```
> restart :
K :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(K(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(K(y) · x/y + y·K(x/y)) elif
        (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) - 1) ·K(y) elif
        (type(x, function) = true) then Kfunction(x) elif
        (type(x, symbol) = true) then Ksymbol(x) else 0 fi end proc:
```

```
> Ksymbol :=proc(x); if x = k# then 0 elif x = P then (-L(k)·P - L(L(k))) elif x = P1 then (
    -L#(k)·P - L(L#(k)) -2·I·Tau(k)) else 'K'(x) fi end proc:
```

```
> recherche :=proc(x) local y; y := op(1, x); while (type(y, function) = true) do y := op(1, y)
    od; y; end proc:
```

```
> Kfunction :=proc(x) local y; y := recherche(x); if y = k then 'K'(x) else Kfunction1(x) fi
    end proc:
```

```
> Kfunction1 :=proc(x) local y, J : if x = k# then 0 elif x = P then (-L(k)·P - L(L(k))) elif x
    = P1 then (-L#(k)·P - L(L#(k)) -2·I·Tau(k)) else J := op(0, x) : y := op(1, x) : if J
    = L then L(Kfunction1(y)) - 'L(k)'·L(y) elif
                                                                        J = L#
    then L#(Kfunction1(y)) - 'L#'(k)·L(y) elif
                                                                        J = K#
    then K#(Kfunction1(y)) elif J = Tau then Tau(Kfunction1(y)) - 'L(k)'·Tau(y) -'Tau(k)'
    ·L(y) fi fi end proc:
```

```
> L :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(L(y) · x/y + y·L(x/y)) elif
        (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) - 1) ·L(y) elif
        (type(x, function) = true) then 'L'(x) elif
        (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:
```

```
> L# :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L#(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(L#(y) · x/y + y·L#(x/y)) elif
        (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) - 1) ·L#(y) elif
        (type(x, function) = true) then 'L#(x)' elif
        (type(x, symbol) = true) then 'L#'(x) else 0 fi end proc:
```

```
> Tau :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Tau(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(Tau(y) · x/y + y·Tau(x/y)) elif
        (type(x, `^`) = true) then op(2, x) ·y^(op(2, x) - 1) ·Tau(y) elif
        (type(x, function) = true) then 'Tau'(x) elif
        (type(x, symbol) = true) then 'Tau'(x) else 0 fi end proc:
```

```
> K# :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(K#(op(i, x)), i = 1
```

```
.. nops(x)) elif
        (type(x, `*`) = true) then expand(K#(y)·x/y + y·K#(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1)·K#(y) elif
        (type(x, function) = true) then Kfunction#(x) elif
        (type(x, symbol) = true) then Ksymbol#(x) else 0 fi end proc:

> Kfunction# :=proc(x) local y; y := recherche(x); if y = k# then 'K#'(x) else Kfunction1#(x)
    fi end proc:

> Kfunction1# :=proc(x) local y, J : if x = k then 0 elif x = P1 then (-L#(k#)·P1
    − L#(L#(k#))) elif x = P then (-L(k#)·P1 - L#(L(k#)) + 2·I·Tau(k#)) else J
    := op(0, x) : y := op(1, x) : if J = L then L(Kfunction1#(y)) − 'L(k#)'·L#(y) elif
                                                                                J = L#
    then L#(Kfunction1#(y)) − 'L#'(k#)·L#(y) elif
                                                                                J = K
    then K(Kfunction1#(y)) elif J = Tau then Tau(Kfunction1#(y)) - 'L#(k#)'·Tau(y) -
    'Tau(k#)'·L#(y) fi fi end proc:
 Ksymbol# :=proc(x); if x = k then 0 elif x = P1 then (-L#(k#)·P1 − L#(L#(k#))) elif x = P
    then (-L(k#)·P1 - L#(L(k#)) + 2·I·Tau(k#)) else 'K#(x)' fi end proc:

>
 Der :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Der(op(i, x)), i = 1
    .. nops(x)) elif
        (type(x, `*`) = true) then expand(x/y·Der(y) + y·Der(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1)·Der(y) elif
        ((type(x, function) = true) or (type(x, symbol) = true)) then Tau(x)·W[1]
    + L(x)·W[2] + K(x)·W[3] + L#(x)·W[4] + K#(x)·W[5]
        else 0 fi end proc:
 derivation :=proc(x) : collect( Der(x), [W[1], W[2], W[3], W[4], W[5]]) : end proc:

> g := ((1/3) I L#(L#(k))) / L#(k) − (1/3) I P1 :

> g1 := - ((1/3) I L(L(k#))) / L(k#) + (1/3) I P :

> dg := derivation(g) :

> dg1 := derivation(g1) :

> with(LinearAlgebra) :

> m := Matrix([[1, 0, 0, 0, 0], [g, 1, 0, 0], [0, 0, L#(k), 0, 0], [g1, 0, 0, 1, 0], [0, 0, 0, 0,
    L(k#)]]) :

> minv := MatrixInverse(m) :
```

```
> dg := derivation(g) :
> dg1 := derivation(g1) :
> with(LinearAlgebra) :
> m := Matrix([[1, 0, 0, 0, 0], [g, 1, 0, 0], [0, 0, L^#(k), 0, 0], [g1, 0, 0, 1, 0], [0, 0, 0, 0,
      L(k^#)]]) :
> minv := MatrixInverse(m) :
> W := minv.Vector([V[1], V[2], V[3], V[4], V[5]]) :

> X := Der(L^#(k)) :
> X1 := Der(L(k^#)) :
> with(DifferentialGeometry) :
> with(Tools) :
> DGsetup([x, y, z, y1, z1], [c, c1, d, d1, e, e1], M, verbose) :
```

