> restart :

> with(LinearAlgebra) :

> with(DifferentialGeometry);

[&minus, &mult, &plus, &tensor, &wedge, Annihilator, ApplyTransformation, ChangeFrame, ComplementaryBasis, ComposeTransformations, DGbasis, DGsetup, DGzip, DeRhamHomotopy, DualBasis, ExteriorDerivative, Flow, FrameData, GetComponents, GroupActions, Hook, InfinitesimalTransformation, IntegrateForm, IntersectSubspaces, InverseTransformation, JetCalculus, Library, LieAlgebras, LieBracket, LieDerivative, Preferences, Pullback, PullbackVector, Pushforward, RemoveFrame, Tensor, Tools, Transformation, evalDG] **(1)**

> DGsetup([c, e, c1, e1, z[1], z[2], w[1], w[2], v], M2, verbose);

The following coordinates have been protected:

$$[c, e, c1, e1, z_1, z_2, w_1, w_2, v]$$

The following vector fields have been defined and protected:

$$[D\_c, D\_e, D\_c1, D\_e1, D\_z_1, D\_z_2, D\_w_1, D\_w_2, D\_v]$$

The following differential 1-forms have been defined and protected:

$$[dc, de, dc1, de1, dz_1, dz_2, dw_1, dw_2, dv]$$

frame name: M2 **(2)**

$$> q[1] := \frac{dc}{c} : q[2] := \frac{-\mathrm{I}\, c1\, de - \mathrm{I}\, e\, dc1}{c\, c1} + \frac{\mathrm{I}\, dc\, e}{c^2} : q[3] := \frac{dc1}{c1} : q[4] := \frac{\mathrm{I}\, e1\, dc + \mathrm{I}\, c\, de1}{c\, c1} - \frac{\mathrm{I}\, dc1\, e1}{c1^2} :$$

$$> Q[1] := -\frac{1}{2}\mathrm{I}\, c\, c1\, (w_1 + w_2 z_1)\, dz_1 + \frac{\frac{1}{4}\mathrm{I}\, c\, c1\, (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2)\, dz_2}{-1 + z_2 w_2} + \frac{1}{2}\mathrm{I}\, c\, c1\, (z_1 + w_1 z_2)\, dw_1 - \frac{\frac{1}{4}\mathrm{I}\, c\, c1\, (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2)\, dw_2}{-1 + z_2 w_2} + \frac{1}{2} c\, c1\, (-1 + z_2 w_2)\, dv :$$

$$> Q[2] := \left(c - \frac{1}{2} w_1\, e\, c1 - \frac{1}{2} e\, w_2 z_1\, c1\right) dz_1 + \frac{1}{4}\frac{(-4 c\, w_1 - 4 c\, w_2 z_1 + e\, w_1^2\, c1 + 2 w_1\, e\, w_2 z_1\, c1 + z_1^2 w_2^2\, e\, c1)\, dz_2}{-1 + z_2 w_2} + \frac{1}{2} e\, c1\, (z_1 + w_1 z_2)\, dw_1 - \frac{1}{4}\frac{e\, c1\, (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2)\, dw_2}{-1 + z_2 w_2} - \frac{1}{2}\mathrm{I}\, e\, c1\, (-1 + z_2 w_2)\, dv :$$

$$> Q[3] := \left(e - \frac{1}{4}\frac{e^2\, c1\, w_1}{c} - \frac{1}{4}\frac{e^2\, c1\, w_2 z_1}{c}\right) dz_1 + \frac{1}{4}\frac{1}{c1\,(-1 + z_2 w_2)}\left(\left(\frac{1}{2}\frac{e^2\, c1^2 z_1^2 w_2^2}{c} - 4 e\, w_2 z_1\, c1 + \frac{e^2\, c1^2\, w_2 z_1 w_1}{c} + 4 c\right.\right.$$

$$-4 w_1 e\, c1 + \frac{1}{2}\frac{e^2 c1^2 w_1^2}{c}\Big) dz_2\Big) + \frac{1}{4}\frac{e^2 c1 (z_1 + w_1 z_2)\, dw_1}{c}$$

$$-\frac{1}{8}\frac{e^2 c1 (z_1^2 + 2 z_2 z_1 w_1 + w_1^2 z_2^2)\, dw_2}{c(-1 + z_2 w_2)} - \frac{\frac{1}{4} \mathrm{I} e^2 c1 (-1 + z_2 w_2)\, dv}{c} :$$

$$> Q[4] := \frac{1}{2} e1\, c (w_1 + w_2 z_1)\, dz_1 - \frac{1}{4}\frac{e1\, c (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2)\, dz_2}{-1 + z_2 w_2} + \Big(c1 - \frac{1}{2} e1\, z_1 c - \frac{1}{2} e1\, w_1 z_2 c\Big) dw_1 - \frac{1}{4}\frac{(4 c1\, z_1 + 4 c1\, w_1 z_2 - e1\, c z_1^2 - 2 e1\, c z_2 z_1 w_1 - e1\, c w_1^2 z_2^2)\, dw_2}{-1 + z_2 w_2} + \frac{1}{2} \mathrm{I} e1\, c (-1 + z_2 w_2)\, dv :$$

$$> Q[5] := \frac{1}{4}\frac{e1^2 c (w_1 + w_2 z_1)\, dz_1}{c1} - \frac{1}{8}\frac{e1^2 c (w_1^2 + 2 w_2 z_1 w_1 + z_1^2 w_2^2)\, dz_2}{c1 (-1 + z_2 w_2)} + \Big(e1 - \frac{1}{4}\frac{e1^2 c z_1}{c1} - \frac{1}{4}\frac{e1^2 c w_1 z_2}{c1}\Big) dw_1 - \frac{1}{4}\frac{1}{c(-1 + z_2 w_2)}\Big(\Big(-\frac{1}{2}\frac{e1^2 c^2 w_1^2 z_2^2}{c1} + 4 e1\, w_1 z_2 c - \frac{e1^2 c^2 z_2 z_1 w_1}{c1} - 4 c1 + 4 e1\, z_1 c - \frac{1}{2}\frac{e1^2 c^2 z_1^2}{c1}\Big) dw_2\Big) + \frac{\frac{1}{4} \mathrm{I} e1^2 c (-1 + z_2 w_2)\, dv}{c1} :$$

