDerivative*(kappa))

$$\alpha_1 \wedge \kappa + \alpha_2 \wedge \rho + \left(\frac{\mathrm{I}\, b\, \overline{z_2}\, \overline{b}\, z_2}{c^2\, \overline{c}^2\, (-1 + z_2 \overline{z_2})} - \frac{\overline{z_2}\, d}{c f \overline{c}\, (-1 + z_2 \overline{z_2})} + \frac{\overline{z_2}\, e\, b}{c^2 f \overline{c}\, (-1 + z_2 \overline{z_2})} \right. \tag{4}$$

$$-\frac{e\,\overline{b}}{c\,f\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}-\frac{I\,b\,\overline{b}}{c^2\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}\Bigg)\rho\wedge\kappa+\frac{\overline{b}\,\rho\wedge\zeta}{f\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}+\Bigg($$

$$-\frac{I\,b^2\,z_2\,\overline{z_2}}{\overline{c}^2\,c^2\,(-1+z_2\,\overline{z_2})}+\frac{\overline{e}\,b\,z_2}{\overline{c}^2\,\overline{f}\,c\,(-1+z_2\,\overline{z_2})}+\frac{e\,b}{\overline{c}^2\,f\,c\,(-1+z_2\,\overline{z_2})}-\frac{d}{\overline{c}^2\,f\,(-1+z_2\,\overline{z_2})}$$

$$+\frac{I\,b^2}{\overline{c}^2\,c^2\,(-1+z_2\,\overline{z_2})}\Bigg)\rho\wedge\kappa^{\#}-\frac{b\,z_2\,\rho\wedge\zeta^{\#}}{\overline{c}\,\overline{f}\,c\,(-1+z_2\,\overline{z_2})}-\frac{\overline{z_2}\,\kappa\wedge\zeta}{f\,(-1+z_2\,\overline{z_2})}$$

$$+\left(\frac{I\,b\,z_2\,\overline{z_2}}{c\,(-1+z_2\,\overline{z_2})\,\overline{c}}-\frac{e}{\overline{c}\,f\,(-1+z_2\,\overline{z_2})}-\frac{I\,b}{c\,(-1+z_2\,\overline{z_2})\,\overline{c}}\right)\kappa\wedge\kappa^{\#}$$

$$+\frac{c\,\zeta\wedge\kappa^{\#}}{(-1+z_2\,\overline{z_2})\,\overline{c}\,f}$$

P > *result*(*ExteriorDerivative*(zeta));

$$\alpha_3\wedge\rho+\alpha_4\wedge\kappa+\alpha_5\wedge\zeta+\Bigg(\frac{I\,b\,d\,\overline{z_2}\,z_2}{c^2\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}-\frac{\overline{b}\,e^2}{c^2\,f\,\overline{c}^2\,(-1+z_2\,\overline{z_2})} \tag{5}$$

$$-\frac{I\,\overline{b}\,d}{c^2\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}\Bigg)\rho\wedge\kappa+\Bigg(-\frac{\overline{z_2}\,d}{c\,f\,\overline{c}\,(-1+z_2\,\overline{z_2})}+\frac{\overline{z_2}\,e\,b}{c^2\,f\,\overline{c}\,(-1+z_2\,\overline{z_2})}$$

$$+\frac{e\,\overline{b}}{c\,f\,\overline{c}^2\,(-1+z_2\,\overline{z_2})}\Bigg)\rho\wedge\zeta+\Bigg(-\frac{I\,d\,b\,z_2\,\overline{z_2}}{\overline{c}^2\,c^2\,(-1+z_2\,\overline{z_2})}+\frac{\overline{e}\,d\,z_2}{\overline{c}^2\,\overline{f}\,c\,(-1+z_2\,\overline{z_2})}$$

$$-\frac{d\,e}{\overline{c}^2\,f\,c\,(-1+z_2\,\overline{z_2})}+\frac{e^2\,b}{\overline{c}^2\,f\,c^2\,(-1+z_2\,\overline{z_2})}+\frac{I\,d\,b}{\overline{c}^2\,c^2\,(-1+z_2\,\overline{z_2})}\Bigg)\rho\wedge\kappa^{\#}$$