The following coordinates have been protected:

[x, y, z, y1, z1, c, c1, d, d1, e, e1]

The following vector fields have been defined and protected:

[D_x, D_y, D_z, D_y1, D_z1, D_c, D_c1, D_d, D_d1, D_e, D_e1]

The following differential 1-forms have been defined and protected:

[dx, dy, dz, dy1, dz1, dc, dc1, dd, dd1, de, de1] **(1)**

$$> gr := Matrix\left(\left[[c\cdot c1, 0, 0, 0, 0], [-I\cdot e\cdot c1, c, 0, 0, 0], \left[d, e, \frac{c}{c1}, 0, 0\right], [+I\cdot e1\cdot c, 0, 0, c1, 0], \left[d1, 0, 0, e1, \frac{c1}{c}\right]\right]\right) :$$

```
M > h := MatrixInverse(gr) :
M > A := map(evalDG, (ExteriorDerivative(gr).h));
```

$$A := \left[\left[\frac{dc}{c} + \frac{dc1}{c1}, 0\,dx, 0\,dx, 0\,dx, 0\,dx\right],\right. \quad \textbf{(2)}$$

$$\left[\frac{I\,e\,dc}{c^2} - \frac{I\,e\,dc1}{c1\,c} - \frac{I\,de}{c}, \frac{dc}{c}, 0\,dx, 0\,dx, 0\,dx\right],$$

$$\left[-\frac{(d\,c + I\,e^2\,c1)\,dc}{c^3\,c1} + \frac{(d\,c + I\,e^2\,c1)\,dc1}{c^2\,c1^2} + \frac{dd}{c\,c1} + \frac{I\,e\,de}{c^2}, -\frac{e\,dc}{c^2} + \frac{e\,dc1}{c1\,c} + \frac{de}{c},\right.$$

$$\left.\frac{dc}{c} - \frac{dc1}{c1}, 0\,dx, 0\,dx\right],$$

$$\left[\frac{I\,e1\,dc}{c1\,c} - \frac{I\,e1\,dc1}{c1^2} + \frac{I\,de1}{c1}, 0\,dx, 0\,dx, \frac{dc1}{c1}, 0\,dx\right],$$

$$\left[-\frac{(-d1\,c1 + I\,e1^2\,c)\,dc}{c1^2\,c^2} + \frac{(-d1\,c1 + I\,e1^2\,c)\,dc1}{c1^3\,c} + \frac{dd1}{c\,c1} - \frac{I\,e1\,de1}{c1^2}, 0\,dx, 0\,dx,\right.$$

$$\left.\left.\frac{e1\,dc}{c\,c1} - \frac{e1\,dc1}{c1^2} + \frac{de1}{c1}, -\frac{dc}{c} + \frac{dc1}{c1}\right]\right]$$

$$M > t[1] := \frac{dc}{c} : t[2] := \frac{I\,e\,dc}{c^2} - \frac{I\,e\,dc1}{c1\,c} - \frac{I\,de}{c} :$$

M > $t[3] := -\frac{(d\,c + \mathrm{I}\,e^2\,c1)\,dc}{c^3\,c1} + \frac{(d\,c + \mathrm{I}\,e^2\,c1)\,dc1}{c^2\,c1^2} + \frac{dd}{c\,c1} + \frac{\mathrm{I}\,e\,de}{c^2}$:

M > $t[4] := \frac{dc1}{c1}$: $t[5] := \frac{\mathrm{I}\,e1\,dc}{c1\,c} - \frac{\mathrm{I}\,e1\,dc1}{c1^2} + \frac{\mathrm{I}\,de1}{c1}$: $t[6] := -\frac{(-d1\,c1 + \mathrm{I}\,e1^2\,c)\,dc}{c1^2\,c^2} + \frac{(-d1\,c1 + \mathrm{I}\,e1^2\,c)\,dc1}{c1^3\,c} + \frac{dd1}{c\,c1} - \frac{\mathrm{I}\,e1\,de1}{c1^2}$:

M > *FD* := *FrameData*([t[1], t[2], t[3], t[4], t[5], t[6], dx, dy, dz, dy1, dz1], *N*) :

M > *DGsetup*(*FD*, [*E*], [alpha[1], alpha[2], alpha[3], $\alpha^{\#}[1]$, $\alpha^{\#}[2]$, $\alpha^{\#}[3]$, rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$]);

frame name: N **(3)**

M > *T* := *Vector*([rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$]) :

N > *V* := *h.T* :

>
$dV[1] := P\cdot(W[1]\&wedge\ W[2]) - L(k)\cdot(W[1]\&wedge\ W[3]) + P1\cdot(W[1]\ \&wedge\ W[4]) - L^{\#}(k^{\#})\cdot(W[1]\ \&wedge\ W[5]) + I\cdot(W[2]\ \&wedge\ W[4])$:

> $dW[2] := -L(k)\cdot(W[2]\&wedge\ W[3]) + L^{\#}(k)\cdot(W[3]\&wedge\ W[4]) - \mathrm{Tau}(k)\cdot(W[1]\ \&wedge\ W[3])$:

> $dV[2] := dW[2] + dg\ \&wedge\ W[1] + g\cdot dV[1]$:

> $dV[3] := (X\ \&wedge\ W[3])$:

> $dW[4] := -L^{\#}(k^{\#})\cdot(W[4]\&wedge\ W[5]) + L(k^{\#})\cdot(W[5]\&wedge\ W[2]) - \mathrm{Tau}(k^{\#})\cdot(W[1]\ \&wedge\ W[5])$:

> $dV[4] := dW[4] + dg1\ \&wedge\ W[1] + g1\ \&wedge\ dV[1]$:

> $dV[5] := (X1\ \&wedge\ W[5])$:

Omega := *map*(*evalDG*, *gr.Vector*([dV[1], dV[2], dV[3], dV[4], dV[5]])) :

N > *A* := (*map*(*evalDG*, (*ExteriorDerivative*(*gr*).*h*))) :

B := *A* &*MatrixWedge* *T* :
SE := *map*(*evalDG*, (*B* &*MatrixPlus* Omega)) :

N > *List* := *GenerateForms*([alpha[1], alpha[2], alpha[3], $\alpha^{\#}[1]$, $\alpha^{\#}[2]$, $\alpha^{\#}[3]$, rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$], 2) :

N > *result* := **proc**(*l*) **local** *k*, *t*, *X*; *X* := 0 : *t* := *expand*(*GetComponents*(*l*, *List*)) : **for** *k* **from** 1 **to** 55 **do** *X* := *X* + *t*[*k*]·*List*[*k*] **od**; *X*; **end proc**:

N > *Res1* := *result*(*SE*[1]) :

N > *Res2* := *result*(*SE*[2]) :

N > *Res3* := *result*(*SE*[3]) :

N > *List2* := *GenerateForms*([rho, kappa, zeta, $\kappa^{\#}$, $\zeta^{\#}$], 2) :

N > *Torsion* := **proc**(*S*, *i*, *j*) **local** *k*, *X*; $k := 5\cdot(i-1) - \frac{i\cdot(i-1)}{2} + j - i$; *X* := *map*(*expand*, *GetComponents*(*S*, *List2*)); *X*[*k*]; **end proc**:

N > *expr* := *expand*(*I*·*Torsion*(Omega[2], 1, 4) − *Torsion*(Omega[3], 2, 4)) : *expr1*

```
:= expand( -I·Torsion(Omega[4], 1, 2) + Torsion(Omega[5], 2, 4)) :
> Normalisation := expand(solve(expr, d)) : Normalisation1 := expand(solve(expr1, d1)) :
> for i from 1 to 3 do Z[i] := result(subs({d = Normalisation, d1 = Normalisation1}, Omega[i])) ; od:
```

$$> H := expand\left(\frac{\left(Normalisation - \left(-I\cdot\frac{e^2\cdot c1}{2\cdot c}\right)\right)\cdot c1}{I\cdot c}\right);$$

