```
> restart :
K :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(K(op(i, x)), i = 1
      .. nops(x)) elif
                  (type(x, `*`) = true) then expand( K(y) · x/y + y·K( x/y ) )elif
                  (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) − 1) ·K(y) elif
                  (type(x, function) = true) then Kfunction(x) elif
                  (type(x, symbol) = true)  then Ksymbol(x) else 0 fi end proc:
> Ksymbol :=proc(x); if x = k^# then 0 elif x = P then ( -L(k)·P − L(L(k))) elif x = P1 then (
      -L^#(k)·P - L(L^#(k)) -2·I·Tau(k)) else 'K'(x) fi end proc:
> recherche :=proc(x) local y; y := op(1, x); while (type(y, function) = true) do y := op(1, y)
      od; y; end proc:
> Kfunction :=proc(x) local y; y := recherche(x); if y = k then 'K'(x) else Kfunction1(x) fi
      end proc:

> Kfunction1 :=proc(x) local y, J : if x = k^# then 0 elif x = P then ( -L(k)·P − L(L(k))) elif x
      = P1 then ( -L^#(k)·P - L(L^#(k)) -2·I·Tau(k)) else J := op(0, x) : y := op(1, x) : if J
      = L then L(Kfunction1(y)) − 'L(k)'·L(y) elif
                                                                               J = L^#
      then L^#(Kfunction1(y)) − 'L^#'(k)·L(y) elif
                                                                               J = K^#
      then K^#(Kfunction1(y)) elif J = Tau then Tau(Kfunction1(y)) - 'L(k)'·Tau(y) -'Tau(k)'
      ·L(y) fi fi end proc:
> L :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
      .. nops(x)) elif
                  (type(x, `*`) = true) then expand( L(y) · x/y + y·L( x/y ) )elif
                  (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) − 1) ·L(y) elif
                  (type(x, function) = true) then 'L'(x) elif
                  (type(x, symbol) = true)  then 'L'(x) else 0 fi end proc:

> L^# :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L^#(op(i, x)), i = 1
      .. nops(x)) elif
                  (type(x, `*`) = true) then expand( L^#(y) · x/y + y·L^#( x/y ) )elif
                  (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) − 1) ·L^#(y) elif
                  (type(x, function) = true) then 'L^#(x)' elif
                  (type(x, symbol) = true)  then 'L^#'(x) else 0 fi end proc:
> Tau :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Tau(op(i, x)), i = 1
      .. nops(x)) elif
                  (type(x, `*`) = true) then expand( Tau(y) · x/y + y·Tau( x/y ) )elif
                  (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) − 1) ·Tau(y) elif
                  (type(x, function) = true) then 'Tau'(x) elif
                  (type(x, symbol) = true)  then 'Tau'(x) else 0 fi end proc:
> K^# :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(K^#(op(i, x)), i = 1
```

```
.. nops(x)) elif
        (type(x, `*`) = true) then expand(K#(y)·x/y + y·K#(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·K#(y) elif
        (type(x, function) = true) then Kfunction#(x) elif
        (type(x, symbol) = true) then Ksymbol#(x) else 0 fi end proc:
> Kfunction# :=proc(x) local y; y := recherche(x); if y = k# then 'K#'(x) else Kfunction1#(x)
    fi end proc:
> Kfunction1# :=proc(x) local y, J : if x = k then 0 elif x = P1 then (-L#(k#)·P1
    - L#(L#(k#))) elif x = P then (-L(k#)·P1 - L#(L(k#)) + 2·I·Tau(k#)) else J
    := op(0, x) : y := op(1, x) : if J = L then L(Kfunction1#(y)) - 'L(k#)'·L#(y) elif
                                                                        J = L#
    then L#(Kfunction1#(y)) - 'L#'(k#)·L#(y) elif
                                                                        J = K
    then K(Kfunction1#(y)) elif J = Tau then Tau(Kfunction1#(y)) - 'L#(k#)'·Tau(y) -
    'Tau(k#)'·L#(y) fi fi end proc:
Ksymbol# :=proc(x); if x = k then 0 elif x = P1 then (-L#(k#)·P1 - L#(L#(k#))) elif x = P
    then (-L(k#)·P1 - L#(L(k#)) + 2·I·Tau(k#)) else 'K#(x)' fi end proc:

>
Der :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Der(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(x/y·Der(y) + y·Der(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·Der(y) elif
        ((type(x, function) = true) or (type(x, symbol) = true)) then Tau(x)·W[1]
    + L(x)·W[2] + K(x)·W[3] + L#(x)·W[4] + K#(x)·W[5]
            else 0 fi end proc:
derivation :=proc(x) : collect( Der(x), [W[1], W[2], W[3], W[4], W[5]]) : end proc:
```

