

> *restart* :
 $K := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x) : \mathbf{if} (\mathit{type}(x, \text{'+'}) = \mathit{true}) \mathbf{then} \mathit{add}(K(\mathit{op}(i, x)), i = 1$
 $\dots \mathit{nops}(x)) \mathbf{elif}$

$(\mathit{type}(x, \text{'*'}) = \mathit{true}) \mathbf{then} \mathit{expand}\left(K(y) \cdot \frac{x}{y} + y \cdot K\left(\frac{x}{y}\right)\right) \mathbf{elif}$

$(\mathit{type}(x, \text{'^'}) = \mathit{true}) \mathbf{then} \mathit{op}(2, x) \cdot y^{(\mathit{op}(2, x) - 1)} \cdot K(y) \mathbf{elif}$

$(\mathit{type}(x, \mathit{function}) = \mathit{true}) \mathbf{then} K\mathit{function}(x) \mathbf{elif}$

$(\mathit{type}(x, \mathit{symbol}) = \mathit{true}) \mathbf{then} K\mathit{symbol}(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $K\mathit{symbol} := \mathbf{proc}(x); \mathbf{if} x = k^\# \mathbf{then} 0 \mathbf{elif} x = P \mathbf{then} (-L(k) \cdot P - L(L(k))) \mathbf{elif} x = P1 \mathbf{then} (-L^\#(k) \cdot P - L(L^\#(k)) - 2 \cdot I \cdot \mathit{Tau}(k)) \mathbf{else} 'K'(x) \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $\mathit{recherche} := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x); \mathbf{while} (\mathit{type}(y, \mathit{function}) = \mathit{true}) \mathbf{do} y := \mathit{op}(1, y) \mathbf{od}; y; \mathbf{end} \mathbf{proc}:$

> $K\mathit{function} := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{recherche}(x); \mathbf{if} y = k \mathbf{then} 'K'(x) \mathbf{else} K\mathit{function}1(x) \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $K\mathit{function}1 := \mathbf{proc}(x) \mathbf{local} y, J; \mathbf{if} x = k^\# \mathbf{then} 0 \mathbf{elif} x = P \mathbf{then} (-L(k) \cdot P - L(L(k))) \mathbf{elif} x = P1 \mathbf{then} (-L^\#(k) \cdot P - L(L^\#(k)) - 2 \cdot I \cdot \mathit{Tau}(k)) \mathbf{else} J := \mathit{op}(0, x) : y := \mathit{op}(1, x) : \mathbf{if} J = L \mathbf{then} L(K\mathit{function}1(y)) - 'L(k)' \cdot L(y) \mathbf{elif}$

$J = L^\#$

$\mathbf{then} L^\#(K\mathit{function}1(y)) - 'L^\#(k)' \cdot L(y) \mathbf{elif}$

$J = K^\#$

$\mathbf{then} K^\#(K\mathit{function}1(y)) \mathbf{elif} J = \mathit{Tau} \mathbf{then} \mathit{Tau}(K\mathit{function}1(y)) - 'L(k)' \cdot \mathit{Tau}(y) - '\mathit{Tau}(k)' \cdot L(y) \mathbf{fi} \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $L := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x) : \mathbf{if} (\mathit{type}(x, \text{'+'}) = \mathit{true}) \mathbf{then} \mathit{add}(L(\mathit{op}(i, x)), i = 1$
 $\dots \mathit{nops}(x)) \mathbf{elif}$

$(\mathit{type}(x, \text{'*'}) = \mathit{true}) \mathbf{then} \mathit{expand}\left(L(y) \cdot \frac{x}{y} + y \cdot L\left(\frac{x}{y}\right)\right) \mathbf{elif}$

$(\mathit{type}(x, \text{'^'}) = \mathit{true}) \mathbf{then} \mathit{op}(2, x) \cdot y^{(\mathit{op}(2, x) - 1)} \cdot L(y) \mathbf{elif}$

$(\mathit{type}(x, \mathit{function}) = \mathit{true}) \mathbf{then} 'L'(x) \mathbf{elif}$

$(\mathit{type}(x, \mathit{symbol}) = \mathit{true}) \mathbf{then} 'L'(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $L^\# := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x) : \mathbf{if} (\mathit{type}(x, \text{'+'}) = \mathit{true}) \mathbf{then} \mathit{add}(L^\#(\mathit{op}(i, x)), i = 1$
 $\dots \mathit{nops}(x)) \mathbf{elif}$

$(\mathit{type}(x, \text{'*'}) = \mathit{true}) \mathbf{then} \mathit{expand}\left(L^\#(y) \cdot \frac{x}{y} + y \cdot L^\#\left(\frac{x}{y}\right)\right) \mathbf{elif}$

$(\mathit{type}(x, \text{'^'}) = \mathit{true}) \mathbf{then} \mathit{op}(2, x) \cdot y^{(\mathit{op}(2, x) - 1)} \cdot L^\#(y) \mathbf{elif}$

$(\mathit{type}(x, \mathit{function}) = \mathit{true}) \mathbf{then} 'L^\#(x)' \mathbf{elif}$

$(\mathit{type}(x, \mathit{symbol}) = \mathit{true}) \mathbf{then} 'L^\#(x)' \mathbf{else} 0 \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $\mathit{Tau} := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x) : \mathbf{if} (\mathit{type}(x, \text{'+'}) = \mathit{true}) \mathbf{then} \mathit{add}(\mathit{Tau}(\mathit{op}(i, x)), i = 1$
 $\dots \mathit{nops}(x)) \mathbf{elif}$

$(\mathit{type}(x, \text{'*'}) = \mathit{true}) \mathbf{then} \mathit{expand}\left(\mathit{Tau}(y) \cdot \frac{x}{y} + y \cdot \mathit{Tau}\left(\frac{x}{y}\right)\right) \mathbf{elif}$

$(\mathit{type}(x, \text{'^'}) = \mathit{true}) \mathbf{then} \mathit{op}(2, x) \cdot y^{(\mathit{op}(2, x) - 1)} \cdot \mathit{Tau}(y) \mathbf{elif}$

$(\mathit{type}(x, \mathit{function}) = \mathit{true}) \mathbf{then} '\mathit{Tau}'(x) \mathbf{elif}$

$(\mathit{type}(x, \mathit{symbol}) = \mathit{true}) \mathbf{then} '\mathit{Tau}'(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end} \mathbf{proc}:$

> $K^\# := \mathbf{proc}(x) \mathbf{local} y; y := \mathit{op}(1, x) : \mathbf{if} (\mathit{type}(x, \text{'+'}) = \mathit{true}) \mathbf{then} \mathit{add}(K^\#(\mathit{op}(i, x)), i = 1$

.. nops(x)) elif

(type(x, `*`) = true) then expand($K^\#(y) \cdot \frac{x}{y} + y \cdot K^\#(\frac{x}{y})$) elif

(type(x, `^`) = true) then op(2, x) · y^{(op(2, x) - 1)} · K[#](y) elif

(type(x, function) = true) then Kfunction[#](x) elif

(type(x, symbol) = true) then Ksymbol[#](x) else 0 fi end proc:

> Kfunction[#] := **proc**(x) **local** y; y := recherche(x); **if** y = k[#] **then** 'K[#]'(x) **else** KfunctionI[#](x) **fi** end **proc**:

> KfunctionI[#] := **proc**(x) **local** y, J: **if** x = k **then** 0 **elif** x = P1 **then** (-L[#](k[#]) · P1 - L[#](L[#](k[#]))) **elif** x = P **then** (-L(k[#]) · P1 - L[#](L(k[#])) + 2 · I · Tau(k[#])) **else** J := op(0, x) : y := op(1, x) : **if** J = L **then** L(KfunctionI[#](y)) - 'L(k[#])' · L[#](y) **elif** J = L[#]

then L[#](KfunctionI[#](y)) - 'L[#]'(k[#]) · L[#](y) **elif**

J = K

then K(KfunctionI[#](y)) **elif** J = Tau **then** Tau(KfunctionI[#](y)) - 'L[#]'(k[#]) · Tau(y) - 'Tau(k[#])' · L[#](y) **fi** **fi** end **proc**:

Ksymbol[#] := **proc**(x); **if** x = k **then** 0 **elif** x = P1 **then** (-L[#](k[#]) · P1 - L[#](L[#](k[#]))) **elif** x = P **then** (-L(k[#]) · P1 - L[#](L(k[#])) + 2 · I · Tau(k[#])) **else** 'K[#]'(x) **fi** end **proc**:

> Der := **proc**(x) **local** y; y := op(1, x) : **if** (type(x, `+`) = true) **then** add(Der(op(i, x)), i = 1 .. nops(x)) **elif**

(type(x, `*`) = true) **then** expand($\frac{x}{y} \cdot Der(y) + y \cdot Der(\frac{x}{y})$) **elif**

(type(x, `^`) = true) **then** op(2, x) · y^{(op(2, x) - 1)} · Der(y) **elif**

((type(x, function) = true) or (type(x, symbol) = true)) **then** Tau(x) · W[1]

+ L(x) · W[2] + K(x) · W[3] + L[#](x) · W[4] + K[#](x) · W[5]

else 0 **fi** end **proc**:

derivation := **proc**(x) : collect(Der(x), [W[1], W[2], W[3], W[4], W[5]]) : end **proc**:

> g := $\frac{\frac{1}{3} IL^\#(L^\#(k))}{L^\#(k)} - \frac{1}{3} IP1$:

> g1 := $-\frac{\frac{1}{3} IL(L(k^\#))}{L(k^\#)} + \frac{1}{3} IP$:

> dg := derivation(g) :

> dg1 := derivation(g1) :

> with(LinearAlgebra) :

> m := Matrix([[1, 0, 0, 0, 0], [g, 1, 0, 0], [0, 0, L[#](k), 0, 0], [g1, 0, 0, 1, 0], [0, 0, 0, 0, L(k[#])]]) :

> minv := MatrixInverse(m) :

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[> dg := derivation(g) :
[> dg1 := derivation(g1) :
[> with(LinearAlgebra) :
[> m := Matrix([[1, 0, 0, 0, 0], [g, 1, 0, 0], [0, 0, L#(k), 0, 0], [g1, 0, 0, 1, 0], [0, 0, 0, 0,
L(k#) ]]) :
[> minv := MatrixInverse(m) :
[> W := minv.Vector([V[1], V[2], V[3], V[4], V[5]]) :