$$H := \frac{2}{9}\frac{L^{\#}(L^{\#}(k))^2}{L^{\#}(k)^2} + \frac{1}{18}\frac{L^{\#}(L^{\#}(k))\,P1}{L^{\#}(k)} - \frac{1}{9}P1^2 - \frac{1}{6}\frac{L^{\#}(L^{\#}(L^{\#}(k)))}{L^{\#}(k)} + \frac{1}{6}L^{\#}(P1) \qquad \textbf{(4)}$$

```
> R := result(evalDG(Der(H)&wedge rho)) :
```

$$> Z[3] := result\left(evalDG\left(Z[3] + \frac{I\cdot R}{c1^2}\right)\right);$$

$$\begin{aligned}
Z_3 := \Bigg(& \frac{1}{3}\frac{\mathrm{T}(k)\,e^2}{L^{\#}(k)\,c^3} - \frac{\frac{1}{9}\,\mathrm{I}\,L(L^{\#}(k))\,P1^2}{c1^2\,L^{\#}(k)\,c} + \frac{\frac{1}{18}\,\mathrm{I}\,L(L(k^{\#}))\,L^{\#}(P1)}{c1^2\,c\,L(k^{\#})} \qquad \textbf{(5)} \\
& - \frac{\frac{1}{27}\,\mathrm{I}\,L(L(k^{\#}))\,P1^2}{c1^2\,c\,L(k^{\#})} - \frac{\frac{1}{9}\,\mathrm{I}\,P\,L^{\#}(L^{\#}(L^{\#}(k)))}{c1^2\,L^{\#}(k)\,c} - \frac{\frac{1}{18}\,\mathrm{I}\,e1\,L^{\#}(L^{\#}(k))\,P1}{c1^3\,L^{\#}(k)} \\
& + \frac{\frac{1}{2}\,\mathrm{I}\,e\,L^{\#}(k^{\#})\,e1^2}{c1^3\,L(k^{\#})} + \frac{\frac{4}{27}\,\mathrm{I}\,P\,L^{\#}(L^{\#}(k))^2}{c1^2\,L^{\#}(k)^2\,c} + \frac{\frac{1}{6}\,\mathrm{I}\,L(L^{\#}(k))\,L^{\#}(P1)}{c1^2\,L^{\#}(k)\,c} \\
& - \frac{\frac{2}{9}\,\mathrm{I}\,e\,L^{\#}(L^{\#}(k))\,L(L(k^{\#}))}{L^{\#}(k)\,L(k^{\#})\,c1\,c^2} - \frac{\frac{1}{3}\,\mathrm{I}\,e\,L(L^{\#}(L^{\#}(k)))}{L^{\#}(k)\,c1\,c^2} + \frac{\frac{1}{3}\,\mathrm{I}\,e\,P1\,e1}{c1^2\,c} \\
& - \frac{\frac{1}{3}\,\mathrm{I}\,e^2\,L^{\#}(L^{\#}(k))\,K(L^{\#}(k))}{L^{\#}(k)^3\,c^3} + \frac{1}{3}\frac{L^{\#}(\mathrm{T}(k))\,e}{c1\,L^{\#}(k)\,c^2} + \frac{\frac{1}{6}\,\mathrm{I}\,L(L^{\#}(L^{\#}(L^{\#}(k))))}{c1^2\,L^{\#}(k)\,c} \\
& + \frac{\frac{2}{9}\,\mathrm{I}\,P1\,L(P1)}{c1^2\,c} - \frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(P)\,e}{c1\,c^2} + \frac{\frac{1}{6}\,\mathrm{I}\,L(P1)\,e}{c1\,c^2} - \frac{e\,\mathrm{T}(L^{\#}(k))}{L^{\#}(k)\,c1\,c^2} - \frac{\frac{1}{6}\,\mathrm{I}\,e1\,L^{\#}(P1)}{c1^3} \\
& + \frac{\frac{1}{9}\,\mathrm{I}\,e1\,P1^2}{c1^3} - \frac{\frac{1}{3}\,\mathrm{I}\,P\,e^2}{c^3} - \frac{\frac{1}{9}\,\mathrm{I}\,e\,P1\,L(L(k^{\#}))}{L(k^{\#})\,c1\,c^2} + \frac{\frac{2}{3}\,\mathrm{I}\,e\,L^{\#}(L^{\#}(k))\,e1}{L^{\#}(k)\,c1^2\,c} \\
& + \frac{\frac{1}{3}\,\mathrm{I}\,e^2\,K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^2\,c^3} - \frac{\frac{1}{6}\,\mathrm{I}\,L(L^{\#}(P1))}{c1^2\,c} - \frac{\frac{1}{18}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,P1\,K(L^{\#}(k))\,e}{c1\,L^{\#}(k)^3\,c^2} \\
& + \frac{1}{9}\frac{L^{\#}(L^{\#}(k))\,\mathrm{T}(k)\,e}{c1\,L^{\#}(k)^2\,c^2} - \frac{4}{9}\frac{P1\,\mathrm{T}(k)\,e}{c1\,L^{\#}(k)\,c^2} + \frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,P\,e}{c1\,L^{\#}(k)\,c^2}
\end{aligned}$$