> $g := \dfrac{\frac{1}{3}\,\mathrm{I}\,L^{\#}(L^{\#}(k))}{L^{\#}(k)} - \frac{1}{3}\,\mathrm{I}\,P1$:

> $g1 := -\dfrac{\frac{1}{3}\,\mathrm{I}\,L(L(k^{\#}))}{L(k^{\#})} + \frac{1}{3}\,\mathrm{I}\,P$:

```
> dg := derivation(g) :
> dg1 := derivation(g1) :
> with(LinearAlgebra) :
> m := Matrix([[1, 0, 0, 0, 0], [g, 1, 0, 0], [0, 0, L#(k), 0, 0], [g1, 0, 0, 1, 0], [0, 0, 0, 0,
    L(k#)]]) :
> minv := MatrixInverse(m) :
> W := minv.Vector([V[1], V[2], V[3], V[4], V[5]]) :
```

> $X := Der(L^{\#}(k))$:

> $X1 := Der(L(k^{\#}))$:

> $with(DifferentialGeometry)$:

> $with(Tools)$:

> $DGsetup([x, y, z, y1, z1], [c, c1, d, d1, e, e1], M, verbose)$:

The following coordinates have been protected:

$[x, y, z, y1, z1, c, c1, d, d1, e, e1]$

The following vector fields have been defined and protected:

$[D\_x, D\_y, D\_z, D\_y1, D\_z1, D\_c, D\_c1, D\_d, D\_d1, D\_e, D\_e1]$

The following differential 1-forms have been defined and protected:

$[dx, dy, dz, dy1, dz1, dc, dc1, dd, dd1, de, de1]$ **(1)**

> $gr := Matrix\left(\left[[c \cdot c1, 0, 0, 0, 0], [-I \cdot e \cdot c1, c, 0, 0, 0], \left[d, e, \frac{c}{c1}, 0, 0\right], [+I \cdot e1 \cdot c, 0, 0, c1, 0], \left[d1, 0, 0, e1, \frac{c1}{c}\right]\right]\right)$:

M > $h := MatrixInverse(gr)$:

M > $A := map(evalDG, (ExteriorDerivative(gr).h))$;

$$A := \left[\left[\frac{dc}{c} + \frac{dc1}{c1}, 0\,dx, 0\,dx, 0\,dx, 0\,dx\right], \left[\frac{I\,e\,dc}{c^2} - \frac{I\,e\,dc1}{c1\,c} - \frac{I\,de}{c}, \frac{dc}{c}, 0\,dx, 0\,dx, 0\,dx\right], \left[-\frac{(d\,c + I\,e^2\,c1)\,dc}{c^3\,c1} + \frac{(d\,c + I\,e^2\,c1)\,dc1}{c^2\,c1^2} + \frac{dd}{c\,c1} + \frac{I\,e\,de}{c^2}, -\frac{e\,dc}{c^2} + \frac{e\,dc1}{c1\,c} + \frac{de}{c}, \frac{dc}{c} - \frac{dc1}{c1}, 0\,dx, 0\,dx\right], \left[\frac{I\,e1\,dc}{c1\,c} - \frac{I\,e1\,dc1}{c1^2} + \frac{I\,de1}{c1}, 0\,dx, 0\,dx, \frac{dc1}{c1}, 0\,dx\right], \left[-\frac{(-d1\,c1 + I\,e1^2\,c)\,dc}{c1^2\,c^2} + \frac{(-d1\,c1 + I\,e1^2\,c)\,dc1}{c1^3\,c} + \frac{dd1}{c\,c1} - \frac{I\,e1\,de1}{c1^2}, 0\,dx, 0\,dx, \frac{e1\,dc}{c\,c1} - \frac{e1\,dc1}{c1^2} + \frac{de1}{c1}, -\frac{dc}{c} + \frac{dc1}{c1}\right]\right] \quad \textbf{(2)}$$

M > $t[1] := \frac{dc}{c} : t[2] := \frac{I\,e\,dc}{c^2} - \frac{I\,e\,dc1}{c1\,c} - \frac{I\,de}{c}$:

M > $t[3] := -\frac{(d\,c + I\,e^2\,c1)\,dc}{c^3\,c1} + \frac{(d\,c + I\,e^2\,c1)\,dc1}{c^2\,c1^2} + \frac{dd}{c\,c1} + \frac{I\,e\,de}{c^2}$:

M > $t[4] := \frac{dc1}{c1} : t[5] := \frac{I\,e1\,dc}{c1\,c} - \frac{I\,e1\,dc1}{c1^2} + \frac{I\,de1}{c1} : t[6] := -\frac{(-d1\,c1 + I\,e1^2\,c)\,dc}{c1^2\,c^2} + \frac{(-d1\,c1 + I\,e1^2\,c)\,dc1}{c1^3\,c} + \frac{dd1}{c\,c1} - \frac{I\,e1\,de1}{c1^2}$:

M > $FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], dx, dy, dz, dy1, dz1], N)$:

M > $DGsetup\left(FD, [E], \left[\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \alpha^{\#}[1], \alpha^{\#}[2], \alpha^{\#}[3], \text{rho}, \text{kappa}, \text{zeta}, \kappa^{\#}, \zeta^{\#}\right]\right);$

$$\textit{frame name: N} \tag{3}$$

M > $T := Vector\left(\left[\text{rho}, \text{kappa}, \text{zeta}, \kappa^{\#}, \zeta^{\#}\right]\right):$

N > $V := h.T:$

>

$dV[1] := P\cdot(W[1]\&wedge\ W[2]) - L(k)\cdot(W[1]\&wedge\ W[3]) + P1\cdot(W[1]\ \&wedge\ W[4]) - L^{\#}(k^{\#})\cdot(W[1]\ \&wedge\ W[5]) + I\cdot(W[2]\ \&wedge\ W[4]);$

$$dV_1 := \frac{P\,\rho \wedge \kappa}{c^2\,c1} - L(k)\left(-\frac{e\,\rho \wedge \kappa}{c^3\,L^{\#}(k)} + \frac{\rho \wedge \zeta}{L^{\#}(k)\,c^2}\right) + \frac{P1\,\rho \wedge \kappa^{\#}}{c\,c1^2} - L^{\#}(k^{\#})\left(-\frac{e1\,\rho \wedge \kappa^{\#}}{c1^3\,L(k^{\#})} + \frac{\rho \wedge \zeta^{\#}}{L(k^{\#})\,c1^2}\right) + \mathrm{I}\left(-\frac{\frac{1}{3}\,\mathrm{I}\left(-3\,e1\,L(k^{\#})\,c + c1\,L(L(k^{\#})) - c1\,P\,L(k^{\#})\right)\rho \wedge \kappa}{c^2\,c1^2\,L(k^{\#})} - \frac{\frac{1}{3}\,\mathrm{I}\left(-3\,e\,L^{\#}(k)\,c1 + c\,L^{\#}(L^{\#}(k)) - c\,P1\,L^{\#}(k)\right)\rho \wedge \kappa^{\#}}{c^2\,L^{\#}(k)\,c1^2} + \frac{\kappa \wedge \kappa^{\#}}{c\,c1}\right) \tag{4}$$

> $dW[2] := -L(k)\cdot(W[2]\&wedge\ W[3]) + L^{\#}(k)\cdot(W[3]\&wedge\ W[4]) - \mathrm{Tau}(k)\cdot(W[1]\ \&wedge\ W[3]):$

> $dV[2] := dW[2] + dg\ \&wedge\ W[1] + g\cdot dV[1]:$

> $dV[3] := (X\ \&wedge\ W[3]):$

> $dW[4] := -L^{\#}(k^{\#})\cdot(W[4]\&wedge\ W[5]) + L(k^{\#})\cdot(W[5]\&wedge\ W[2]) - \mathrm{Tau}(k^{\#})\cdot(W[1]\ \&wedge\ W[5]):$

> $dV[4] := dW[4] + dg1\ \&wedge\ W[1] + g1\ \&wedge\ dV[1]:$

> $dV[5] := (X1\ \&wedge\ W[5]):$

$\mathrm{Omega} := map(evalDG, gr.Vector([dV[1], dV[2], dV[3], dV[4], dV[5]])):$

N > $A := (map(evalDG, (ExteriorDerivative(gr).h))):$

$B := A\ \&MatrixWedge\ T:$
$SE := map(evalDG, (B\ \&MatrixPlus\ \mathrm{Omega})):$

N > $List := GenerateForms\left(\left[\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \alpha^{\#}[1], \alpha^{\#}[2], \alpha^{\#}[3], \text{rho}, \text{kappa}, \text{zeta}, \kappa^{\#}, \zeta^{\#}\right], 2\right):$

N > *result* := **proc**(*l*) **local** *k, t, X*; *X* := 0 : *t* := *expand*(*GetComponents*(*l, List*)) : **for** *k* **from** 1 **to** 55 **do** *X* := *X* + *t*[*k*]·*List*[*k*] **od**; *X*; **end proc**:

N > $Res1 := result(SE[1]);$

$$Res1 := \alpha_1 \wedge \rho + \alpha^{\#}_1 \wedge \rho + \left(-\frac{e1}{c1} + \frac{1}{3}\,\frac{L(L(k^{\#}))}{c\,L(k^{\#})} + \frac{2}{3}\,\frac{P}{c} + \frac{e\,L(k)\,c1}{c^2\,L^{\#}(k)}\right)\rho \wedge \kappa - \frac{c1\,L(k)\,\rho \wedge \zeta}{c\,L^{\#}(k)} + \left(-\frac{e}{c} + \frac{1}{3}\,\frac{L^{\#}(L^{\#}(k))}{c1\,L^{\#}(k)} + \frac{2}{3}\,\frac{P1}{c1} + \frac{e1\,L^{\#}(k^{\#})\,c}{c1^2\,L(k^{\#})}\right)\rho \wedge \kappa^{\#} - \frac{L^{\#}(k^{\#})\,c\,\rho \wedge \zeta^{\#}}{c1\,L(k^{\#})} + \mathrm{I}\,\kappa \wedge \kappa^{\#} \tag{5}$$

N > $Res2 := result(SE[2]);$

$$Res2 := \alpha_1 \wedge \kappa + \alpha_2 \wedge \rho + \Bigg(-\frac{I\, c1\, e^2\, L(k)}{L^{\#}(k)\, c^3} - \frac{\frac{1}{9}\, I\, P1\, L(L(k^{\#}))}{c1\, L(k^{\#})\, c} - \frac{\frac{2}{9}\, I\, P1\, P}{c1\, c} + \frac{\frac{1}{3}\, I\, L^{\#}(L^{\#}(k))\, L(L^{\#}(k))}{L^{\#}(k)^2\, c1\, c} + \frac{\frac{1}{3}\, I\, P1\, e1}{c1^2} - \frac{\frac{2}{3}\, I\, e\, P}{c^2} - \frac{\frac{1}{3}\, I\, L(L^{\#}(L^{\#}(k)))}{L^{\#}(k)\, c1\, c} + \frac{\frac{1}{9}\, I\, L^{\#}(L^{\#}(k))\, L(L(k^{\#}))}{L^{\#}(k)\, c1\, L(k^{\#})\, c} + \frac{\frac{2}{9}\, I\, L^{\#}(L^{\#}(k))\, P}{L^{\#}(k)\, c1\, c} + \frac{1}{3}\, \frac{e\, \mathrm{T}(k)}{L^{\#}(k)\, c^2} + \frac{\frac{1}{3}\, I\, e\, L(L^{\#}(k))}{L^{\#}(k)\, c^2} - \frac{\frac{1}{3}\, I\, e\, L^{\#}(L^{\#}(k))\, K(L^{\#}(k))}{L^{\#}(k)^3\, c^2} + \frac{\frac{1}{3}\, I\, e\, K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2\, c^2} - \frac{L(k)\, d}{L^{\#}(k)\, c^2} - \frac{\frac{1}{3}\, I\, L^{\#}(L^{\#}(k))\, e1}{L^{\#}(k)\, c1^2} + \frac{\frac{1}{3}\, I\, L(P1)}{c1\, c} \Bigg) \rho \wedge \kappa + \Bigg(-\frac{\frac{1}{3}\, I\, L(L^{\#}(k))}{c\, L^{\#}(k)} + \frac{\frac{1}{3}\, I\, L^{\#}(L^{\#}(k))\, K(L^{\#}(k))}{c\, L^{\#}(k)^3} - \frac{\frac{1}{3}\, I\, K(L^{\#}(L^{\#}(k)))}{c\, L^{\#}(k)^2} - \frac{1}{3}\, \frac{\mathrm{T}(k)}{c\, L^{\#}(k)} + \frac{I\, e1}{c1} - \frac{\frac{1}{3}\, I\, L(L(k^{\#}))}{c\, L(k^{\#})} \Bigg) \rho \wedge \zeta + \Bigg(-\frac{\frac{2}{3}\, I\, L^{\#}(L^{\#}(k))\, e}{c\, c1\, L^{\#}(k)} + \frac{\frac{4}{9}\, I\, L^{\#}(L^{\#}(k))^2}{c1^2\, L^{\#}(k)^2} + \frac{\frac{1}{9}\, I\, L^{\#}(L^{\#}(k))\, P1}{c1^2\, L^{\#}(k)} - \frac{\frac{1}{3}\, I\, P1\, e}{c\, c1} - \frac{\frac{2}{9}\, I\, P1^2}{c1^2} + \frac{\frac{1}{3}\, I\, L^{\#}(P1)}{c1^2} - \frac{\frac{1}{3}\, I\, L^{\#}(L^{\#}(L^{\#}(k)))}{c1^2\, L^{\#}(k)} - \frac{d}{c\, c1} - \frac{I\, e\, L^{\#}(k^{\#})\, e1}{c1^2\, L(k^{\#})} \Bigg) \rho \wedge \kappa^{\#} + \frac{I\, e\, L^{\#}(k^{\#})\, \rho \wedge \zeta^{\#}}{c1\, L(k^{\#})} - \frac{c1\, L(k)\, \kappa \wedge \zeta}{c\, L^{\#}(k)} + \Bigg(-\frac{1}{3}\, \frac{L^{\#}(L^{\#}(k))}{c1\, L^{\#}(k)} + \frac{1}{3}\, \frac{P1}{c1} \Bigg) \kappa \wedge \kappa^{\#} + \zeta \wedge \kappa^{\#} \quad \mathbf{(6)}$$