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[> X := Der(L#(k)) :
[> X1 := Der(L(k#)) :
[> with(DifferentialGeometry) :
[> with(Tools) :
[> DGsetup([x, y, z, y1, z1], [c, c1, d, d1, e, e1], M, verbose) :
The following coordinates have been protected:
[x, y, z, y1, z1, c, c1, d, d1, e, e1]
The following vector fields have been defined and protected:
[D_x, D_y, D_z, D_y1, D_z1, D_c, D_c1, D_d, D_d1, D_e, D_e1]
The following differential 1-forms have been defined and protected:
[dx, dy, dz, dy1, dz1, dc, dc1, dd, dd1, de, de1]
(1)

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> gr := Matrix([[ [c·c1, 0, 0, 0, 0], [-I·e·c1, c, 0, 0, 0], [d, e, c/c1, 0, 0], [ +I·e1·c, 0, 0, c1, 0],
[d1, 0, 0, e1, c1/c ]]]) :

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M > h := MatrixInverse(gr) :
M > A := map(evalDG, (ExteriorDerivative(gr).h));
A := [[ [dc/c + dc1/c1, 0 dx, 0 dx, 0 dx, 0 dx],
(2)

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$$\begin{aligned}
& \left[\frac{Ie dc}{c^2} - \frac{Ie dc1}{c1 c} - \frac{Ide}{c}, \frac{dc}{c}, 0 dx, 0 dx, 0 dx \right], \\
& \left[-\frac{(dc + Ie^2 c1) dc}{c^3 c1} + \frac{(dc + Ie^2 c1) dc1}{c^2 c1^2} + \frac{dd}{c c1} + \frac{Ie de}{c^2}, -\frac{e dc}{c^2} + \frac{e dc1}{c1 c} + \frac{de}{c}, \right. \\
& \left. \frac{dc}{c} - \frac{dc1}{c1}, 0 dx, 0 dx \right], \\
& \left[\frac{Ie1 dc}{c1 c} - \frac{Ie1 dc1}{c1^2} + \frac{Ide1}{c1}, 0 dx, 0 dx, \frac{dc1}{c1}, 0 dx \right], \\
& \left[-\frac{(-d1 c1 + Ie1^2 c) dc}{c1^2 c^2} + \frac{(-d1 c1 + Ie1^2 c) dc1}{c1^3 c} + \frac{dd1}{c c1} - \frac{Ie1 de1}{c1^2}, 0 dx, 0 dx, \right. \\
& \left. \frac{e1 dc}{c c1} - \frac{e1 dc1}{c1^2} + \frac{de1}{c1}, -\frac{dc}{c} + \frac{dc1}{c1} \right]
\end{aligned}$$

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M > t[1] := dc/c : t[2] := Ide dc/c^2 - Ide dc1/c1 c - Ide/c :

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M > t[3] := -  $\frac{(dc + Ie^2 cl) dc}{c^3 cl}$  +  $\frac{(dc + Ie^2 cl) dc l}{c^2 cl^2}$  +  $\frac{dd}{c cl}$  +  $\frac{Ie de}{c^2}$  :
M > t[4] :=  $\frac{dc l}{cl}$  : t[5] :=  $\frac{Ie l dc}{cl c}$  -  $\frac{Ie l dc l}{cl^2}$  +  $\frac{I del}{cl}$  : t[6] := -  $\frac{(-dl cl + Ie l^2 c) dc}{cl^2 c^2}$ 
+  $\frac{(-dl cl + Ie l^2 c) dc l}{cl^3 c}$  +  $\frac{ddl}{c cl}$  -  $\frac{Ie l del}{cl^2}$  :
M > FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], dx, dy, dz, dy1, dz1], N) :
M > DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3],  $\alpha^{\#}[1]$ ,  $\alpha^{\#}[2]$ ,  $\alpha^{\#}[3]$ , rho, kappa, zeta,
 $\kappa^{\#}$ ,  $\zeta^{\#}$ ]);
frame name: N (3)
M > T := Vector([rho, kappa, zeta,  $\kappa^{\#}$ ,  $\zeta^{\#}$ ]) :
N > V := h.T :
>
dV[1] := P · (W[1] &wedge W[2]) - L(k) · (W[1] &wedge W[3]) + P1 · (W[1]
&wedge W[4]) - L#(k#) · (W[1] &wedge W[5]) + I · (W[2] &wedge W[4]) :
> dW[2] := -L(k) · (W[2] &wedge W[3]) + L#(k) · (W[3] &wedge W[4]) - Tau(k) · (W[1]
&wedge W[3]) :
> dV[2] := dW[2] + dg &wedge W[1] + g · dV[1] :
> dV[3] := (X &wedge W[3]) :
> dW[4] := -L#(k#) · (W[4] &wedge W[5]) + L(k#) · (W[5] &wedge W[2]) - Tau(k#) · (W[1]
&wedge W[5]) :
> dV[4] := dW[4] + dg1 &wedge W[1] + g1 &wedge dV[1] :
> dV[5] := (X1 &wedge W[5]) :

Omega := map(evalDG, gr.Vector([dV[1], dV[2], dV[3], dV[4], dV[5]])) :

N > A := (map(evalDG, (ExteriorDerivative(gr).h))) :

B := A &MatrixWedge T :
SE := map(evalDG, (B &MatrixPlus Omega)) :
N > List := GenerateForms([alpha[1], alpha[2], alpha[3],  $\alpha^{\#}[1]$ ,  $\alpha^{\#}[2]$ ,  $\alpha^{\#}[3]$ , rho, kappa,
zeta,  $\kappa^{\#}$ ,  $\zeta^{\#}$ ], 2) :
N > result := proc(l) local k, t, X; X := 0 : t := expand(GetComponents(l, List)) : for k
from 1 to 55 do X := X + t[k] · List[k] od; X; end proc:
N > Res1 := result(SE[1]) :

N > Res2 := result(SE[2]) :
N > Res3 := result(SE[3]) :
N > List2 := GenerateForms([rho, kappa, zeta,  $\kappa^{\#}$ ,  $\zeta^{\#}$ ], 2) :
N > Torsion := proc(S, i, j) local k, X; k :=  $5 \cdot (i - 1) - \frac{i \cdot (i - 1)}{2} + j - i$ ; X
:= map(expand, GetComponents(S, List2)); X[k]; end proc:
N > expr := expand(I · Torsion(Omega[2], 1, 4) - Torsion(Omega[3], 2, 4)) : expr1

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$:= \text{expand}(-I \cdot \text{Torsion}(\text{Omega}[4], 1, 2) + \text{Torsion}(\text{Omega}[5], 2, 4)) :$

N > $\text{Normalisation} := \text{expand}(\text{solve}(\text{expr}, d)) : \text{Normalisation1} := \text{expand}(\text{solve}(\text{expr1}, d1)) :$

N > **for** i **from** 1 **to** 3 **do** $Z[i] := \text{result}(\text{subs}(\{d = \text{Normalisation}, d1 = \text{Normalisation1}\}, \text{Omega}[i])) ;$ **od**:

N > $To := \text{result}(\text{evalDG}(\text{Der}(H) \& \text{wedge } \rho)) :$

N > $G[3] := \text{result}\left(\text{evalDG}\left(Z[3] + \frac{I \cdot To}{cI^2}\right)\right) :$

N > $H := \text{expand}\left(\frac{\left(\text{Normalisation} - \left(-I \cdot \frac{e^2 \cdot cI}{2 \cdot c}\right)\right) \cdot cI}{I \cdot c}\right) ;$

$$H := \frac{2}{9} \frac{L^\#(L^\#(k))^2}{L^\#(k)^2} + \frac{1}{18} \frac{L^\#(L^\#(k)) PI}{L^\#(k)} - \frac{1}{9} PI^2 + \frac{1}{6} L^\#(PI) - \frac{1}{6} \frac{L^\#(L^\#(L^\#(k)))}{L^\#(k)} \quad (4)$$

