

> restart :
 > with(LinearAlgebra) :
 > with(DifferentialGeometry) :
 > DGsetup([c, e, c1, e1, z[1], z[2], w[1], w[2], v, l], M2, verbose);
 The following coordinates have been protected:
 [c, e, c1, e1, z1, z2, w1, w2, v, l]
 The following vector fields have been defined and protected:
 [D_c, D_e, D_c1, D_e1, D_z1, D_z2, D_w1, D_w2, D_v, D_l]
 The following differential 1-forms have been defined and protected:
 [dc, de, dc1, de1, dz1, dz2, dw1, dw2, dv, dl]
 frame name: M2 (1)

> $q[1] := \frac{dc}{c} : q[2] := \frac{-Ic1 de - Ie dc1}{c c1} + \frac{I dc e}{c^2} : q[3] := \frac{dc1}{c1} : q[4]$
 $:= \frac{Ie1 dc + Ic de1}{c c1} - \frac{I dc1 e1}{c1^2} :$

> $Q[1] := -\frac{1}{2} I c c1 (w_1 + w_2 z_1) dz_1 + \frac{\frac{1}{4} I c c1 (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2}$
 $+ \frac{1}{2} I c c1 (z_1 + w_1 z_2) dw_1 - \frac{\frac{1}{4} I c c1 (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} + \frac{1}{2} c c1 (-1$
 $+ z_2 w_2) dv :$

> $Q[2] := \left(c - \frac{1}{2} w_1 e c1 - \frac{1}{2} e w_2 z_1 c1 \right) dz_1$
 $+ \frac{1}{4} \frac{(-4 c w_1 - 4 c w_2 z_1 + e w_1^2 c1 + 2 w_1 e w_2 z_1 c1 + z_1^2 w_2^2 e c1) dz_2}{-1 + z_2 w_2} + \frac{1}{2} e c1 (z_1$
 $+ w_1 z_2) dw_1 - \frac{1}{4} \frac{e c1 (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} - \frac{1}{2} I e c1 (-1 + z_2 w_2) dv :$

> $Q[3] := \left(e - \frac{1}{4} \frac{e^2 c1 w_1}{c} - \frac{1}{4} \frac{e^2 c1 w_2 z_1}{c} \right) dz_1$
 $+ \frac{1}{4} \frac{1}{c1 (-1 + z_2 w_2)} \left(\left(\frac{1}{2} \frac{e^2 c1^2 z_1^2 w_2^2}{c} - 4 e w_2 z_1 c1 + \frac{e^2 c1^2 w_2 z_1 w_1}{c} + 4 c \right. \right.$
 $\left. - 4 w_1 e c1 + \frac{1}{2} \frac{e^2 c1^2 w_1^2}{c} \right) dz_2 \Big) + \frac{1}{4} \frac{e^2 c1 (z_1 + w_1 z_2) dw_1}{c}$
 $- \frac{1}{8} \frac{e^2 c1 (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2) dw_2}{c (-1 + z_2 w_2)} - \frac{1}{4} \frac{I e^2 c1 (-1 + z_2 w_2) dv}{c} :$

> $Q[4] := \frac{1}{2} e1 c (w_1 + w_2 z_1) dz_1 - \frac{1}{4} \frac{e1 c (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{-1 + z_2 w_2} + \left(c1 \right.$

$$\begin{aligned}
& -\frac{1}{2} el z_1 c - \frac{1}{2} el w_1 z_2 c) dw_1 \\
& -\frac{1}{4} \frac{(4 cl z_1 + 4 cl w_1 z_2 - el c z_1^2 - 2 el c z_2 z_1 w_1 - el c w_1^2 z_2^2) dw_2}{-1 + z_2 w_2} + \frac{1}{2} I el c (-1 \\
& + z_2 w_2) dv :
\end{aligned}$$