N > $Res3 := result(SE[3]);$

$$Res3 := \alpha_1 \wedge \zeta + I\, \alpha_2 \wedge \kappa + \alpha_3 \wedge \rho - \alpha^{\#}_1 \wedge \zeta + \Bigg(\frac{\frac{1}{3}\, I\, e^2\, K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2\, c^3} + \frac{\frac{2}{3}\, I\, e\, L^{\#}(L^{\#}(k))\, e1}{L^{\#}(k)\, c\, c1^2} + \frac{\frac{1}{3}\, I\, e\, P1\, e1}{c\, c1^2} + \frac{\frac{1}{3}\, I\, e\, L(P1)}{c^2\, c1} + \frac{I\, e\, L^{\#}(k^{\#})\, e1^2}{c1^3\, L(k^{\#})} \quad \mathbf{(7)}$$

$$+\frac{1}{3}\,\frac{e^{2}\,\mathrm{T}(k)}{L^{\#}(k)\,c^{3}}+\frac{\frac{2}{3}\,\mathrm{I}\,e\,L\left(L^{\#}(k)\right)\,L^{\#}\left(L^{\#}(k)\right)}{L^{\#}(k)^{2}\,c^{2}\,c1}+\frac{1}{3}\,\frac{d\,L\left(L(k^{\#})\right)}{c^{2}\,c1\,L(k^{\#})}+\frac{L\left(L^{\#}(k)\right)\,d}{L^{\#}(k)\,c^{2}\,c1}$$

$$-\frac{e\,L^{\#}(k^{\#})\,d1}{c\,c1^{2}\,L(k^{\#})}-\frac{\frac{2}{9}\,\mathrm{I}\,e\,P1\,P}{c^{2}\,c1}-\frac{e\,\mathrm{T}\left(L^{\#}(k)\right)}{L^{\#}(k)\,c^{2}\,c1}+\frac{\frac{5}{9}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}(k)\right)\,P}{L^{\#}(k)\,c^{2}\,c1}$$

$$-\frac{\frac{1}{3}\,\mathrm{I}\,e\,L\left(L^{\#}\left(L^{\#}(k)\right)\right)}{L^{\#}(k)\,c^{2}\,c1}-\frac{\frac{2}{9}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}(k)\right)\,L\left(L(k^{\#})\right)}{L^{\#}(k)\,c^{2}\,c1\,L(k^{\#})}-\frac{d\,e1}{c\,c1^{2}}+\frac{2}{3}\,\frac{d\,P}{c^{2}\,c1}$$

$$+\frac{\frac{1}{3}\,\mathrm{I}\,e^{2}\,L\left(L(k^{\#})\right)}{c^{3}\,L(k^{\#})}-\frac{\frac{1}{9}\,\mathrm{I}\,e\,P1\,L\left(L(k^{\#})\right)}{c^{2}\,c1\,L(k^{\#})}-\frac{\frac{1}{3}\,\mathrm{I}\,e^{2}\,L^{\#}\left(L^{\#}(k)\right)\,K\left(L^{\#}(k)\right)}{L^{\#}(k)^{3}\,c^{3}}-\frac{\mathrm{I}\,e^{2}\,e1}{c^{2}\,c1}$$