N > $Z[1]; Z[2]; \text{result}(\text{evalDG}(G[3])) ;$

$$\begin{aligned} & \left(-\frac{eI}{cI} + \frac{1}{3} \frac{L(L(k^\#))}{cL(k^\#)} + \frac{2}{3} \frac{P}{c} + \frac{eL(k) cI}{c^2 L^\#(k)} \right) \rho \wedge \kappa - \frac{cI L(k) \rho \wedge \zeta}{cL^\#(k)} + \left(-\frac{e}{c} \right. \\ & \quad \left. + \frac{1}{3} \frac{L^\#(L^\#(k))}{L^\#(k) cI} + \frac{2}{3} \frac{PI}{cI} + \frac{eI L^\#(k^\#) c}{cI^2 L(k^\#)} \right) \rho \wedge \kappa^\# - \frac{L^\#(k^\#) c \rho \wedge \zeta^\#}{cI L(k^\#)} + I \kappa \wedge \kappa^\# \\ & \left(-\frac{\frac{2}{9} IL(k) L^\#(L^\#(k))^2}{L^\#(k)^3 cI c} - \frac{\frac{1}{18} IL(k) L^\#(L^\#(k)) PI}{L^\#(k)^2 cI c} + \frac{\frac{1}{3} IeK(L^\#(L^\#(k)))}{L^\#(k)^2 c^2} \right. \\ & \quad \left. + \frac{\frac{1}{3} IPI eI}{cI^2} + \frac{\frac{1}{3} IL(PI)}{cI c} - \frac{\frac{1}{6} IL(k) L^\#(PI)}{L^\#(k) cI c} - \frac{\frac{1}{3} IL^\#(L^\#(k)) eI}{L^\#(k) cI^2} \right. \\ & \quad \left. - \frac{\frac{1}{9} IPI L(L(k^\#))}{L(k^\#) cI c} - \frac{\frac{2}{3} IeP}{c^2} + \frac{\frac{1}{3} eT(k)}{L^\#(k) c^2} + \frac{\frac{1}{9} IL(k) PI^2}{L^\#(k) cI c} \right. \\ & \quad \left. + \frac{\frac{1}{3} IL^\#(L^\#(k)) L(L^\#(k))}{L^\#(k)^2 cI c} + \frac{\frac{1}{3} IeL(L^\#(k))}{L^\#(k) c^2} - \frac{\frac{1}{3} IeL^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 c^2} \right. \\ & \quad \left. - \frac{\frac{1}{3} IL(L^\#(L^\#(k)))}{L^\#(k) cI c} + \frac{\frac{2}{9} IL^\#(L^\#(k)) P}{L^\#(k) cI c} + \frac{\frac{1}{6} IL(k) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 cI c} \right. \\ & \quad \left. + \frac{\frac{1}{9} IL^\#(L^\#(k)) L(L(k^\#))}{L^\#(k) L(k^\#) cI c} - \frac{\frac{2}{9} IPIP}{cI c} - \frac{\frac{1}{2} I cI e^2 L(k)}{L^\#(k) c^3} \right) \rho \wedge \kappa + \left(\right. \\ & \quad \left. - \frac{\frac{1}{3} IK(L^\#(L^\#(k)))}{cL^\#(k)^2} + \frac{\frac{1}{3} IL^\#(L^\#(k)) K(L^\#(k))}{cL^\#(k)^3} - \frac{\frac{1}{3} IL(L^\#(k))}{cL^\#(k)} - \frac{1}{3} \frac{T(k)}{cL^\#(k)} \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{IeI}{cl} - \frac{\frac{1}{3} IL(L(k^\#))}{cL(k^\#)} \Big) \rho \wedge \zeta + \left(-\frac{\frac{2}{3} IL^\#(L^\#(k)) e}{cclL^\#(k)} + \frac{\frac{2}{9} IL^\#(L^\#(k))^2}{cI^2L^\#(k)^2} \right. \\
& + \frac{\frac{1}{18} IL^\#(L^\#(k)) PI}{cI^2L^\#(k)} - \frac{\frac{1}{3} IPI e}{ccl} - \frac{\frac{1}{9} IPI^2}{cI^2} + \frac{\frac{1}{6} IL^\#(PI)}{cI^2} \\
& \left. - \frac{\frac{1}{6} IL^\#(L^\#(L^\#(k)))}{cI^2L^\#(k)} + \frac{\frac{1}{2} Ie^2}{c^2} - \frac{IeL^\#(k^\#) el}{cI^2L(k^\#)} \right) \rho \wedge \kappa^\# + \frac{IeL^\#(k^\#) \rho \wedge \zeta^\#}{cIL(k^\#)} \\
& - \frac{cIL(k) \kappa \wedge \zeta}{cL^\#(k)} + \left(-\frac{1}{3} \frac{L^\#(L^\#(k))}{L^\#(k) cl} + \frac{1}{3} \frac{PI}{cl} \right) \kappa \wedge \kappa^\# + \zeta \wedge \kappa^\# \\
& \left(-\frac{\frac{1}{6} IL(L^\#(PI))}{cI^2c} + \frac{\frac{1}{9} IL(L^\#(k)) L^\#(L^\#(k)) PI}{cI^2L^\#(k)^2c} - \frac{\frac{1}{9} IeL(L^\#(k)) PI}{cIL^\#(k)c^2} \right. \\
& + \frac{\frac{1}{3} IeL^\#(L^\#(k)) P}{cIL^\#(k)c^2} + \frac{\frac{11}{18} IeL(L^\#(k)) L^\#(L^\#(k))}{cIL^\#(k)^2c^2} \\
& + \frac{\frac{4}{9} IeL^\#(L^\#(k)) K(L^\#(L^\#(k)))}{cIL^\#(k)^3c^2} + \frac{\frac{1}{18} IeK(L^\#(L^\#(k))) PI}{cIL^\#(k)^2c^2} \\
& + \frac{\frac{1}{6} IeK(L^\#(k)) L^\#(L^\#(L^\#(k)))}{cIL^\#(k)^3c^2} - \frac{\frac{4}{9} IeK(L^\#(k)) L^\#(L^\#(k))^2}{cIL^\#(k)^4c^2} \\
& + \frac{\frac{1}{6} IeIL^\#(L^\#(L^\#(k)))}{cI^3L^\#(k)} - \frac{\frac{1}{2} Ie^2 el}{cI^2c} - \frac{\frac{2}{27} IPI^2}{cI^2c} + \frac{\frac{1}{9} IPI^\#(PI)}{cI^2c} \\
& + \frac{\frac{1}{6} Ie^2L(L(k^\#))}{c^3L(k^\#)} - \frac{\frac{1}{6} Ie^2L(L^\#(k))}{L^\#(k)c^3} - \frac{\frac{2}{9} IeIL^\#(L^\#(k))^2}{cI^3L^\#(k)^2} + \frac{\frac{1}{3} IePI el}{cI^2} \\
& - \frac{\frac{1}{3} Ie^2L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3c^3} - \frac{\frac{1}{3} IeL(L^\#(L^\#(k)))}{L^\#(k)c^2cl} - \frac{\frac{2}{9} IeL^\#(L^\#(k)) L(L(k^\#))}{L^\#(k)c^2cIL(k^\#)} \\
& + \frac{\frac{1}{3} Ie^2K(L^\#(L^\#(k)))}{L^\#(k)^2c^3} - \frac{\frac{1}{6} IeL^\#(L(L^\#(k)))}{cIL^\#(k)c^2} - \frac{\frac{1}{6} IeK(L^\#(L^\#(L^\#(k))))}{cIL^\#(k)^2c^2} \\
& - \frac{\frac{4}{9} IL^\#(L^\#(k)) L(L^\#(L^\#(k)))}{cI^2L^\#(k)^2c} - \frac{\frac{1}{18} IL(L^\#(L^\#(k))) PI}{cI^2L^\#(k)c}
\end{aligned} \tag{5}$$