$$-\frac{d\,z_2\,\rho\wedge\zeta^{\#}}{\overline{f}\,\overline{c}\,c\,(-1+z_2\,\overline{z_2})}-\frac{e\,\overline{z_2}\,\kappa\wedge\zeta}{c\,f\,(-1+z_2\,\overline{z_2})}+\Bigg(\frac{I\,d\,\overline{z_2}\,z_2}{c\,\overline{c}\,(-1+z_2\,\overline{z_2})}-\frac{e^2}{c\,\overline{c}\,(-1+z_2\,\overline{z_2})\,f}$$

$$-\frac{I\,d}{c\,\overline{c}\,(-1+z_2\,\overline{z_2})}\Bigg)\kappa\wedge\kappa^{\#}+\frac{e\,\zeta\wedge\kappa^{\#}}{\overline{c}\,f\,(-1+z_2\,\overline{z_2})}$$

P > $new := \textbf{proc}\ (l);\ subs\left(\left\{a=c\cdot c1,\ f=\frac{c}{c1\cdot(-1+z[2]\cdot w[2])},\ f1=\frac{c1}{c\cdot(-1+z[2]\cdot w[2])}\right\},\ l\right);$ **end proc**;

new := **proc**(*l*)
subs({*a* = *c1* * *c*, *f* = *c* / (*c1* * (− 1 + *z*[2] * *w*[2])), *f1* = *c1* / (*c* * (− 1 + *z*[2] * *w*[2]))}, *l*)
end proc **(6)**

P > $g := Matrix\left(\left[[c\cdot c1, 0, 0, 0],\ [b, c, 0, 0, 0],\ \left[d, e, \frac{c}{c1}, 0, 0\right],\ [b1, 0, 0, c1, 0],\ \left[d1, 0, 0, e1, \frac{c1}{c}\right]\right]\right);$

$$g := \begin{bmatrix} c1\,c & 0 & 0 & 0 & 0 \\ b & c & 0 & 0 & 0 \\ d & e & \frac{c}{c1} & 0 & 0 \\ b1 & 0 & 0 & c1 & 0 \\ d1 & 0 & 0 & e1 & \frac{c1}{c} \end{bmatrix} \quad \textbf{(7)}$$

M > $s := ExteriorDerivative(g);$

$$s := \textbf{(8)}$$

$$\Big[\Big[c1\,c\,\alpha_1 + c1\,c\,\alpha^{\#}_{1}, 0\,\alpha_1, 0\,\alpha_1, 0\,\alpha_1, 0\,\alpha_1\Big],$$

$$\Big[b\,\alpha_1 + c1\,c\,\alpha_2, c\,\alpha_1, 0\,\alpha_1, 0\,\alpha_1, 0\,\alpha_1\Big],$$

$$\Big[c1\,c\,\alpha_3 + b\,\alpha_4 + d\,\alpha_5, c\,\alpha_4 + e\,\alpha_5, \frac{c\,\alpha_1}{c1} - \frac{c\,\alpha^{\#}_{1}}{c1}, 0\,\alpha_1, 0\,\alpha_1\Big],$$

$$\Big[b1\,\alpha^{\#}_{1} + c1\,c\,\alpha^{\#}_{2}, 0\,\alpha_1, 0\,\alpha_1, c1\,\alpha^{\#}_{1}, 0\,\alpha_1\Big],$$

$$\Big[c1\,c\,\alpha^{\#}_{3} + b1\,\alpha^{\#}_{4} + d1\,\alpha^{\#}_{5}, 0\,\alpha_1, 0\,\alpha_1, c1\,\alpha^{\#}_{4} + e1\,\alpha^{\#}_{5}, -\frac{c1\,\alpha_1}{c} + \frac{c1\,\alpha^{\#}_{1}}{c}\Big]\Big]$$

M > $dg := Matrix\left(\left[[c1\,dc + c\,dc1, 0, 0, 0, 0], [db, dc, 0, 0, 0], \left[dd, de, \frac{dc}{c1} - \frac{c\,dc1}{c1^2}, 0, 0\right], [db1, 0, 0, dc1, 0], \left[dd1, 0, 0, de1, -\frac{c1 \cdot dc}{c^2} + \frac{dc1}{c}\right]\right]\right);$

$$dg := \begin{bmatrix} c1\,dc + c\,dc1 & 0 & 0 & 0 & 0 \\ db & dc & 0 & 0 & 0 \\ dd & de & \frac{dc}{c1} - \frac{c\,dc1}{c1^2} & 0 & 0 \\ db1 & 0 & 0 & dc1 & 0 \\ dd1 & 0 & 0 & de1 & -\frac{c1\,dc}{c^2} + \frac{dc1}{c} \end{bmatrix} \quad \textbf{(9)}$$