$$\begin{aligned}
> Q[5] := & \frac{1}{4} \frac{el^2 c (w_1 + w_2 z_1) dz_1}{cl} - \frac{1}{8} \frac{el^2 c (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2) dz_2}{cl (-1 + z_2 w_2)} + \left(el \right. \\
& \left. - \frac{1}{4} \frac{el^2 c z_1}{cl} - \frac{1}{4} \frac{el^2 c w_1 z_2}{cl} \right) dw_1 \\
& - \frac{1}{4} \frac{1}{c (-1 + z_2 w_2)} \left(\left(-\frac{1}{2} \frac{el^2 c^2 w_1^2 z_2^2}{cl} + 4 el w_1 z_2 c - \frac{el^2 c^2 z_2 z_1 w_1}{cl} - 4 cl \right. \right. \\
& \left. \left. + 4 el z_1 c - \frac{1}{2} \frac{el^2 c^2 z_1^2}{cl} \right) dw_2 \right) + \frac{1}{4} \frac{I el^2 c (-1 + z_2 w_2) dv}{cl} :
\end{aligned}$$

$$\begin{aligned}
\mathbf{P} > r[1] := & evalDG \left(q[1] - \frac{\frac{1}{4} I (cl^4 e^2 w_2 - 2 el e c^2 cl^2 + c^4 el^2 z_2)}{c^3 cl^3} \cdot Q[1] \right. \\
& \left. + \frac{(-e cl^2 w_2 + el c^2)}{cl c^2} \cdot Q[2] + \frac{w[2] \cdot cl}{c} \cdot Q[3] \right) :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > r[2] := & evalDG \left(q[2] - I \cdot \frac{\frac{1}{2} I e el (-e cl^2 + el c^2 z_2)}{c^2 cl^3} \cdot Q[1] - \left(-\frac{\frac{1}{2} I e^2 cl w_2}{c^3} \right. \right. \\
& \left. \left. + \frac{\frac{1}{4} I (cl^4 e^2 w_2 - 2 el e c^2 cl^2 + c^4 el^2 z_2)}{c^3 cl^3} \right) \cdot Q[2] - \frac{I \cdot el}{cl} \cdot Q[3] \right. \\
& \left. + \frac{\frac{1}{2} I e (-e cl^2 + 2 el c^2 z_2)}{c^2 cl^2} \cdot Q[4] - \frac{I e z_2}{cl} \cdot Q[5] \right) :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > r[3] := & evalDG \left(q[3] + \frac{\frac{1}{4} I (cl^4 e^2 w_2 - 2 el e c^2 cl^2 + c^4 el^2 z_2)}{c^3 cl^3} \cdot Q[1] \right. \\
& \left. - \frac{-e cl^2 + el c^2 z_2}{c cl^2} \cdot Q[4] + \frac{z_2 c}{cl} \cdot Q[5] \right) :
\end{aligned}$$

$$\begin{aligned}
\mathbf{M2} > r[4] := & evalDG \left(q[4] + I \cdot \frac{\frac{1}{2} I e el (-e cl^2 w_2 + el c^2)}{cl^2 c^3} \cdot Q[1] - \left(\frac{\frac{1}{2} I el^2 c z_2}{cl^3} \right. \right. \\
& \left. \left. - \frac{\frac{1}{4} I (cl^4 e^2 w_2 - 2 el e c^2 cl^2 + c^4 el^2 z_2)}{c^3 cl^3} \right) \cdot Q[4] + \frac{I e}{c} \cdot Q[5] \right) :
\end{aligned}$$

$$+ \frac{\frac{1}{2} \text{I} e l (-2 e c l^2 w_2 + e l c^2)}{c l^2 c^2} \cdot Q[2] + \frac{\text{I} e l w_2}{c} \cdot Q[3] \Bigg) :$$

> $s[1] := \text{evalDG}(r[1] + l \cdot Q[1]) ; s[2] := \text{evalDG}(r[2] + l \cdot Q[2]) ; s[3] := \text{evalDG}(r[3] + l \cdot Q[1]) ; s[4] := \text{evalDG}(r[4] + l \cdot Q[4]) :$