$$\begin{aligned}
& - \frac{\frac{1}{18} \text{IL}^\#(L^\#(k)) L(PI)}{cI^2 L^\#(k) c} - \frac{\frac{1}{3} \text{IL}(L^\#(k)) L^\#(L^\#(L^\#(k)))}{cI^2 L^\#(k)^2 c} \\
& + \frac{\frac{2}{3} \text{IL}(L^\#(k)) L^\#(L^\#(k))^2}{cI^2 L^\#(k)^3 c} - \frac{\frac{1}{18} \text{IeK}(L^\#(k)) L^\#(L^\#(k)) PI}{cI L^\#(k)^3 c^2} + \frac{1}{3} \frac{e^2 \text{T}(k)}{L^\#(k) c^3} \\
& - \frac{e \text{T}(L^\#(k))}{L^\#(k) c^2 cI} - \frac{4}{9} \frac{e PI \text{T}(k)}{cI L^\#(k) c^2} + \frac{1}{9} \frac{e L^\#(L^\#(k)) \text{T}(k)}{cI L^\#(k)^2 c^2} \\
& + \frac{\frac{1}{54} \text{IL}(L(k^\#)) L^\#(L^\#(k)) PI}{cI^2 L^\#(k) c L(k^\#)} + \frac{\frac{1}{18} \text{Ie} L^\#(k^\#) L(L(k^\#)) P}{cI c^2 L(k^\#)^2} - \frac{\frac{1}{9} \text{Ie} L^\#(k^\#) P^2}{cI c^2 L(k^\#)} \\
& - \frac{\frac{1}{6} \text{Ie} L^\#(k^\#) L(L(L(k^\#)))}{cI c^2 L(k^\#)^2} + \frac{\frac{1}{6} \text{Ie} L^\#(k^\#) L(P)}{cI c^2 L(k^\#)} + \frac{\frac{1}{27} \text{IPL}^\#(L^\#(k)) PI}{cI^2 L^\#(k) c} \\
& + \frac{\frac{2}{9} \text{Ie} L^\#(k^\#) L(L(k^\#))^2}{cI c^2 L(k^\#)^3} + \frac{\frac{2}{27} \text{IL}(L(k^\#)) L^\#(L^\#(k))^2}{cI^2 L^\#(k)^2 c L(k^\#)} \\
& - \frac{\frac{1}{18} \text{IL}(L(k^\#)) L^\#(L^\#(L^\#(k)))}{cI^2 L^\#(k) c L(k^\#)} - \frac{\frac{1}{18} \text{Ie} l L^\#(L^\#(k)) PI}{cI^3 L^\#(k)} - \frac{\frac{1}{27} \text{IL}(L(k^\#)) PI^2}{cI^2 c L(k^\#)} \\
& + \frac{\frac{1}{18} \text{IL}(L(k^\#)) L^\#(PI)}{cI^2 c L(k^\#)} - \frac{\frac{1}{9} \text{IL}(L^\#(k)) PI^2}{cI^2 L^\#(k) c} + \frac{\frac{1}{6} \text{IL}(L^\#(k)) L^\#(PI)}{cI^2 L^\#(k) c} \\
& + \frac{\frac{4}{27} \text{IPL}^\#(L^\#(k))^2}{cI^2 L^\#(k)^2 c} - \frac{\frac{1}{9} \text{IPL}^\#(L^\#(L^\#(k)))}{cI^2 L^\#(k) c} + \frac{\frac{1}{2} \text{Ie} L^\#(k^\#) eI^2}{cI^3 L(k^\#)} \\
& + \frac{1}{3} \frac{e L^\#(\text{T}(k))}{cI L^\#(k) c^2} - \frac{\frac{1}{3} \text{IPE}^2}{c^3} + \frac{\frac{1}{9} \text{Ie} l PI^2}{cI^3} - \frac{\frac{1}{6} \text{Ie} l L^\#(PI)}{cI^3} \\
& + \frac{\frac{2}{3} \text{Ie} L^\#(L^\#(k)) eI}{L^\#(k) c cI^2} - \frac{\frac{1}{9} \text{Ie} P I L(L(k^\#))}{c^2 cI L(k^\#)} - \frac{\frac{1}{6} \text{Ie} L^\#(P)}{cI c^2} + \frac{\frac{2}{9} \text{IPI} L(PI)}{cI^2 c} \\
& + \left. \frac{\frac{1}{6} \text{Ie} L(PI)}{cI c^2} + \frac{\frac{1}{6} \text{IL}(L^\#(L^\#(L^\#(k))))}{cI^2 L^\#(k) c} \right\} \rho \wedge \kappa + \left(\frac{\frac{1}{6} \text{IL}^\#(P)}{cI c} + \frac{\frac{1}{6} \text{IL}(PI)}{cI c} \right) \\
& + \frac{\frac{1}{18} \text{IK}(L^\#(k)) L^\#(L^\#(k)) PI}{cI L^\#(k)^3 c} - \frac{1}{3} \frac{L^\#(\text{T}(k))}{cI L^\#(k) c} + \frac{\frac{1}{6} \text{IL}^\#(L(L^\#(k)))}{cI L^\#(k) c}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\frac{1}{6} \text{IK}(L^\#(L^\#(L^\#(k))))}{c l L^\#(k)^2 c} + \frac{\frac{1}{3} \text{IL}^\#(L^\#(k)) L(L(k^\#))}{L^\#(k) L(k^\#) c l c} \\
& + \frac{\frac{1}{3} \text{Ie} L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 c^2} - \frac{\frac{2}{9} \text{IPI} P}{c l c} + \frac{\text{T}(L^\#(k))}{L^\#(k) c c l} + \frac{\text{Ie} e l}{c c l} \\
& - \frac{\frac{1}{3} \text{Ie} L(L(k^\#))}{c^2 L(k^\#)} - \frac{\text{IL}^\#(L^\#(k)) e l}{L^\#(k) c l^2} + \frac{\frac{2}{3} \text{IL}(L^\#(k)) e}{L^\#(k) c^2} - \frac{\frac{1}{3} \text{Ie} K(L^\#(L^\#(k)))}{L^\#(k)^2 c^2} \\
& + \frac{4}{9} \frac{\text{PI} \text{T}(k)}{c l L^\#(k) c} - \frac{1}{9} \frac{L^\#(L^\#(k)) \text{T}(k)}{c l L^\#(k)^2 c} - \frac{\frac{4}{9} \text{IL}^\#(L^\#(k)) K(L^\#(L^\#(k)))}{c l L^\#(k)^3 c} \\
& - \frac{\frac{1}{18} \text{IK}(L^\#(L^\#(k))) \text{PI}}{c l L^\#(k)^2 c} - \frac{\frac{1}{6} \text{IK}(L^\#(k)) L^\#(L^\#(L^\#(k)))}{c l L^\#(k)^3 c} \\
& - \frac{\frac{5}{18} \text{IL}(L^\#(k)) L^\#(L^\#(k))}{c l L^\#(k)^2 c} - \frac{\frac{1}{9} \text{IL}^\#(L^\#(k)) P}{c l L^\#(k) c} + \frac{\frac{1}{9} \text{IL}(L^\#(k)) \text{PI}}{c l L^\#(k) c} \\
& + \frac{\frac{4}{9} \text{IK}(L^\#(k)) L^\#(L^\#(k))^2}{c l L^\#(k)^4 c} - \frac{\frac{1}{18} \text{IL}(k) L^\#(L^\#(k)) \text{PI}}{L^\#(k)^2 c l c} - \frac{\frac{1}{18} \text{IL}^\#(k^\#) L(L(k^\#)) P}{c l c L(k^\#)^2} \\
& - \frac{\frac{2}{9} \text{IL}^\#(k^\#) L(L(k^\#))^2}{c l c L(k^\#)^3} + \frac{\frac{1}{6} \text{IL}^\#(k^\#) L(L(L(k^\#)))}{c l c L(k^\#)^2} - \frac{\frac{1}{2} \text{Ic} L^\#(k^\#) e l^2}{c l^3 L(k^\#)} \\
& + \frac{\frac{1}{9} \text{IL}^\#(k^\#) P^2}{c l c L(k^\#)} - \frac{\frac{1}{6} \text{IL}^\#(k^\#) L(P)}{c l c L(k^\#)} - \frac{\frac{1}{3} \frac{e \text{T}(k)}{L^\#(k) c^2}}{L^\#(k)^3 c l c} - \frac{\frac{2}{9} \text{IL}(k) L^\#(L^\#(k))^2}{L^\#(k)^3 c l c} \\
& - \frac{\frac{1}{6} \text{IL}(k) L^\#(\text{PI})}{L^\#(k) c l c} + \frac{\frac{1}{9} \text{IL}(k) \text{PI}^2}{L^\#(k) c l c} + \frac{\frac{1}{6} \text{IL}(k) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 c l c} \\
& - \frac{\frac{1}{2} \text{Ic} l e^2 L(k)}{L^\#(k) c^3} \Big) \rho \wedge \zeta + \left(\frac{\frac{1}{6} \text{IL}^\#(L^\#(k)) L^\#(\text{PI})}{c l^3 L^\#(k)} - \frac{\frac{1}{9} \text{IL}^\#(L^\#(k)) \text{PI}^2}{L^\#(k) c l^3} \right. \\
& + \frac{\frac{5}{18} \text{IL}^\#(L^\#(k))^2 \text{PI}}{c l^3 L^\#(k)^2} - \frac{\frac{5}{6} \text{IL}^\#(L^\#(k)) L^\#(L^\#(L^\#(k)))}{c l^3 L^\#(k)^2} - \frac{\frac{1}{6} \text{IL}^\#(L^\#(L^\#(k))) \text{PI}}{c l^3 L^\#(k)} \\
& \left. + \frac{\frac{1}{3} \text{IPI} L^\#(\text{PI})}{c l^3} + \frac{\frac{20}{27} \text{IL}^\#(L^\#(k))^3}{c l^3 L^\#(k)^3} - \frac{\frac{2}{27} \text{IPI}^3}{c l^3} - \frac{\frac{1}{6} \text{IL}^\#(L^\#(\text{PI}))}{c l^3} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} \frac{1L^\#(L^\#(L^\#(L^\#(k))))}{cI^3 L^\#(k)} \Big) \rho \wedge \kappa^\# + \left(\frac{L(L^\#(k))}{cL^\#(k)} - \frac{eL(k) cI}{c^2 L^\#(k)} \right) \kappa \wedge \zeta \\
& + \left(\frac{2}{3} \frac{L^\#(L^\#(k)) e}{c cI L^\#(k)} - \frac{2}{9} \frac{L^\#(L^\#(k))^2}{cI^2 L^\#(k)^2} - \frac{1}{18} \frac{L^\#(L^\#(k)) PI}{cI^2 L^\#(k)} + \frac{1}{3} \frac{PI e}{c cI} + \frac{1}{9} \frac{PI^2}{cI^2} \right. \\
& - \left. \frac{1}{6} \frac{L^\#(PI)}{cI^2} + \frac{1}{6} \frac{L^\#(L^\#(L^\#(k)))}{cI^2 L^\#(k)} - \frac{1}{2} \frac{e^2}{c^2} + \frac{eL^\#(k^\#) eI}{cI^2 L(k^\#)} \right) \kappa \wedge \kappa^\# \\
& - \frac{L^\#(k^\#) e \kappa \wedge \zeta^\#}{cI L(k^\#)} + \left(-\frac{eI L^\#(k^\#) c}{cI^2 L(k^\#)} - \frac{L^\#(L^\#(k))}{L^\#(k) cI} + \frac{e}{c} \right) \zeta \wedge \kappa^\# + \frac{L^\#(k^\#) c \zeta \wedge \zeta^\#}{cI L(k^\#)}
\end{aligned}$$

N > *result(evalDG(Der(Wu)) &wedge rho); sub := proc(l); subs({e = epsilon·Wu, eI = εI·WuI}, l); end proc;*

$$\begin{aligned}
& \left(\frac{K(Wu) e cI}{L^\#(k) c^2} - \frac{L(Wu)}{c} \right) \rho \wedge \kappa - \frac{K(Wu) cI \rho \wedge \zeta}{L^\#(k) c} + \left(\frac{K^\#(Wu) eI c}{L(k^\#) cI^2} \right. \\
& \left. - \frac{L^\#(Wu)}{cI} \right) \rho \wedge \kappa^\# - \frac{K^\#(Wu) c \rho \wedge \zeta^\#}{L(k^\#) cI} \\
& \text{sub := proc(l) subs({e = ε * Wu, eI = εI * WuI}, l) end proc} \tag{6}
\end{aligned}$$

N > *Q[1] := sub((evalDG(subs({c = Wu, cI = WuI}, (Der(Wu) / Wu + Der(WuI) / WuI)) &wedge rho + Z[1])))) :*

$$\begin{aligned}
& \text{N} > \text{result}(Q[1]); \\
& \left(-\varepsilon I + \frac{1}{3} \frac{L(L(k^\#))}{Wu L(k^\#)} + \frac{2}{3} \frac{P}{Wu} + \frac{WuI \varepsilon L(k)}{Wu L^\#(k)} + \frac{K(WuI) \varepsilon}{Wu L^\#(k)} - \frac{L(WuI)}{Wu WuI} \right. \\
& + \frac{WuI K(Wu) \varepsilon}{Wu^2 L^\#(k)} - \frac{L(Wu)}{Wu^2} \Big) \rho \wedge \kappa + \left(-\frac{WuI L(k)}{Wu L^\#(k)} - \frac{K(WuI)}{Wu L^\#(k)} \right. \\
& - \left. \frac{K(Wu) WuI}{Wu^2 L^\#(k)} \right) \rho \wedge \zeta + \left(-\varepsilon + \frac{1}{3} \frac{L^\#(L^\#(k))}{WuI L^\#(k)} + \frac{2}{3} \frac{PI}{WuI} + \frac{Wu \varepsilon I L^\#(k^\#)}{WuI L(k^\#)} \right. \\
& + \frac{Wu K^\#(WuI) \varepsilon I}{WuI^2 L(k^\#)} - \frac{L^\#(WuI)}{WuI^2} + \frac{K^\#(Wu) \varepsilon I}{WuI L(k^\#)} - \frac{L^\#(Wu)}{WuI Wu} \Big) \rho \wedge \kappa^\# + \left(-\frac{L^\#(k^\#) Wu}{WuI L(k^\#)} \right. \\
& - \left. \frac{K^\#(WuI) Wu}{WuI^2 L(k^\#)} - \frac{K^\#(Wu)}{WuI L(k^\#)} \right) \rho \wedge \zeta^\# + I \kappa \wedge \kappa^\# \tag{7}
\end{aligned}$$