M > $simplify(dg.MatrixInverse(g));$

$$\left[\left[\frac{c1\,dc + c\,dc1}{c1\,c}, 0, 0, 0, 0\right], \quad \textbf{(10)}$$

$$\left[-\frac{-db\,c + b\,dc}{c^2\,c1}, \frac{dc}{c}, 0, 0, 0\right],$$

$$\left[\frac{dd\,c1\,c^2 - b\,de\,c1\,c - c1\,dc\,d\,c + c1\,dc\,e\,b + dc1\,c^2\,d - dc1\,c\,e\,b}{c1^2\,c^3},\right.$$
$$\left.-\frac{-de\,c1\,c + e\,c1\,dc - e\,dc1\,c}{c1\,c^2}, \frac{c1\,dc - c\,dc1}{c\,c1}, 0, 0\right],$$
$$\left[-\frac{-db1\,c1 + b1\,dc1}{c1^2\,c}, 0, 0, \frac{dc1}{c1}, 0\right],$$
$$\left[\frac{dd1\,c\,c1^2 - b1\,de1\,c\,c1 + c1^2\,dc\,d1 - c1\,dc\,e1\,b1 - dc1\,c\,d1\,c1 + dc1\,c\,e1\,b1}{c^2\,c1^3}, 0, 0,\right.$$
$$\left.\frac{de1\,c1\,c + e1\,c1\,dc - e1\,dc1\,c}{c\,c1^2}, -\frac{c1\,dc - c\,dc1}{c\,c1}\right]\Bigg]$$

M > $u[1] := \frac{dc}{c}; u[2] := -\frac{-db\,c + dc\,b}{c^2\,c1}; u[3] := \frac{dd\,c1\,c^2 - de\,b\,c1\,c - c1\,dc\,d\,c + c1\,dc\,e\,b + dc1\,c^2\,d - dc1\,c\,e\,b}{c1^2\,c^3}; u[4] := -\frac{-de\,c\,c1 + e\,c1\,dc - e\,dc1\,c}{c1\,c^2}; u[5] := \frac{dc1}{c1}; u[7] := -\frac{-dd1\,c\,c1^2 + de1\,b1\,c\,c1 - c1^2\,dc\,d1 + c1\,dc\,e1\,b1 + dc1\,c\,d1\,c1 - dc1\,c\,e1\,b1}{c^2\,c1^3}; u[6] := -\frac{-db1\,c1 + dc1\,b1}{c1^2\,c}; u[8] := \frac{de1\,c\,c1 + e1\,c1\,dc - e1\,dc1\,c}{c\,c1^2}:$

$$u_1 := \frac{dc}{c}$$
$$u_2 := -\frac{-db\,c + b\,dc}{c^2\,c1}$$
$$u_3 := \frac{dd\,c1\,c^2 - b\,de\,c1\,c - c1\,dc\,d\,c + c1\,dc\,e\,b + dc1\,c^2\,d - dc1\,c\,e\,b}{c1^2\,c^3}$$
$$u_4 := -\frac{-de\,c1\,c + e\,c1\,dc - e\,dc1\,c}{c1\,c^2}$$
$$u_5 := \frac{dc1}{c1}$$
$$u_7 := -\frac{-dd1\,c\,c1^2 + b1\,de1\,c\,c1 - c1^2\,dc\,d1 + c1\,dc\,e1\,b1 + dc1\,c\,d1\,c1 - dc1\,c\,e1\,b1}{c^2\,c1^3}$$
$$u_6 := -\frac{-db1\,c1 + b1\,dc1}{c1^2\,c} \quad \textbf{(11)}$$