$$-\frac{\frac{1}{3}\,\mathrm{I}\,e\,L\left(L^{\#}(k)\right)\,P1}{L^{\#}(k)\,c^{2}\,c1}+\frac{\frac{1}{3}\,\mathrm{I}\,e^{2}\,L\left(L^{\#}(k)\right)}{L^{\#}(k)\,c^{3}}\Bigg)\,\rho\wedge\kappa+\Bigg(\frac{L^{\#}(k^{\#})\,d1}{c1^{2}\,L(k^{\#})}-\frac{\mathrm{I}\,c1\,e^{2}\,L(k)}{L^{\#}(k)\,c^{3}}$$

$$+\frac{\mathrm{I}\,e\,e1}{c\,c1}-\frac{\frac{1}{3}\,\mathrm{I}\,L\left(L^{\#}(k)\right)\,L^{\#}\left(L^{\#}(k)\right)}{L^{\#}(k)^{2}\,c\,c1}-\frac{\frac{1}{3}\,\mathrm{I}\,e\,L\left(L(k^{\#})\right)}{c^{2}\,L(k^{\#})}+\frac{\frac{2}{3}\,\mathrm{I}\,L\left(L^{\#}(k)\right)\,e}{L^{\#}(k)\,c^{2}}$$

$$+\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}\left(L^{\#}(k)\right)\,L\left(L(k^{\#})\right)}{L^{\#}(k)\,c\,c1\,L(k^{\#})}-\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}\left(L^{\#}(k)\right)\,P}{L^{\#}(k)\,c\,c1}+\frac{\mathrm{T}\left(L^{\#}(k)\right)}{L^{\#}(k)\,c\,c1}$$

$$+\frac{\frac{1}{3}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}(k)\right)\,K\left(L^{\#}(k)\right)}{L^{\#}(k)^{3}\,c^{2}}-\frac{\mathrm{I}\,L^{\#}\left(L^{\#}(k)\right)\,e1}{L^{\#}(k)\,c1^{2}}-\frac{1}{3}\,\frac{e\,\mathrm{T}(k)}{L^{\#}(k)\,c^{2}}$$

$$-\frac{\frac{1}{3}\,\mathrm{I}\,e\,K\left(L^{\#}\left(L^{\#}(k)\right)\right)}{L^{\#}(k)^{2}\,c^{2}}-\frac{\mathrm{I}\,c\,L^{\#}(k^{\#})\,e1^{2}}{c1^{3}\,L(k^{\#})}+\frac{\frac{1}{3}\,\mathrm{I}\,L\left(L^{\#}(k)\right)\,P1}{L^{\#}(k)\,c\,c1}-\frac{L(k)\,d}{L^{\#}(k)\,c^{2}}\Bigg)\,\rho\wedge\zeta$$

$$+\Bigg(\frac{2\,d\,L^{\#}(k^{\#})\,e1}{L(k^{\#})\,c1^{3}}+\frac{\mathrm{I}\,L^{\#}(k^{\#})\,e1\,e^{2}}{L(k^{\#})\,c1^{2}\,c}+\frac{4}{3}\,\frac{d\,L^{\#}\left(L^{\#}(k)\right)}{L^{\#}(k)\,c1^{2}\,c}+\frac{\frac{2}{3}\,\mathrm{I}\,e^{2}\,L^{\#}\left(L^{\#}(k)\right)}{L^{\#}(k)\,c1\,c^{2}}$$

$$+\frac{\frac{4}{9}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}(k)\right)^{2}}{L^{\#}(k)^{2}\,c1^{2}\,c}+\frac{\frac{1}{9}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}(k)\right)\,P1}{L^{\#}(k)\,c1^{2}\,c}+\frac{\frac{1}{3}\,\mathrm{I}\,e^{2}\,P1}{c1\,c^{2}}-\frac{\frac{2}{9}\,\mathrm{I}\,e\,P1^{2}}{c1^{2}\,c}$$

$$+\frac{\frac{1}{3}\,\mathrm{I}\,e\,L^{\#}(P1)}{c1^{2}\,c}-\frac{\frac{1}{3}\,\mathrm{I}\,e\,L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right)}{L^{\#}(k)\,c1^{2}\,c}-\frac{2\,e\,d}{c1\,c^{2}}-\frac{\mathrm{I}\,e^{3}}{c^{3}}+\frac{2}{3}\,\frac{d\,P1}{c1^{2}\,c}\Bigg)\,\rho\wedge\kappa^{\#}+\Bigg($$