N > *Q[3] := evalDG(sub(subs({c = Wu, cI = WuI, d = Normalisation, dI = NormalisationI}, (e·Der(WuI) / Wu·WuI &wedge kappa) + (Der(Wu) / Wu - Der(WuI) / WuI) &wedge zeta + G[3]))) : result(Q[3]);*

$$\begin{aligned}
& \left(\frac{\frac{1}{3} \text{I} \varepsilon L^\#(Wu1) L(L(k^\#))}{Wu Wu1^2 L(k^\#)} - \frac{\frac{1}{9} \text{I} \varepsilon L^\#(k^\#) P^2}{Wu Wu1 L(k^\#)} - \frac{\frac{4}{9} \text{I} \varepsilon K(L^\#(k)) L^\#(L^\#(k))^2}{L^\#(k)^4 Wu Wu1} \right. \\
& + \frac{\frac{4}{9} \text{I} \varepsilon L^\#(L^\#(k)) K(L^\#(L^\#(k)))}{L^\#(k)^3 Wu Wu1} + \frac{\frac{1}{6} \text{I} \varepsilon K(L^\#(k)) L^\#(L^\#(L^\#(k)))}{L^\#(k)^3 Wu Wu1} \\
& + \frac{\frac{1}{18} \text{I} \varepsilon K(L^\#(L^\#(k))) PI}{L^\#(k)^2 Wu Wu1} - \frac{\frac{1}{3} \text{I} \varepsilon L(Wu1) L^\#(L^\#(k))}{L^\#(k) Wu Wu1^2} + \frac{\frac{1}{2} \text{I} c1 \varepsilon^3 K(Wu1)}{L^\#(k) Wu1 c} \\
& + \frac{\text{I} Wu \varepsilon K^\#(Wu1) \varepsilon I^2}{Wu I^2 L(k^\#)} + \frac{\frac{1}{2} \text{I} Wu \varepsilon L^\#(k^\#) \varepsilon I^2}{Wu1 L(k^\#)} - \frac{\frac{1}{18} \text{I} L(L(k^\#)) L^\#(L^\#(L^\#(k)))}{L^\#(k) Wu Wu1^2 L(k^\#)} \\
& + \frac{\frac{1}{27} \text{I} P L^\#(L^\#(k)) PI}{L^\#(k) Wu Wu1^2} + \frac{\frac{1}{9} \text{I} L(L^\#(k)) L^\#(L^\#(k)) PI}{L^\#(k)^2 Wu Wu1^2} \\
& + \frac{\frac{11}{18} \text{I} \varepsilon L(L^\#(k)) L^\#(L^\#(k))}{L^\#(k)^2 Wu Wu1} + \frac{\frac{1}{3} \text{I} \varepsilon L^\#(L^\#(k)) P}{L^\#(k) Wu Wu1} + \frac{\frac{2}{27} \text{I} L(L(k^\#)) L^\#(L^\#(k))^2}{L^\#(k)^2 Wu Wu1^2 L(k^\#)} \\
& - \frac{\frac{1}{9} \text{I} \varepsilon PI L(L(k^\#))}{Wu Wu1 L(k^\#)} - \frac{\frac{1}{9} \text{I} \varepsilon L(L^\#(k)) PI}{L^\#(k) Wu Wu1} + \frac{\frac{2}{9} \text{I} \varepsilon L^\#(k^\#) L(L(k^\#))^2}{Wu Wu1 L(k^\#)^3} \\
& + \frac{\frac{1}{6} \text{I} \varepsilon L^\#(k^\#) L(P)}{Wu Wu1 L(k^\#)} - \frac{\frac{1}{6} \text{I} \varepsilon L^\#(k^\#) L(L(L(k^\#)))}{Wu Wu1 L(k^\#)^2} - \frac{\text{I} \varepsilon^3 K(Wu1)}{L^\#(k) Wu} + \frac{\text{I} \varepsilon^2 L(Wu1)}{Wu Wu1} \\
& - \frac{\frac{2}{27} \text{I} P PI^2}{Wu Wu1^2} - \frac{\frac{1}{6} \text{I} \varepsilon L^\#(P)}{Wu Wu1} + \frac{\frac{1}{6} \text{I} L(L^\#(L^\#(L^\#(k))))}{L^\#(k) Wu Wu1^2} + \frac{\frac{1}{9} \text{I} P L^\#(PI)}{Wu Wu1^2} \\
& + \frac{\frac{1}{3} \text{I} \varepsilon L(Wu1) PI}{Wu Wu1^2} - \frac{\frac{1}{3} \text{I} \varepsilon L(L^\#(L^\#(k)))}{L^\#(k) Wu Wu1} - \frac{\frac{1}{3} \text{I} \varepsilon^2 L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 Wu} \\
& + \frac{\frac{2}{3} \text{I} L(L^\#(k)) L^\#(L^\#(k))^2}{L^\#(k)^3 Wu Wu1^2} - \frac{\frac{1}{9} \text{I} P L^\#(L^\#(L^\#(k)))}{L^\#(k) Wu Wu1^2} + \frac{\frac{4}{27} \text{I} P L^\#(L^\#(k))^2}{L^\#(k)^2 Wu Wu1^2} \\
& - \frac{\frac{1}{3} \text{I} L(L^\#(k)) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 Wu Wu1^2} + \frac{\frac{1}{6} \text{I} L(L^\#(k)) L^\#(PI)}{L^\#(k) Wu Wu1^2} - \frac{\frac{1}{9} \text{I} L(L^\#(k)) PI^2}{L^\#(k) Wu Wu1^2} \\
& + \frac{\frac{1}{18} \text{I} L(L(k^\#)) L^\#(PI)}{Wu Wu1^2 L(k^\#)} - \frac{\frac{1}{27} \text{I} L(L(k^\#)) PI^2}{Wu Wu1^2 L(k^\#)} - \frac{\frac{1}{6} \text{I} \varepsilon K(L^\#(L^\#(L^\#(k))))}{L^\#(k)^2 Wu Wu1}
\end{aligned} \tag{8}$$

$$\begin{aligned}
& - \frac{\frac{1}{6} \text{I} \varepsilon L^\#(L(L^\#(k)))}{L^\#(k) Wu Wu l} - \frac{\frac{1}{3} \text{I} \varepsilon L^\#(Wu l) P}{Wu Wu l^2} - \frac{\frac{1}{18} \text{I} L^\#(L^\#(k)) L(P l)}{L^\#(k) Wu Wu l^2} \\
& - \frac{\frac{1}{18} \text{I} L(L^\#(L^\#(k))) P l}{L^\#(k) Wu Wu l^2} - \frac{\frac{4}{9} \text{I} L^\#(L^\#(k)) L(L^\#(L^\#(k)))}{L^\#(k)^2 Wu Wu l^2} + \frac{\frac{2}{3} \text{I} \varepsilon L^\#(L^\#(k)) \varepsilon l}{L^\#(k) Wu l} \\
& - \frac{\frac{1}{18} \text{I} \varepsilon l L^\#(L^\#(k)) P l}{L^\#(k) Wu l^2} - \frac{\frac{1}{6} \text{I} \varepsilon l L^\#(P l)}{Wu l^2} + \frac{\frac{1}{9} \text{I} \varepsilon l P l^2}{Wu l^2} - \frac{\frac{1}{3} \text{I} P \varepsilon^2}{Wu} \\
& - \frac{\frac{1}{6} \text{I} L(L^\#(P l))}{Wu Wu l^2} + \frac{\frac{1}{6} \text{I} \varepsilon l L^\#(L^\#(L^\#(k)))}{L^\#(k) Wu l^2} - \frac{\frac{2}{9} \text{I} \varepsilon l L^\#(L^\#(k))^2}{L^\#(k)^2 Wu l^2} \\
& - \frac{\frac{1}{6} \text{I} \varepsilon^2 L(L^\#(k))}{L^\#(k) Wu} + \frac{\frac{1}{6} \text{I} \varepsilon^2 L(L(k^\#))}{Wu L(k^\#)} + \frac{\frac{2}{9} \text{I} P l L(P l)}{Wu Wu l^2} + \frac{\frac{1}{3} \text{I} \varepsilon P l \varepsilon l}{Wu l} \\
& + \frac{\frac{1}{6} \text{I} \varepsilon L(P l)}{Wu Wu l} + \frac{\frac{1}{3} \text{I} \varepsilon^2 K(L^\#(L^\#(k)))}{L^\#(k)^2 Wu} - \frac{\text{I} \varepsilon L^\#(Wu l) \varepsilon l}{Wu l^2} - \frac{4}{9} \frac{\varepsilon P l T(k)}{L^\#(k) Wu Wu l} \\
& + \frac{1}{9} \frac{\varepsilon L^\#(L^\#(k)) T(k)}{L^\#(k)^2 Wu Wu l} + \frac{1}{3} \frac{\varepsilon^2 T(k)}{L^\#(k) Wu} - \frac{\varepsilon T(L^\#(k))}{L^\#(k) Wu Wu l} + \frac{\varepsilon T(Wu l)}{Wu Wu l^2} \\
& + \frac{\frac{1}{54} \text{I} L(L(k^\#)) L^\#(L^\#(k)) P l}{L^\#(k) Wu Wu l^2 L(k^\#)} - \frac{1}{2} \text{I} \varepsilon^2 \varepsilon l + \frac{1}{3} \frac{\varepsilon L^\#(T(k))}{L^\#(k) Wu Wu l} \\
& - \frac{\frac{2}{9} \text{I} \varepsilon L^\#(L^\#(k)) L(L(k^\#))}{L^\#(k) Wu Wu l L(k^\#)} + \frac{\frac{1}{18} \text{I} \varepsilon L^\#(k^\#) L(L(k^\#)) P}{Wu Wu l L(k^\#)^2} \\
& - \frac{\frac{1}{18} \text{I} \varepsilon K(L^\#(k)) L^\#(L^\#(k)) P l}{L^\#(k)^3 Wu Wu l} + \frac{\frac{1}{18} \text{I} c l \varepsilon K^\#(Wu l) L(L(k^\#)) P}{Wu l^3 L(k^\#)^2 c} \\
& - \frac{\frac{1}{9} \text{I} c l \varepsilon K^\#(Wu l) P^2}{Wu l^3 L(k^\#) c} - \frac{\frac{1}{6} \text{I} c l \varepsilon K^\#(Wu l) L(L(L(k^\#)))}{Wu l^3 L(k^\#)^2 c} \\
& + \frac{\frac{1}{6} \text{I} c l \varepsilon K^\#(Wu l) L(P)}{Wu l^3 L(k^\#) c} - \frac{\frac{1}{2} \text{I} c \varepsilon K^\#(Wu l) \varepsilon l^2}{Wu l L(k^\#) c l} - \frac{\frac{2}{9} \text{I} c \varepsilon K(Wu l) L^\#(L^\#(k))^2}{L^\#(k)^3 Wu^2 Wu l c l} \\
& - \frac{\frac{1}{18} \text{I} c \varepsilon K(Wu l) L^\#(L^\#(k)) P l}{L^\#(k)^2 Wu^2 Wu l c l} + \frac{\frac{1}{6} \text{I} c \varepsilon K(Wu l) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 Wu^2 Wu l c l}
\end{aligned}$$