M > **for** i **from** 1 **to** 5 **do** $\Lambda[i] := simplify(new(\Theta[i]));$ **od**;

$$\Lambda_1 := -\frac{1}{2}\,\mathrm{I}\,c\,c1\,(w_1 + w_2 z_1)\,dz_1 + \frac{\frac{1}{4}\,\mathrm{I}\,c\,c1\,(w_1^2 + 2\,w_2 z_1 w_1 + z_1^2 w_2^2)\,dz_2}{-1 + z_2 w_2} + \frac{1}{2}\,\mathrm{I}\,c\,c1\,(z_1$$

$$+ w_1 z_2)\, dw_1 - \frac{\frac{1}{4}\, I\, c\, c1\, \left(z_1^2 + 2\, z_2\, z_1\, w_1 + w_1^2\, z_2^2\right) dw_2}{-1 + z_2\, w_2} + \frac{1}{2}\, c\, c1\, \left(-1 + z_2\, w_2\right) dv$$

$$\Lambda_2 := \left(c - \frac{1}{2}\, I\, b\, w_1 - \frac{1}{2}\, I\, b\, w_2\, z_1\right) dz_1$$

$$+ \frac{1}{4}\, \frac{\left(-4\, c\, w_1 - 4\, c\, w_2\, z_1 + I\, b\, w_1^2 + 2\, I\, b\, w_2\, z_1\, w_1 + I\, b\, z_1^2\, w_2^2\right) dz_2}{-1 + z_2\, w_2} + \frac{1}{2}\, I\, b\, (z_1$$

$$+ w_1 z_2)\, dw_1 - \frac{\frac{1}{4}\, I\, b\, \left(z_1^2 + 2\, z_2\, z_1\, w_1 + w_1^2\, z_2^2\right) dw_2}{-1 + z_2\, w_2} + \frac{1}{2}\, b\, \left(-1 + z_2\, w_2\right) dv$$

$$\Lambda_3 := \left(e - \frac{1}{2}\, I\, d\, w_1 - \frac{1}{2}\, I\, d\, w_2\, z_1\right) dz_1$$

$$+ \frac{1}{4}\, \frac{\left(I\, d\, z_1^2\, w_2^2\, c1 - 4\, e\, w_2\, z_1\, c1 + 2\, I\, d\, w_2\, z_1\, w_1\, c1 + 4\, c - 4\, e\, w_1\, c1 + I\, d\, w_1^2\, c1\right) dz_2}{c1\, \left(-1 + z_2\, w_2\right)}$$

$$+ \frac{1}{2}\, I\, d\, \left(z_1 + w_1\, z_2\right) dw_1 - \frac{\frac{1}{4}\, I\, d\, \left(z_1^2 + 2\, z_2\, z_1\, w_1 + w_1^2\, z_2^2\right) dw_2}{-1 + z_2\, w_2} + \frac{1}{2}\, d\, \left(-1 + z_2\, w_2\right) dv$$

$$\Lambda_4 := -\frac{1}{2}\, I\, b1\, \left(w_1 + w_2\, z_1\right) dz_1 + \frac{\frac{1}{4}\, I\, b1\, \left(w_1^2 + 2\, w_2\, z_1\, w_1 + z_1^2\, w_2^2\right) dz_2}{-1 + z_2\, w_2} + \left(c1 + \frac{1}{2}\, I\, b1\, z_1\right.$$

$$\left. + \frac{1}{2}\, I\, b1\, w_1\, z_2\right) dw_1$$

$$- \frac{1}{4}\, \frac{\left(4\, c1\, z_1 + 4\, c1\, w_1\, z_2 + I\, b1\, z_1^2 + 2\, I\, b1\, z_2\, z_1\, w_1 + I\, b1\, w_1^2\, z_2^2\right) dw_2}{-1 + z_2\, w_2} + \frac{1}{2}\, b1\, (-1$$

$$+ z_2\, w_2)\, dv$$

$$\Lambda_5 := -\frac{1}{2}\, I\, d1\, \left(w_1 + w_2\, z_1\right) dz_1 + \frac{\frac{1}{4}\, I\, d1\, \left(w_1^2 + 2\, w_2\, z_1\, w_1 + z_1^2\, w_2^2\right) dz_2}{-1 + z_2\, w_2} + \left(e1 + \frac{1}{2}\, I\, d1\, z_1\right. \qquad \textbf{(12)}$$

$$\left. + \frac{1}{2}\, I\, d1\, w_1\, z_2\right) dw_1$$

$$- \frac{1}{4}\, \frac{\left(I\, d1\, w_1^2\, z_2^2\, c + 4\, e1\, w_1\, z_2\, c + 2\, I\, d1\, z_2\, z_1\, w_1\, c - 4\, c1 + 4\, e1\, z_1\, c + I\, d1\, z_1^2\, c\right) dw_2}{c\, \left(-1 + z_2\, w_2\right)}$$

$$+ \frac{1}{2}\, d1\, \left(-1 + z_2\, w_2\right) dv$$

M >