P $$> r[1] := evalDG\Big(q[1] - \frac{\frac{1}{4}\mathrm{I}(c1^4 e^2 w_2 - 2 e1\, e c^2 c1^2 + c^4 e1^2 z_2)}{c^3 c1^3} \cdot Q[1] + \frac{(-e\, c1^2 w_2 + e1\, c^2)}{c1\, c^2} \cdot Q[2] + \frac{w[2] \cdot c1}{c} \cdot Q[3]\Big) :$$

M2 $$> r[2] := evalDG\Big(q[2] - I \cdot \frac{\frac{1}{2}\mathrm{I} e\, e1 (-e\, c1^2 + e1\, c^2 z_2)}{c^2 c1^3} \cdot Q[1] - \Big(-\frac{\frac{1}{2}\mathrm{I} e^2 c1\, w_2}{c^3} + \frac{\frac{1}{4}\mathrm{I}(c1^4 e^2 w_2 - 2 e1\, e c^2 c1^2 + c^4 e1^2 z_2)}{c^3 c1^3}\Big) \cdot Q[2] - \frac{I \cdot e1}{c1} \cdot Q[3] + \frac{\frac{1}{2}\mathrm{I} e (-e\, c1^2 + 2 e1\, c^2 z_2)}{c^2 c1^2} \cdot Q[4] - \frac{\mathrm{I} e z_2}{c1} \cdot Q[5]\Big) :$$

M2 $$> r[3] := evalDG\Big(q[3] + \frac{\frac{1}{4}\mathrm{I}(c1^4 e^2 w_2 - 2 e1\, e c^2 c1^2 + c^4 e1^2 z_2)}{c^3 c1^3} \cdot Q[1]$$

$$- \frac{-e\,c1^2 + e1\,c^2 z_2}{c\,c1^2} \cdot Q[4] + \frac{z_2\,c}{c1} \cdot Q[5] \Big) :$$

M2 > $r[4] := evalDG\Big(q[4] + I \cdot \frac{\frac{1}{2}\,\mathrm{I}\,e1\,e\,(-e\,c1^2 w_2 + e1\,c^2)}{c1^2\,c^3} \cdot Q[1] - \Big(\frac{\frac{1}{2}\,\mathrm{I}\,e1^2\,c\,z_2}{c1^3} - \frac{\frac{1}{4}\,\mathrm{I}\,(c1^4 e^2 w_2 - 2\,e1\,e\,c^2\,c1^2 + c^4\,e1^2 z_2)}{c^3\,c1^3} \Big) \cdot Q[4] + \frac{\mathrm{I}\,e}{c} \cdot Q[5] + \frac{\frac{1}{2}\,\mathrm{I}\,e1\,(-2\,e\,c1^2 w_2 + e1\,c^2)}{c1^2\,c^2} \cdot Q[2] + \frac{\mathrm{I}\,e1\,w_2}{c} \cdot Q[3] \Big) :$

P >

M2 > $Fr := FrameData([r[1], r[2], r[3], r[4], Q[1], Q[2], Q[3], Q[4], Q[5]], P) :$

M2 > $DGsetup(Fr, [E], [\mathrm{eta}[1], \mathrm{eta}[2], \mathrm{eta}[3], \mathrm{eta}[4], \mathrm{rho}, \mathrm{kappa}, \mathrm{zeta}, \kappa^{\#}, \zeta^{\#}]);$

frame name: P **(3)**

M2 > $visualisation :=$ **proc** $(l);\ subs(\{w[1] = conjugate(z[1]), w[2] = conjugate(z[2]), c1 = conjugate(c), e1 = conjugate(e), \kappa^{\#} = conjugate(\mathrm{kappa}), \zeta^{\#} = conjugate(\mathrm{zeta})\}, l);$ **end proc**:

P > $visualisation(ExteriorDerivative(\mathrm{rho}));$

$$\eta_1 \wedge \rho + \eta_3 \wedge \rho + \mathrm{I}\,\kappa \wedge \kappa^{\#} \quad (4)$$

P > $visualisation(ExteriorDerivative(\mathrm{kappa}));$

$$\eta_1 \wedge \kappa + \eta_2 \wedge \rho + \zeta \wedge \kappa^{\#} \quad (5)$$

P > $visualisation(ExteriorDerivative(\mathrm{zeta}));$

$$\eta_1 \wedge \zeta + \mathrm{I}\,\eta_2 \wedge \kappa - \eta_3 \wedge \zeta \quad (6)$$

P > $visualisation(ExteriorDerivative(\kappa^{\#}));$

$$\eta_3 \wedge \kappa^{\#} + \eta_4 \wedge \rho - \kappa \wedge \zeta^{\#} \quad (7)$$

P > $visualisation(ExteriorDerivative(\zeta^{\#}));$

$$-\eta_1 \wedge \zeta^{\#} + \eta_3 \wedge \zeta^{\#} - \mathrm{I}\,\eta_4 \wedge \kappa^{\#} \quad (8)$$

P >

P >