$$\begin{aligned}
& + \left. \frac{\frac{1}{9} \text{Ic} \varepsilon K(Wu1) P I^2}{L^\#(k) Wu^2 Wu1 c1} - \frac{\frac{1}{6} \text{Ic} \varepsilon K(Wu1) L^\#(P1)}{L^\#(k) Wu^2 Wu1 c1} + \frac{\frac{2}{9} \text{Ic} l \varepsilon K^\#(Wu1) L(L(k^\#))^2}{Wu1^3 L(k^\#)^3 c} \right) \\
& \rho \wedge \kappa + \left(\frac{\frac{1}{3} \text{IL}(Wu) P I}{Wu1 Wu^2} - \frac{\frac{1}{3} \text{IL}(Wu1) P I}{Wu1^2 Wu} - \frac{\text{IL}(Wu1) \varepsilon}{Wu1 Wu} \right. \\
& + \frac{\frac{1}{18} \text{IK}(L^\#(k)) L^\#(L^\#(k)) P I}{Wu1 L^\#(k)^3 Wu} + \frac{\frac{1}{3} \text{IL}^\#(L^\#(k)) L(L(k^\#))}{Wu1 L^\#(k) Wu L(k^\#)} - \frac{\frac{2}{9} \text{IPI} P}{Wu1 Wu} \\
& + \frac{\text{IK}^\#(Wu) \varepsilon l^2}{Wu1 L(k^\#)} - \frac{\frac{1}{3} \text{IL}^\#(Wu) P}{Wu1 Wu^2} + \frac{\frac{1}{6} \text{IK}(L^\#(L^\#(L^\#(k))))}{Wu1 L^\#(k)^2 Wu} + \frac{\frac{1}{6} \text{IL}^\#(L(L^\#(k)))}{Wu1 L^\#(k) Wu} \\
& - \frac{\frac{1}{3} \text{I} \varepsilon L(L(k^\#))}{Wu L(k^\#)} - \frac{\frac{1}{3} \text{I} \varepsilon K(L^\#(L^\#(k)))}{L^\#(k)^2 Wu} + \frac{\frac{2}{3} \text{IL}(L^\#(k)) \varepsilon}{L^\#(k) Wu} - \frac{\text{IL}^\#(L^\#(k)) \varepsilon l}{Wu1 L^\#(k)} \\
& + \frac{\frac{1}{3} \text{IL}^\#(Wu1) P}{Wu1^2 Wu} - \frac{\text{IL}^\#(Wu) \varepsilon l}{Wu1 Wu} + \frac{\text{IK}(Wu1) \varepsilon^2}{L^\#(k) Wu} + \text{I} \varepsilon \varepsilon l + \frac{\text{T}(L^\#(k))}{Wu1 L^\#(k) Wu} \\
& - \frac{1}{3} \frac{L^\#(\text{T}(k))}{Wu1 L^\#(k) Wu} - \frac{\text{T}(Wu1)}{Wu1^2 Wu} + \frac{\text{T}(Wu)}{Wu1 Wu^2} - \frac{\frac{1}{18} \text{IL}(k) L^\#(L^\#(k)) P I}{Wu1 L^\#(k)^2 Wu} \\
& - \frac{\frac{1}{18} \text{IL}^\#(k^\#) L(L(k^\#)) P}{Wu1 Wu L(k^\#)^2} + \frac{\frac{1}{9} \text{Ic} l K^\#(Wu1) P^2}{Wu1^3 L(k^\#) c} - \frac{\frac{2}{9} \text{Ic} K(Wu) L^\#(L^\#(k))^2}{L^\#(k)^3 Wu^3 c1} \\
& + \frac{\frac{1}{9} \text{Ic} K(Wu) P I^2}{L^\#(k) Wu^3 c1} - \frac{\frac{1}{6} \text{Ic} l K^\#(Wu1) L(P)}{Wu1^3 L(k^\#) c} + \frac{\frac{1}{6} \text{Ic} l K^\#(Wu1) L(L(L(k^\#)))}{Wu1^3 L(k^\#)^2 c} \\
& - \frac{\frac{2}{9} \text{Ic} l K^\#(Wu1) L(L(k^\#))^2}{Wu1^3 L(k^\#)^3 c} + \frac{\frac{1}{6} \text{Ic} K(Wu) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 Wu^3 c1} \\
& - \frac{\frac{1}{6} \text{Ic} K(Wu) L^\#(P1)}{L^\#(k) Wu^3 c1} + \frac{\frac{1}{2} \text{Ic} K^\#(Wu1) \varepsilon l^2}{Wu1 L(k^\#) c1} - \frac{\frac{1}{2} \text{Ic} l K(Wu1) \varepsilon^2}{Wu1 L^\#(k) c} \\
& + \frac{\frac{1}{2} \text{Ic} l K(Wu) \varepsilon^2}{L^\#(k) Wu c} - \frac{\frac{1}{2} \text{Ic} K^\#(Wu) \varepsilon l^2}{Wu L(k^\#) c1} + \frac{\text{IL}^\#(Wu1) \varepsilon l}{Wu1^2} + \frac{\frac{1}{6} \text{IL}(P1)}{Wu1 Wu} \\
& + \frac{\frac{1}{6} \text{IL}^\#(P)}{Wu1 Wu} + \frac{\text{IL}(Wu) \varepsilon}{Wu^2} + \frac{4}{9} \frac{P I \text{T}(k)}{Wu1 L^\#(k) Wu} - \frac{1}{9} \frac{L^\#(L^\#(k)) \text{T}(k)}{Wu1 L^\#(k)^2 Wu}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{3} \frac{\varepsilon T(k)}{L^\#(k) Wu} - \frac{1}{18} \frac{IcI K^\#(WuI) L(L(k^\#)) P}{WuI^3 L(k^\#)^2 c} - \frac{1}{18} \frac{IcK(Wu) L^\#(L^\#(k)) PI}{L^\#(k)^2 Wu^3 cI} \\
& + \frac{2}{9} \frac{IcI K^\#(Wu) L(L(k^\#))^2}{WuI^2 WuL(k^\#)^3 c} + \frac{1}{18} \frac{IcK(WuI) L^\#(L^\#(k)) PI}{WuI L^\#(k)^2 Wu^2 cI} - \frac{1}{9} \frac{IcK(WuI) PI^2}{WuI L^\#(k) Wu^2 cI} \\
& - \frac{1}{18} \frac{IK(L^\#(L^\#(k))) PI}{WuI L^\#(k)^2 Wu} - \frac{1}{6} \frac{IK(L^\#(k)) L^\#(L^\#(L^\#(k)))}{WuI L^\#(k)^3 Wu} \\
& - \frac{4}{9} \frac{IL^\#(L^\#(k)) K(L^\#(L^\#(k)))}{WuI L^\#(k)^3 Wu} + \frac{1}{3} \frac{IL(WuI) L^\#(L^\#(k))}{WuI^2 L^\#(k) Wu} \\
& - \frac{1}{3} \frac{IL^\#(WuI) L(L(k^\#))}{WuI^2 WuL(k^\#)} - \frac{2}{9} \frac{IL^\#(k^\#) L(L(k^\#))^2}{WuI WuL(k^\#)^3} - \frac{1}{6} \frac{IL^\#(k^\#) L(P)}{WuI WuL(k^\#)} \\
& + \frac{1}{9} \frac{IL^\#(k^\#) P^2}{WuI WuL(k^\#)} + \frac{1}{6} \frac{IL^\#(k^\#) L(L(L(k^\#)))}{WuI WuL(k^\#)^2} - \frac{1}{6} \frac{IL(k) L^\#(PI)}{WuI L^\#(k) Wu} \\
& - \frac{1}{2} \frac{IWuI \varepsilon^2 L(k)}{L^\#(k) Wu} - \frac{1}{2} \frac{IWuL^\#(k^\#) \varepsilon I^2}{WuI L(k^\#)} + \frac{1}{3} \frac{I\varepsilon L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 Wu} \\
& - \frac{2}{9} \frac{IL(k) L^\#(L^\#(k))^2}{WuI L^\#(k)^3 Wu} + \frac{1}{9} \frac{IL(k) PI^2}{WuI L^\#(k) Wu} + \frac{1}{9} \frac{IL(L^\#(k)) PI}{WuI L^\#(k) Wu} \\
& + \frac{4}{9} \frac{IK(L^\#(k)) L^\#(L^\#(k))^2}{WuI L^\#(k)^4 Wu} + \frac{1}{6} \frac{IL(k) L^\#(L^\#(L^\#(k)))}{WuI L^\#(k)^2 Wu} - \frac{1}{9} \frac{IL^\#(L^\#(k)) P}{WuI L^\#(k) Wu} \\
& - \frac{5}{18} \frac{IL(L^\#(k)) L^\#(L^\#(k))}{WuI L^\#(k)^2 Wu} - \frac{IWuI K(Wu) \varepsilon^2}{L^\#(k) Wu^2} + \frac{1}{3} \frac{IL^\#(Wu) L(L(k^\#))}{WuI Wu^2 L(k^\#)} \\
& - \frac{IWuK^\#(WuI) \varepsilon I^2}{WuI^2 L(k^\#)} - \frac{1}{3} \frac{IL(Wu) L^\#(L^\#(k))}{WuI L^\#(k) Wu^2} + \frac{1}{6} \frac{IcK(WuI) L^\#(PI)}{WuI L^\#(k) Wu^2 cI} \\
& - \frac{1}{6} \frac{IcK(WuI) L^\#(L^\#(L^\#(k)))}{WuI L^\#(k)^2 Wu^2 cI} + \frac{1}{18} \frac{IcI K^\#(Wu) L(L(k^\#)) P}{WuI^2 WuL(k^\#)^2 c} \\
& - \frac{1}{9} \frac{IcI K^\#(Wu) P^2}{WuI^2 WuL(k^\#) c} - \frac{1}{6} \frac{IcI K^\#(Wu) L(L(L(k^\#)))}{WuI^2 WuL(k^\#)^2 c} + \frac{1}{6} \frac{IcI K^\#(Wu) L(P)}{WuI^2 WuL(k^\#) c}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\frac{2}{9} I c K(Wu1) L^\#(L^\#(k))^2}{Wu1 L^\#(k)^3 Wu^2 c1} \Big) \rho \wedge \zeta + \left(-\frac{\frac{2}{27} I P I^3}{Wu1^3} + \frac{\frac{5}{18} I L^\#(L^\#(k))^2 P I}{Wu1^3 L^\#(k)^2} \right. \\
& - \frac{\frac{1}{9} I L^\#(L^\#(k)) P I^2}{Wu1^3 L^\#(k)} + \frac{\frac{1}{6} I L^\#(L^\#(k)) L^\#(P I)}{Wu1^3 L^\#(k)} - \frac{\frac{5}{6} I L^\#(L^\#(k)) L^\#(L^\#(L^\#(k)))}{Wu1^3 L^\#(k)^2} \\
& + \frac{\frac{1}{3} I P I L^\#(P I)}{Wu1^3} - \frac{\frac{1}{6} I L^\#(L^\#(L^\#(k))) P I}{Wu1^3 L^\#(k)} + \frac{\frac{20}{27} I L^\#(L^\#(k))^3}{Wu1^3 L^\#(k)^3} - \frac{\frac{1}{6} I L^\#(L^\#(P I))}{Wu1^3} \\
& \left. + \frac{\frac{1}{6} I L^\#(L^\#(L^\#(L^\#(k))))}{Wu1^3 L^\#(k)} \right) \rho \wedge \kappa^\# + \left(\frac{L(L^\#(k))}{Wu L^\#(k)} - \frac{Wu1 \varepsilon L(k)}{Wu L^\#(k)} - \frac{L(Wu1)}{Wu Wu1} \right. \\
& - \frac{Wu1 K(Wu) \varepsilon}{Wu^2 L^\#(k)} + \frac{L(Wu)}{Wu^2} \Big) \kappa \wedge \zeta + \left(\frac{2}{3} \frac{L^\#(L^\#(k)) \varepsilon}{L^\#(k) Wu1} - \frac{2}{9} \frac{L^\#(L^\#(k))^2}{L^\#(k)^2 Wu1^2} \right. \\
& - \frac{1}{18} \frac{L^\#(L^\#(k)) P I}{L^\#(k) Wu1^2} + \frac{1}{3} \frac{P I \varepsilon}{Wu1} + \frac{1}{9} \frac{P I^2}{Wu1^2} - \frac{1}{6} \frac{L^\#(P I)}{Wu1^2} + \frac{1}{6} \frac{L^\#(L^\#(L^\#(k)))}{L^\#(k) Wu1^2} \\
& - \frac{1}{2} \varepsilon^2 + \frac{\varepsilon Wu L^\#(k^\#) \varepsilon l}{Wu1 L(k^\#)} + \frac{\varepsilon K^\#(Wu1) \varepsilon l Wu}{Wu1^2 L(k^\#)} - \frac{\varepsilon L^\#(Wu1)}{Wu1^2} \Big) \kappa \wedge \kappa^\# + \left(\right. \\
& - \frac{\varepsilon Wu L^\#(k^\#)}{Wu1 L(k^\#)} - \frac{\varepsilon Wu K^\#(Wu1)}{Wu1^2 L(k^\#)} \Big) \kappa \wedge \zeta^\# + \left(-\frac{Wu \varepsilon l L^\#(k^\#)}{Wu1 L(k^\#)} - \frac{L^\#(L^\#(k))}{Wu1 L^\#(k)} + \varepsilon \right. \\
& - \frac{Wu K^\#(Wu1) \varepsilon l}{Wu1^2 L(k^\#)} + \frac{L^\#(Wu1)}{Wu1^2} + \frac{K^\#(Wu) \varepsilon l}{Wu1 L(k^\#)} - \frac{L^\#(Wu)}{Wu1 Wu} \Big) \zeta \wedge \kappa^\# + \left(\frac{L^\#(k^\#) Wu}{Wu1 L(k^\#)} \right. \\
& \left. + \frac{K^\#(Wu1) Wu}{Wu1^2 L(k^\#)} - \frac{K^\#(Wu)}{Wu1 L(k^\#)} \right) \zeta \wedge \zeta^\#
\end{aligned}$$