$$
\begin{aligned}
& - \frac{\frac{1}{9}\, \mathrm{I}\, \mathit{P1}\, L\left(L^{\#}(k)\right) e}{\mathit{c1}\, L^{\#}(k)\, c^{2}} + \frac{\frac{4}{9}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right) K\left(L^{\#}\left(L^{\#}(k)\right)\right) e}{\mathit{c1}\, L^{\#}(k)^{3}\, c^{2}} \\
& + \frac{\frac{1}{9}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right) \mathit{P1}\, L\left(L^{\#}(k)\right)}{\mathit{c1}^{2}\, L^{\#}(k)^{2}\, c} + \frac{\frac{1}{6}\, \mathrm{I}\, L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right) K\left(L^{\#}(k)\right) e}{\mathit{c1}\, L^{\#}(k)^{3}\, c^{2}} \\
& + \frac{\frac{1}{18}\, \mathrm{I}\, \mathit{P1}\, K\left(L^{\#}\left(L^{\#}(k)\right)\right) e}{\mathit{c1}\, L^{\#}(k)^{2}\, c^{2}} + \frac{\frac{11}{18}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right) L\left(L^{\#}(k)\right) e}{\mathit{c1}\, L^{\#}(k)^{2}\, c^{2}} \\
& - \frac{\frac{4}{9}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right)^{2} K\left(L^{\#}(k)\right) e}{\mathit{c1}\, L^{\#}(k)^{4}\, c^{2}} - \frac{\frac{1}{18}\, \mathrm{I}\, L\left(L\left(k^{\#}\right)\right) L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right)}{\mathit{c1}^{2}\, L^{\#}(k)\, c\, L\left(k^{\#}\right)} \\
& + \frac{\frac{2}{27}\, \mathrm{I}\, L\left(L\left(k^{\#}\right)\right) L^{\#}\left(L^{\#}(k)\right)^{2}}{\mathit{c1}^{2}\, L^{\#}(k)^{2}\, c\, L\left(k^{\#}\right)} + \frac{\frac{2}{9}\, \mathrm{I}\, e\, L^{\#}\left(k^{\#}\right) L\left(L\left(k^{\#}\right)\right)^{2}}{\mathit{c1}\, c^{2}\, L\left(k^{\#}\right)^{3}} \\
& - \frac{\frac{1}{6}\, \mathrm{I}\, e\, L^{\#}\left(k^{\#}\right) L\left(L\left(L\left(k^{\#}\right)\right)\right)}{\mathit{c1}\, c^{2}\, L\left(k^{\#}\right)^{2}} - \frac{\frac{1}{9}\, \mathrm{I}\, e\, L^{\#}\left(k^{\#}\right) P^{2}}{\mathit{c1}\, c^{2}\, L\left(k^{\#}\right)} + \frac{\frac{1}{27}\, \mathrm{I}\, P\, L^{\#}\left(L^{\#}(k)\right) \mathit{P1}}{\mathit{c1}^{2}\, L^{\#}(k)\, c} \\
& + \frac{\frac{1}{6}\, \mathrm{I}\, e\, L^{\#}\left(k^{\#}\right) L(P)}{\mathit{c1}\, c^{2}\, L\left(k^{\#}\right)} + \frac{\frac{1}{18}\, \mathrm{I}\, e\, L^{\#}\left(k^{\#}\right) L\left(L\left(k^{\#}\right)\right) P}{\mathit{c1}\, c^{2}\, L\left(k^{\#}\right)^{2}} + \frac{\frac{1}{54}\, \mathrm{I}\, L\left(L\left(k^{\#}\right)\right) L^{\#}\left(L^{\#}(k)\right) \mathit{P1}}{\mathit{c1}^{2}\, L^{\#}(k)\, c\, L\left(k^{\#}\right)} \\
& - \frac{\frac{4}{9}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right) L\left(L^{\#}\left(L^{\#}(k)\right)\right)}{\mathit{c1}^{2}\, L^{\#}(k)^{2}\, c} - \frac{\frac{1}{18}\, \mathrm{I}\, \mathit{P1}\, L\left(L^{\#}\left(L^{\#}(k)\right)\right)}{\mathit{c1}^{2}\, L^{\#}(k)\, c} \\
& - \frac{\frac{1}{18}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right) L(\mathit{P1})}{\mathit{c1}^{2}\, L^{\#}(k)\, c} - \frac{\frac{1}{3}\, \mathrm{I}\, L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right) L\left(L^{\#}(k)\right)}{\mathit{c1}^{2}\, L^{\#}(k)^{2}\, c} \\
& + \frac{\frac{2}{3}\, \mathrm{I}\, L^{\#}\left(L^{\#}(k)\right)^{2} L\left(L^{\#}(k)\right)}{\mathit{c1}^{2}\, L^{\#}(k)^{3}\, c} - \frac{\frac{1}{6}\, \mathrm{I}\, L^{\#}\left(L\left(L^{\#}(k)\right)\right) e}{\mathit{c1}\, L^{\#}(k)\, c^{2}} - \frac{\frac{1}{6}\, \mathrm{I}\, K\left(L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right)\right) e}{\mathit{c1}\, L^{\#}(k)^{2}\, c^{2}} \\
& - \frac{\frac{1}{2}\, \mathrm{I}\, e^{2}\, \mathit{e1}}{\mathit{c1}\, c^{2}} + \frac{\frac{1}{9}\, \mathrm{I}\, P\, L^{\#}(\mathit{P1})}{\mathit{c1}^{2}\, c} - \frac{\frac{2}{27}\, \mathrm{I}\, P\, \mathit{P1}^{2}}{\mathit{c1}^{2}\, c} + \frac{\frac{1}{6}\, \mathrm{I}\, \mathit{e1}\, L^{\#}\left(L^{\#}\left(L^{\#}(k)\right)\right)}{\mathit{c1}^{3}\, L^{\#}(k)} \\
& \left. - \frac{\frac{1}{6}\, \mathrm{I}\, e^{2}\, L\left(L^{\#}(k)\right)}{L^{\#}(k)\, c^{3}} + \frac{\frac{1}{6}\, \mathrm{I}\, e^{2}\, L\left(L\left(k^{\#}\right)\right)}{c^{3}\, L\left(k^{\#}\right)} - \frac{\frac{2}{9}\, \mathrm{I}\, \mathit{e1}\, L^{\#}\left(L^{\#}(k)\right)^{2}}{\mathit{c1}^{3}\, L^{\#}(k)^{2}} \right) \rho \wedge \kappa \\
& + \left(\frac{\frac{1}{9}\, \mathrm{I}\, L^{\#}\left(k^{\#}\right) P^{2}}{\mathit{c1}\, c\, L\left(k^{\#}\right)} - \frac{\frac{2}{9}\, \mathrm{I}\, L^{\#}\left(k^{\#}\right) L\left(L\left(k^{\#}\right)\right)^{2}}{\mathit{c1}\, c\, L\left(k^{\#}\right)^{3}} - \frac{\frac{1}{6}\, \mathrm{I}\, L^{\#}\left(k^{\#}\right) L(P)}{\mathit{c1}\, c\, L\left(k^{\#}\right)} \right.
\end{aligned}
$$