$$-\frac{2\,L^{\#}(k^{\#})\,d}{c1^{2}\,L(k^{\#})}-\frac{\mathrm{I}\,L^{\#}(k^{\#})\,e^{2}}{c\,c1\,L(k^{\#})}\Bigg)\,\rho\wedge\zeta^{\#}+\Bigg(\frac{L\left(L^{\#}(k)\right)}{c\,L^{\#}(k)}-\frac{e\,L(k)\,c1}{c^{2}\,L^{\#}(k)}\Bigg)\,\kappa\wedge\zeta$$

$$+\left(\frac{e\,L^{\#}(k^{\#})\,e1}{c1^2\,L(k^{\#})}+\frac{2}{3}\,\frac{L^{\#}(L^{\#}(k))\,e}{c\,c1\,L^{\#}(k)}+\frac{1}{3}\,\frac{P1\,e}{c\,c1}-\frac{e^2}{c^2}+\frac{\mathrm{I}\,d}{c1\,c}\right)\kappa\wedge\kappa^{\#}$$

$$-\frac{L^{\#}(k^{\#})\,e\,\kappa\wedge\zeta^{\#}}{c1\,L(k^{\#})}+\left(-\frac{e1\,L^{\#}(k^{\#})\,c}{c1^2\,L(k^{\#})}-\frac{L^{\#}(L^{\#}(k))}{c1\,L^{\#}(k)}+\frac{e}{c}\right)\zeta\wedge\kappa^{\#}+\frac{L^{\#}(k^{\#})\,c\,\zeta\wedge\zeta^{\#}}{c1\,L(k^{\#})}$$

N > $List2 := GenerateForms\left([\text{rho, kappa, zeta}, \kappa^{\#}, \zeta^{\#}], 2\right):$

N > $Torsion := \mathbf{proc}(S, i, j)\ \mathbf{local}\, k, X;\ k := 5\cdot(i-1) - \frac{i\cdot(i-1)}{2} + j - i;\ X := map(expand, GetComponents(S, List2));\ X[k];\ \mathbf{end\ proc}:$

N > $expr := expand(I\cdot Torsion(\text{Omega}[2], 1, 4) - Torsion(\text{Omega}[3], 2, 4)) : expr1 := expand(-I\cdot Torsion(\text{Omega}[4], 1, 2) + Torsion(\text{Omega}[5], 2, 4)) :$

N > $Normalisation := expand(solve(expr, d)) : Normalisation1 := expand(solve(expr1, d1)) :$

N > $H := expand\left(\left(Normalisation - \left(-I\cdot\frac{e^2\cdot c1}{2\cdot c}\right)\right)\right);$

$$H := \frac{\frac{2}{9}\,\mathrm{I}\,c\,L^{\#}(L^{\#}(k))^2}{c1\,L^{\#}(k)^2}+\frac{\frac{1}{18}\,\mathrm{I}\,c\,L^{\#}(L^{\#}(k))\,P1}{c1\,L^{\#}(k)}-\frac{\frac{1}{9}\,\mathrm{I}\,c\,P1^2}{c1}+\frac{\frac{1}{6}\,\mathrm{I}\,c\,L^{\#}(P1)}{c1}-\frac{\frac{1}{6}\,\mathrm{I}\,c\,L^{\#}(L^{\#}(L^{\#}(k)))}{c1\,L^{\#}(k)} \quad \textbf{(8)}$$

N > $Wu := expand(c\cdot simplify(Torsion(\text{Omega}[3], 2, 3) + Torsion(\text{Omega}[5], 2, 5) - I\cdot Torsion(\text{Omega}[2], 1, 3)));$

$$Wu := \frac{2}{3}\,\frac{L(L^{\#}(k))}{L^{\#}(k)}+\frac{2}{3}\,\frac{L(L(k^{\#}))}{L(k^{\#})}+\frac{1}{3}\,\frac{L^{\#}(L^{\#}(k))\,K(L^{\#}(k))}{L^{\#}(k)^3}-\frac{1}{3}\,\frac{K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2}+\frac{\frac{1}{3}\,\mathrm{I}\,\mathrm{T}(k)}{L^{\#}(k)} \quad \textbf{(9)}$$

N > $Wu1 := expand(c1\cdot simplify(Torsion(\text{Omega}[5], 4, 5) - Torsion(\text{Omega}[3], 3, 4) + I\cdot Torsion(\text{Omega}[4], 1, 5)));$

$$Wu1 := \frac{2}{3}\,\frac{L^{\#}(L(k^{\#}))}{L(k^{\#})}+\frac{2}{3}\,\frac{L^{\#}(L^{\#}(k))}{L^{\#}(k)}-\frac{1}{3}\,\frac{K^{\#}(L(L(k^{\#})))}{L(k^{\#})^2}-\frac{\frac{1}{3}\,\mathrm{I}\,\mathrm{T}(k^{\#})}{L(k^{\#})}+\frac{1}{3}\,\frac{L(L(k^{\#}))\,K^{\#}(L(k^{\#}))}{L(k^{\#})^3} \quad \textbf{(10)}$$