N > $Q[2] := evalDG\left(sub\left(subs\left(\{c = Wu, c1 = Wu1, d = Normalisation, d1 = Normalisation1\}, Z[2] + \frac{Der(Wu)}{Wu} \&wedge \kappa - e \cdot \frac{I \cdot Der(Wu1)}{Wu1 \cdot Wu} \&wedge \rho\right)\right) : result(Q[2]);$

$$\left(\frac{\frac{1}{3} I L(Wu) P I}{Wu1 Wu^2} + \frac{\frac{1}{3} I P I \varepsilon l}{Wu1} - \frac{\frac{2}{3} I \varepsilon P}{Wu} - \frac{\frac{2}{9} I P I P}{Wu1 Wu} + \frac{I K^\#(Wu) \varepsilon l^2}{Wu1 L(k^\#)} \right.$$

$$- \frac{\frac{1}{3} I L^\#(Wu) P}{Wu1 Wu^2} - \frac{I L^\#(Wu) \varepsilon l}{Wu1 Wu} + \frac{T(Wu)}{Wu1 Wu^2} - \frac{\frac{1}{18} I L(k) L^\#(L^\#(k)) P I}{Wu1 L^\#(k)^2 Wu}$$

(9)

$$\begin{aligned}
& - \frac{\frac{2}{9} \text{IcK}(Wu) L^\#(L^\#(k))^2}{L^\#(k)^3 Wu^3 cl} + \frac{\frac{1}{9} \text{IcK}(Wu) Pl^2}{L^\#(k) Wu^3 cl} + \frac{\frac{1}{6} \text{IcK}(Wu) L^\#(L^\#(L^\#(k)))}{L^\#(k)^2 Wu^3 cl} \\
& - \frac{\frac{1}{6} \text{IcK}(Wu) L^\#(Pl)}{L^\#(k) Wu^3 cl} + \frac{\frac{1}{3} \text{IL}(Pl)}{Wu l Wu} + \frac{\frac{1}{2} \text{IclK}(Wu) \varepsilon^2}{L^\#(k) Wuc} - \frac{\frac{1}{2} \text{IcK}^\#(Wu) \varepsilon l^2}{Wu L(k^\#) cl} \\
& + \frac{\text{IL}(Wu) \varepsilon}{Wu^2} + \frac{1}{3} \frac{\varepsilon \text{T}(k)}{L^\#(k) Wu} + \frac{\frac{1}{3} \text{I}\varepsilon L(L^\#(k))}{L^\#(k) Wu} + \frac{\frac{1}{3} \text{I}\varepsilon K(L^\#(L^\#(k)))}{L^\#(k)^2 Wu} \\
& - \frac{\frac{1}{3} \text{IL}^\#(L^\#(k)) \varepsilon l}{L^\#(k) Wu l} - \frac{\frac{1}{18} \text{IcK}(Wu) L^\#(L^\#(k)) Pl}{L^\#(k)^2 Wu^3 cl} + \frac{\frac{2}{9} \text{IclK}^\#(Wu) L(L(k^\#))^2}{Wu l^2 Wu L(k^\#)^3 c} \\
& - \frac{\frac{1}{6} \text{IL}(k) L^\#(Pl)}{Wu l L^\#(k) Wu} - \frac{\frac{1}{2} \text{IWu l} \varepsilon^2 L(k)}{L^\#(k) Wu} - \frac{\frac{2}{9} \text{IL}(k) L^\#(L^\#(k))^2}{Wu l L^\#(k)^3 Wu} + \frac{\frac{1}{9} \text{IL}(k) Pl^2}{Wu l L^\#(k) Wu} \\
& + \frac{\frac{1}{6} \text{IL}(k) L^\#(L^\#(L^\#(k)))}{Wu l L^\#(k)^2 Wu} - \frac{\text{IWu l K}(Wu) \varepsilon^2}{L^\#(k) Wu^2} + \frac{\frac{1}{3} \text{IL}^\#(Wu) L(L(k^\#))}{Wu l Wu^2 L(k^\#)} \\
& - \frac{\frac{1}{3} \text{IL}(Wu) L^\#(L^\#(k))}{Wu l L^\#(k) Wu^2} - \frac{\frac{1}{9} \text{IPl} L(L(k^\#))}{L(k^\#) Wu l Wu} + \frac{\frac{1}{3} \text{IL}^\#(L^\#(k)) L(L^\#(k))}{L^\#(k)^2 Wu l Wu} \\
& + \frac{\frac{2}{9} \text{IL}^\#(L^\#(k)) P}{L^\#(k) Wu l Wu} + \frac{\frac{1}{18} \text{IclK}^\#(Wu) L(L(k^\#)) P}{Wu l^2 Wu L(k^\#)^2 c} - \frac{\frac{1}{9} \text{IclK}^\#(Wu) P^2}{Wu l^2 Wu L(k^\#) c} \\
& - \frac{\frac{1}{6} \text{IclK}^\#(Wu) L(L(L(k^\#)))}{Wu l^2 Wu L(k^\#)^2 c} + \frac{\frac{1}{6} \text{IclK}^\#(Wu) L(P)}{Wu l^2 Wu L(k^\#) c} \\
& + \frac{\frac{1}{9} \text{IL}^\#(L^\#(k)) L(L(k^\#))}{L^\#(k) L(k^\#) Wu l Wu} - \frac{\frac{1}{3} \text{I}\varepsilon L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 Wu} + \frac{\text{IL}(Wu l) \varepsilon}{Wu l Wu} \\
& - \frac{\text{IK}(Wu l) \varepsilon^2}{L^\#(k) Wu} - \frac{\frac{1}{3} \text{IL}(L^\#(L^\#(k)))}{L^\#(k) Wu l Wu} \Big) \rho \wedge \kappa + \left(\frac{\text{I}\varepsilon K(Wu l)}{Wu L^\#(k)} - \frac{\frac{1}{3} \text{IK}(L^\#(L^\#(k)))}{Wu L^\#(k)^2} \right. \\
& + \frac{\frac{1}{3} \text{IL}^\#(L^\#(k)) K(L^\#(k))}{Wu L^\#(k)^3} - \frac{\frac{1}{3} \text{IL}(L^\#(k))}{Wu L^\#(k)} - \frac{1}{3} \frac{\text{T}(k)}{Wu L^\#(k)} + \text{I}\varepsilon l \\
& \left. - \frac{\frac{1}{3} \text{IL}(L(k^\#))}{Wu L(k^\#)} \right) \rho \wedge \zeta + \left(- \frac{\frac{2}{3} \text{IL}^\#(L^\#(k)) \varepsilon}{Wu l L^\#(k)} + \frac{\frac{2}{9} \text{IL}^\#(L^\#(k))^2}{Wu l^2 L^\#(k)^2} \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{\frac{1}{18} \text{IL}^\#(L^\#(k)) \text{PI}}{\text{WuI}^2 L^\#(k)} - \frac{\frac{1}{3} \text{IPI} \varepsilon}{\text{WuI}} - \frac{\frac{1}{9} \text{IPI}^2}{\text{WuI}^2} + \frac{\frac{1}{6} \text{IL}^\#(\text{PI})}{\text{WuI}^2} \\
& - \frac{\frac{1}{6} \text{IL}^\#(L^\#(L^\#(k)))}{\text{WuI}^2 L^\#(k)} + \frac{1}{2} \text{I}\varepsilon^2 - \frac{\text{I}\varepsilon \text{Wu} L^\#(k^\#) \varepsilon \text{I}}{\text{WuI} L(k^\#)} - \frac{\text{I}\varepsilon K^\#(\text{WuI}) \varepsilon \text{I} \text{Wu}}{\text{WuI}^2 L(k^\#)} \\
& + \frac{\text{I}\varepsilon L^\#(\text{WuI})}{\text{WuI}^2} \left. \right) \rho \wedge \kappa^\# + \left(\frac{\text{I}\varepsilon \text{Wu} K^\#(\text{WuI})}{\text{WuI}^2 L(k^\#)} + \frac{\text{I}\varepsilon \text{Wu} L^\#(k^\#)}{\text{WuI} L(k^\#)} \right) \rho \wedge \zeta^\# + \left(\right. \\
& - \frac{K(\text{Wu}) \text{WuI}}{\text{Wu}^2 L^\#(k)} - \frac{\text{WuI} L(k)}{\text{Wu} L^\#(k)} \left. \right) \kappa \wedge \zeta + \left(\frac{K^\#(\text{Wu}) \varepsilon \text{I}}{\text{WuI} L(k^\#)} - \frac{L^\#(\text{Wu})}{\text{WuI} \text{Wu}} - \frac{1}{3} \frac{L^\#(L^\#(k))}{\text{WuI} L^\#(k)} \right. \\