$$+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(k^{\#})\,L(L(L(k^{\#})))}{c1\,c\,L(k^{\#})^{2}}-\frac{\frac{1}{2}\,\mathrm{I}\,c\,L^{\#}(k^{\#})\,e1^{2}}{c1^{3}\,L(k^{\#})}-\frac{1}{3}\,\frac{L^{\#}(\mathrm{T}(k))}{c1\,L^{\#}(k)\,c}$$

$$+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L(L^{\#}(k)))}{c1\,L^{\#}(k)\,c}+\frac{\frac{1}{6}\,\mathrm{I}\,K(L^{\#}(L^{\#}(L^{\#}(k))))}{c1\,L^{\#}(k)^{2}\,c}+\frac{\frac{1}{6}\,\mathrm{I}\,L(k)\,L^{\#}(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^{2}\,c1\,c}$$

$$-\frac{\frac{1}{2}\,\mathrm{I}\,c1\,e^{2}\,L(k)}{L^{\#}(k)\,c^{3}}-\frac{\frac{2}{9}\,\mathrm{I}\,L(k)\,L^{\#}(L^{\#}(k))^{2}}{L^{\#}(k)^{3}\,c1\,c}+\frac{\frac{1}{9}\,\mathrm{I}\,L(k)\,P1^{2}}{L^{\#}(k)\,c1\,c}-\frac{\frac{1}{6}\,\mathrm{I}\,L(k)\,L^{\#}(P1)}{L^{\#}(k)\,c1\,c}$$