> $\text{Sigma} := \text{evalDG} \left(dl + l \cdot s[1] + \frac{1}{2} \frac{(-e c l^2 w_2 + e l c^2)}{c^2 c l} s[2] + l \cdot s[3] \right. \\ - \frac{1}{2} \frac{(-e c l^2 + e l c^2 z_2)}{c c l^2} s[4] + \frac{1}{16} \frac{1}{c^6 c l^6} \left((c^8 e l^4 z_2^2 + c l^8 e^4 w_2^2 - 16 l^2 c^6 c l^6 \right. \\ \left. - 4 e l^3 c^6 z_2 e c l^2 - 4 c l^6 e^3 w_2 e l c^2 + 4 c^4 e l^2 c l^4 e^2 + 2 e l^2 c^4 z_2 c l^4 e^2 w_2) \right) \cdot Q[1] \\ + \frac{1}{8} \frac{1}{c^5 c l^4} \left((-4 l c^5 c l^3 e l + 4 l c^3 c l^5 e w_2 - 2 \text{I} e l^2 e c^4 c l^2 - \text{I} e^3 c l^6 w_2^2 \right. \\ \left. + 3 \text{I} e l e^2 c^2 c l^4 w_2 + \text{I} e l^3 c^6 z_2 - \text{I} e l^2 c^4 z_2 e c l^2 w_2) \right) \cdot Q[2] \\ + \frac{\frac{1}{4} \text{I} (e^2 c l^4 w_2^2 - 2 w_2 e l e c^2 c l^2 + c^4 e l^2)}{c l^2 c^4} Q[3] + \frac{1}{8} \frac{1}{c l^5 c^4} (2 \text{I} e l e^2 c^2 c l^4 \\ + \text{I} e l^3 c^6 z_2^2 - 3 \text{I} e l^2 e c^4 c l^2 z_2 - 4 l c^3 c l^5 e - \text{I} e^3 c l^6 w_2 + 4 l c^5 c l^3 e l z_2 \\ \left. + \text{I} e^2 c l^4 w_2 e l c^2 z_2) \cdot Q[4] - \frac{\frac{1}{4} \text{I} (-2 z_2 e l e c^2 c l^2 + c l^4 e^2 + c^4 e l^2 z_2^2)}{c l^4 c^2} \cdot Q[5] \right) :$

M2 > $Fr := \text{FrameData}([s[1], s[2], s[3], s[4], Q[1], Q[2], Q[3], Q[4], Q[5], \text{Sigma}], P) :$

M2 > $DGsetup(Fr, [E], [\text{pi}[1], \text{pi}[2], \text{pi}[3], \text{pi}[4], \text{rho}, \text{kappa}, \text{zeta}, \kappa^\#, \zeta^\#, \text{Lambda}]);$
frame name: P (2)

P >

P > $\text{ExteriorDerivative}(\text{rho});$

$$\pi_1 \wedge \rho + \pi_3 \wedge \rho + \text{I} \kappa \wedge \kappa^\# \quad (3)$$

P > $\text{ExteriorDerivative}(\text{kappa});$

$$\pi_1 \wedge \kappa + \pi_2 \wedge \rho + \zeta \wedge \kappa^\# \quad (4)$$

P > $\text{ExteriorDerivative}(\text{zeta});$

$$\pi_1 \wedge \zeta + \text{I} \pi_2 \wedge \kappa - \pi_3 \wedge \zeta \quad (5)$$

P > $\text{ExteriorDerivative}(\kappa^\#);$

(6)

$$\pi_3 \wedge \kappa^\# + \pi_4 \wedge \rho - \kappa \wedge \zeta^\# \quad (6)$$

P > *ExteriorDerivative*($\zeta^\#$);

$$-\pi_1 \wedge \zeta^\# + \pi_3 \wedge \zeta^\# - I\pi_4 \wedge \kappa^\# \quad (7)$$

P > *ExteriorDerivative*($\text{pi}[1]$);

$$-I\pi_4 \wedge \kappa - \rho \wedge \Lambda + \zeta \wedge \zeta^\# \quad (8)$$

P > *ExteriorDerivative*($\text{pi}[2]$);

$$\pi_2 \wedge \pi_3 - \pi_4 \wedge \zeta - \kappa \wedge \Lambda \quad (9)$$

P > *ExteriorDerivative*($\text{pi}[3]$);

$$I\pi_2 \wedge \kappa^\# - \rho \wedge \Lambda - \zeta \wedge \zeta^\# \quad (10)$$

P > *ExteriorDerivative*($\text{pi}[4]$);

$$-\pi_1 \wedge \pi_4 - \pi_2 \wedge \zeta^\# - \kappa^\# \wedge \Lambda \quad (11)$$

P > *ExteriorDerivative*(Lambda);

$$-\pi_1 \wedge \Lambda + I\pi_2 \wedge \pi_4 - \pi_3 \wedge \Lambda \quad (12)$$

P >