& \left. + \frac{1}{3} \frac{\text{PI}}{\text{WuI}} \right) \kappa \wedge \kappa^\# - \frac{K^\#(\text{Wu}) \kappa \wedge \zeta^\#}{\text{WuI} L(k^\#)} + \zeta \wedge \kappa^\#
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{Sub} := \mathbf{proc}(l); \text{subs}(\{d = \text{Normalisation}, dI = \text{NormalisationI}\}, l); \mathbf{end proc}; \\
\text{Sub} := \mathbf{proc}(l) \text{subs}(\{d = \text{Normalisation}, dI = \text{NormalisationI}\}, l) \mathbf{end proc} \quad (10)
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{Co1} := \text{Torsion}(Q[1], 1, 4); \\
\text{Co1} := -\varepsilon + \frac{1}{3} \frac{L^\#(L^\#(k))}{\text{WuI} L^\#(k)} + \frac{2}{3} \frac{\text{PI}}{\text{WuI}} + \frac{\text{Wu} \varepsilon \text{I} L^\#(k^\#)}{\text{WuI} L(k^\#)} + \frac{\text{Wu} K^\#(\text{WuI}) \varepsilon \text{I}}{\text{WuI}^2 L(k^\#)} - \frac{L^\#(\text{WuI})}{\text{WuI}^2} \\
+ \frac{K^\#(\text{Wu}) \varepsilon \text{I}}{\text{WuI} L(k^\#)} - \frac{L^\#(\text{Wu})}{\text{WuI} \text{Wu}} \quad (11)
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{Co2} := \text{Torsion}(Q[2], 2, 4); \\
\text{Co2} := \frac{K^\#(\text{Wu}) \varepsilon \text{I}}{\text{WuI} L(k^\#)} - \frac{L^\#(\text{Wu})}{\text{WuI} \text{Wu}} - \frac{1}{3} \frac{L^\#(L^\#(k))}{\text{WuI} L^\#(k)} + \frac{1}{3} \frac{\text{PI}}{\text{WuI}} \quad (12)
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{Co3} := \text{Torsion}(Q[3], 3, 4); \\
\text{Co3} := -\frac{\text{Wu} \varepsilon \text{I} L^\#(k^\#)}{\text{WuI} L(k^\#)} - \frac{L^\#(L^\#(k))}{\text{WuI} L^\#(k)} + \varepsilon - \frac{\text{Wu} K^\#(\text{WuI}) \varepsilon \text{I}}{\text{WuI}^2 L(k^\#)} + \frac{L^\#(\text{WuI})}{\text{WuI}^2} + \frac{K^\#(\text{Wu}) \varepsilon \text{I}}{\text{WuI} L(k^\#)} \\
- \frac{L^\#(\text{Wu})}{\text{WuI} \text{Wu}} \quad (13)
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{expand}\left(\text{simplify}\left(\frac{(\text{Co1} + \text{Co3}) \cdot \text{WuI}}{2}\right)\right); \\
- \frac{1}{3} \frac{L^\#(L^\#(k))}{L^\#(k)} + \frac{1}{3} \text{PI} + \frac{K^\#(\text{Wu}) \varepsilon \text{I}}{L(k^\#)} - \frac{L^\#(\text{Wu})}{\text{Wu}} \quad (14)
\end{aligned}$$

$$\begin{aligned}
\mathbf{N} > \text{expand}(\text{Co2} \cdot \text{WuI}); \\
- \frac{1}{3} \frac{L^\#(L^\#(k))}{L^\#(k)} + \frac{1}{3} \text{PI} + \frac{K^\#(\text{Wu}) \varepsilon \text{I}}{L(k^\#)} - \frac{L^\#(\text{Wu})}{\text{Wu}} \quad (15)
\end{aligned}$$

$$\mathbf{N} > \text{Fo3} := \text{expand}\left(\frac{(\text{Co1} - \text{Co3})}{2}\right);$$

$$Fo3 := -\varepsilon + \frac{2}{3} \frac{L^\#(L^\#(k))}{WuL^\#(k)} + \frac{1}{3} \frac{PI}{WuL} + \frac{Wu \varepsilon l L^\#(k^\#)}{WuL L(k^\#)} + \frac{Wu K^\#(WuL) \varepsilon l}{WuL^2 L(k^\#)} - \frac{L^\#(WuL)}{WuL^2} \quad (16)$$

N > Co4 := expand(I·Torsion(Q[2], 1, 3) - Torsion(Q[3], 2, 3));

$$Co4 := -\frac{\varepsilon K(WuL)}{L^\#(k) Wu} + \frac{1}{3} \frac{K(L^\#(L^\#(k)))}{L^\#(k)^2 Wu} - \frac{1}{3} \frac{L^\#(L^\#(k)) K(L^\#(k))}{L^\#(k)^3 Wu} - \frac{2}{3} \frac{L(L^\#(k))}{WuL^\#(k)} \\ - \frac{\frac{1}{3} I\Gamma(k)}{WuL^\#(k)} - \varepsilon l + \frac{1}{3} \frac{L(L(k^\#))}{WuL(k^\#)} + \frac{WuL \varepsilon L(k)}{WuL^\#(k)} + \frac{L(WuL)}{WuWuL} + \frac{WuL K(Wu) \varepsilon}{Wu^2 L^\#(k)} \\ - \frac{L(Wu)}{Wu^2} \quad (17)$$

N > expand(Torsion(Q[2], 1, 4) + I·Torsion(Q[3], 2, 4));

(18)

N > expand(Torsion(Q[2], 1, 5) + I·Torsion(Q[3], 2, 5));

(19)

N > conjugue1 := **proc**(expr); subs({epsilon = AA, k = BB, L = CC, K = DD, Wu = EE, epsilonL = AAA, k# = B#, L# = C#, K# = D#, WuL = E#}, expr); **end proc**;

conjugue1 := **proc**(expr)

subs({K = DD, L = CC, Wu = EE, WuL = E#, k = BB, ε = AA, εl = AAA, K# = D#, L# = C#, k# = B#}, expr)

(20)

end proc

N > conjugue := **proc**(expr); subs({AA = epsilonL, BB = k#, CC = L#, DD = K#, EE = WuL, AAA = epsilon, B# = k, C# = L, D# = K, E# = Wu}, conjugue1(expr)); **end proc**

conjugue := **proc**(expr)

subs({AA = εl, AAA = ε, BB = k#, CC = L#, DD = K#, EE = WuL, B# = k, C# = L, E# = Wu, D# = K}, conjugue1(expr))

(21)

end proc

N > Fo4 := conjugue(Co4);

$$Fo4 := -\frac{K^\#(Wu) \varepsilon l}{WuL L(k^\#)} + \frac{1}{3} \frac{K^\#(L(L(k^\#)))}{L(k^\#)^2 WuL} - \frac{1}{3} \frac{L(L(k^\#)) K^\#(L(k^\#))}{L(k^\#)^3 WuL} - \frac{2}{3} \frac{L^\#(L(k^\#))}{WuL L(k^\#)} \\ - \frac{\frac{1}{3} I\Gamma(k^\#)}{WuL L(k^\#)} - \varepsilon + \frac{1}{3} \frac{L^\#(L^\#(k))}{WuL L^\#(k)} + \frac{Wu \varepsilon l L^\#(k^\#)}{WuL L(k^\#)} + \frac{L^\#(Wu)}{WuL Wu} + \frac{Wu K^\#(WuL) \varepsilon l}{WuL^2 L(k^\#)} \\ - \frac{L^\#(WuL)}{WuL^2} \quad (22)$$

N > expand(WuL·(Fo3 - Fo4 - Co2));

$$\frac{2}{3} \frac{L^\#(L^\#(k))}{L^\#(k)} - \frac{1}{3} \frac{K^\#(L(L(k^\#)))}{L(k^\#)^2} + \frac{1}{3} \frac{L(L(k^\#)) K^\#(L(k^\#))}{L(k^\#)^3} + \frac{2}{3} \frac{L^\#(L(k^\#))}{L(k^\#)} \quad (23)$$

$$\left[+ \frac{\frac{1}{3} \text{IT}(k^\#)}{L(k^\#)} \right]$$