> $Hr := expand\left(\frac{H\cdot c1}{I\cdot c}\right);$

$$Hr := \frac{2}{9}\,\frac{L^{\#}(L^{\#}(k))^2}{L^{\#}(k)^2}+\frac{1}{18}\,\frac{L^{\#}(L^{\#}(k))\,P1}{L^{\#}(k)}-\frac{1}{9}\,P1^2+\frac{1}{6}\,L^{\#}(P1)-\frac{1}{6}\,\frac{L^{\#}(L^{\#}(L^{\#}(k)))}{L^{\#}(k)} \quad \textbf{(11)}$$

N > $simplify\left(K^{\#}(Hr) + 2\cdot L^{\#}(k^{\#})\cdot Hr\right);$

$$0 \quad \textbf{(12)}$$

N > $J^{\#} := expand\left(\frac{2}{3}\cdot\left(\frac{2\cdot L^{\#}(L^{\#}(k))}{L^{\#}(k)} + P1\right)\cdot Hr - L^{\#}(Hr)\right);$

$$J^{\#} := \frac{20}{27}\frac{L^{\#}(L^{\#}(k))^3}{L^{\#}(k)^3} + \frac{5}{18}\frac{L^{\#}(L^{\#}(k))^2 P1}{L^{\#}(k)^2} - \frac{1}{9}\frac{L^{\#}(L^{\#}(k))\,P1^2}{L^{\#}(k)} + \frac{1}{6}\frac{L^{\#}(L^{\#}(k))\,L^{\#}(P1)}{L^{\#}(k)} - \frac{5}{6}\frac{L^{\#}(L^{\#}(k))\,L^{\#}(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2} - \frac{2}{27}P1^3 + \frac{1}{3}P1\,L^{\#}(P1) - \frac{1}{6}\frac{L^{\#}(L^{\#}(L^{\#}(k)))\,P1}{L^{\#}(k)} - \frac{1}{6}L^{\#}(L^{\#}(P1)) + \frac{1}{6}\frac{L^{\#}(L^{\#}(L^{\#}(L^{\#}(k))))}{L^{\#}(k)} \quad \textbf{(13)}$$

N > $simplify(K^{\#}(J^{\#}) + 3\cdot J^{\#}\cdot L^{\#}(k^{\#}));$

$$0 \quad \textbf{(14)}$$

N > $K^{\#}(Wu);$

$$-L(L^{\#}(k^{\#})) - \frac{4}{3}\frac{L(k^{\#})\,L^{\#}(L^{\#}(k))}{L^{\#}(k)} - \frac{2}{3}\frac{K^{\#}(L(k^{\#}))\,L(L(k^{\#}))}{L(k^{\#})^2} + \frac{2}{3}\frac{K^{\#}(L(L(k^{\#})))}{L(k^{\#})} - \frac{1}{3}L^{\#}(L(k^{\#})) - \frac{1}{3}\,\mathrm{I\,T}(k^{\#}) \quad \textbf{(15)}$$

N > $Wu;$

$$\frac{2}{3}\frac{L(L^{\#}(k))}{L^{\#}(k)} + \frac{2}{3}\frac{L(L(k^{\#}))}{L(k^{\#})} + \frac{1}{3}\frac{L^{\#}(L^{\#}(k))\,K(L^{\#}(k))}{L^{\#}(k)^3} - \frac{1}{3}\frac{K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2} + \frac{\frac{1}{3}\,\mathrm{I\,T}(k)}{L^{\#}(k)} \quad \textbf{(16)}$$

N > $K(Wu1) + 2\cdot expand(L^{\#}(k)\cdot Wu);$

$$-L^{\#}(L(k)) + L(L^{\#}(k)) + \mathrm{I\,T}(k) \quad \textbf{(17)}$$

N > Tau :=**proc**(x) ; $I\cdot\,(L(L^{\#}(x)) - L^{\#}(L(x)))$; **end proc**;

$$\mathrm{T} := \mathbf{proc}(x)\ \ \mathrm{I}*\left(L(L^{\#}(x)) - L^{\#}(L(x))\right)\ \ \mathbf{end\ proc} \quad \textbf{(18)}$$

N > $K(Wu1) + 2\cdot expand(L^{\#}(k)\cdot Wu);$

$$0 \quad \textbf{(19)}$$