$$-\frac{\frac{2}{9}\,\mathrm{I}\,P1\,P}{c1\,c}+\frac{\frac{1}{3}\,\mathrm{I}\,e\,L^{\#}(L^{\#}(k))\,K(L^{\#}(k))}{L^{\#}(k)^{3}\,c^{2}}-\frac{\frac{1}{18}\,\mathrm{I}\,L(k)\,L^{\#}(L^{\#}(k))\,P1}{L^{\#}(k)^{2}\,c1\,c}$$

$$-\frac{1}{3}\,\frac{e\,\mathrm{T}(k)}{L^{\#}(k)\,c^{2}}+\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,L(L(k^{\#}))}{L^{\#}(k)\,L(k^{\#})\,c1\,c}-\frac{\frac{1}{3}\,\mathrm{I}\,e\,L(L(k^{\#}))}{L(k^{\#})\,c^{2}}-\frac{\mathrm{I}\,L^{\#}(L^{\#}(k))\,e1}{L^{\#}(k)\,c1^{2}}$$

$$+\frac{\frac{2}{3}\,\mathrm{I}\,L(L^{\#}(k))\,e}{L^{\#}(k)\,c^{2}}-\frac{\frac{1}{3}\,\mathrm{I}\,e\,K(L^{\#}(L^{\#}(k)))}{L^{\#}(k)^{2}\,c^{2}}+\frac{\mathrm{I}\,e\,e1}{c1\,c}+\frac{\mathrm{T}(L^{\#}(k))}{L^{\#}(k)\,c1\,c}+\frac{\frac{1}{6}\,\mathrm{I}\,L(P1)}{c1\,c}$$

$$+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(P)}{c1\,c}+\frac{\frac{1}{18}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,P1\,K(L^{\#}(k))}{c1\,L^{\#}(k)^{3}\,c}-\frac{\frac{1}{18}\,\mathrm{I}\,L^{\#}(k^{\#})\,L(L(k^{\#}))\,P}{c1\,c\,L(k^{\#})^{2}}$$

$$-\frac{1}{9}\,\frac{L^{\#}(L^{\#}(k))\,\mathrm{T}(k)}{c1\,L^{\#}(k)^{2}\,c}+\frac{4}{9}\,\frac{P1\,\mathrm{T}(k)}{c1\,L^{\#}(k)\,c}-\frac{\frac{1}{18}\,\mathrm{I}\,P1\,K(L^{\#}(L^{\#}(k)))}{c1\,L^{\#}(k)^{2}\,c}$$

$$+\frac{\frac{4}{9}\,\mathrm{I}\,L^{\#}(L^{\#}(k))^{2}\,K(L^{\#}(k))}{c1\,L^{\#}(k)^{4}\,c}-\frac{\frac{1}{9}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,P}{c1\,L^{\#}(k)\,c}-\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L^{\#}(L^{\#}(k)))\,K(L^{\#}(k))}{c1\,L^{\#}(k)^{3}\,c}$$

$$+\frac{\frac{1}{9}\,\mathrm{I}\,P1\,L(L^{\#}(k))}{c1\,L^{\#}(k)\,c}-\frac{\frac{5}{18}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,L(L^{\#}(k))}{c1\,L^{\#}(k)^{2}\,c}-\frac{\frac{4}{9}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,K(L^{\#}(L^{\#}(k)))}{c1\,L^{\#}(k)^{3}\,c}\Bigg)$$

$$\rho\wedge\zeta+\Bigg(\frac{\frac{20}{27}\,\mathrm{I}\,L^{\#}(L^{\#}(k))^{3}}{c1^{3}\,L^{\#}(k)^{3}}-\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L^{\#}(P1))}{c1^{3}}+\frac{\frac{5}{18}\,\mathrm{I}\,L^{\#}(L^{\#}(k))^{2}\,P1}{c1^{3}\,L^{\#}(k)^{2}}$$

$$-\frac{\frac{1}{9}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,P1^{2}}{L^{\#}(k)\,c1^{3}}-\frac{\frac{5}{6}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,L^{\#}(L^{\#}(L^{\#}(k)))}{c1^{3}\,L^{\#}(k)^{2}}+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L^{\#}(k))\,L^{\#}(P1)}{c1^{3}\,L^{\#}(k)}$$

$$-\frac{\frac{1}{6}\,\mathrm{I}\,P1\,L^{\#}(L^{\#}(L^{\#}(k)))}{c1^{3}\,L^{\#}(k)}+\frac{\frac{1}{3}\,\mathrm{I}\,P1\,L^{\#}(P1)}{c1^{3}}+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L^{\#}(L^{\#}(L^{\#}(k))))}{c1^{3}\,L^{\#}(k)}$$

$$-\frac{\frac{2}{27}\,\mathrm{I}\,P1^3}{c1^3}\Bigg)\rho\wedge\kappa^\# + \left(\frac{L(L^\#(k))}{L^\#(k)\,c} - \frac{e\,L(k)\,c1}{c^2\,L^\#(k)}\right)\kappa\wedge\zeta + \Bigg(\frac{e\,L^\#(k^\#)\,e1}{c1^2\,L(k^\#)}$$

$$+\frac{2}{3}\frac{e\,L^\#(L^\#(k))}{c\,c1\,L^\#(k)} + \frac{1}{3}\frac{e\,P1}{c\,c1} - \frac{1}{2}\frac{e^2}{c^2} - \frac{2}{9}\frac{L^\#(L^\#(k))^2}{c1^2\,L^\#(k)^2} - \frac{1}{18}\frac{L^\#(L^\#(k))\,P1}{c1^2\,L^\#(k)}$$

$$+\frac{1}{9}\frac{P1^2}{c1^2} + \frac{1}{6}\frac{L^\#(L^\#(L^\#(k)))}{c1^2\,L^\#(k)} - \frac{1}{6}\frac{L^\#(P1)}{c1^2}\Bigg)\kappa\wedge\kappa^\# - \frac{L^\#(k^\#)\,e\,\kappa\wedge\zeta^\#}{c1\,L(k^\#)} + \Bigg($$

$$-\frac{e1\,L^\#(k^\#)\,c}{c1^2\,L(k^\#)} - \frac{L^\#(L^\#(k))}{c1\,L^\#(k)} + \frac{e}{c}\Bigg)\zeta\wedge\kappa^\# + \frac{L^\#(k^\#)\,c\,\zeta\wedge\zeta^\#}{c1\,L(k^\#)}$$

> $result(evalDG(Der(J))\ \&wedge\ \mathrm{rho}):$

> $Q[1] := \left(evalDG\left(subs\left(\left\{c = J^{\frac{1}{3}}, c1 = J1^{\frac{1}{3}}\right\}, \left(\frac{1}{3}\frac{Der(J)}{(J)} + \frac{1}{3}\frac{Der(J1)}{J1}\right)\&wedge\ \mathrm{rho} + Z[1]\right)\right)\right):$

N > $Torsion(Q[1], 1, 4);$

$$-\frac{e}{J^{1/3}} + \frac{1}{3}\frac{L^\#(L^\#(k))}{J1^{1/3}\,L^\#(k)} + \frac{2}{3}\frac{P1}{J1^{1/3}} + \frac{J^{1/3}\,e1\,L^\#(k^\#)}{J1^{2/3}\,L(k^\#)} + \frac{1}{3}\frac{J^{1/3}\,K^\#(J1)\,e1}{J1^{5/3}\,L(k^\#)} - \frac{1}{3}\frac{L^\#(J1)}{J1^{4/3}} + \frac{1}{3}\frac{K^\#(J)\,e1}{J^{2/3}\,J1^{2/3}\,L(k^\#)} - \frac{1}{3}\frac{L^\#(J)}{J\,J1^{1/3}} \tag{6}$$

N > $Y[1] := subs\left(\left\{e = \frac{e}{J^{\frac{-1}{3}}}, e1 = \frac{e1}{J1^{\frac{-1}{3}}}\right\}, Q[1]\right):$

N > $Co1 := Torsion(Y[1], 1, 4);$

$$Co1 := -e + \frac{1}{3}\frac{L^\#(L^\#(k))}{J1^{1/3}\,L^\#(k)} + \frac{2}{3}\frac{P1}{J1^{1/3}} + \frac{J^{1/3}\,e1\,L^\#(k^\#)}{J1^{1/3}\,L(k^\#)} + \frac{1}{3}\frac{J^{1/3}\,K^\#(J1)\,e1}{J1^{4/3}\,L(k^\#)} - \frac{1}{3}\frac{L^\#(J1)}{J1^{4/3}} + \frac{1}{3}\frac{K^\#(J)\,e1}{J^{2/3}\,J1^{1/3}\,L(k^\#)} - \frac{1}{3}\frac{L^\#(J)}{J\,J1^{1/3}} \tag{7}$$

N > $Q[3] := \left(evalDG\left(subs\left(\left\{c = J^{\frac{1}{3}}, c1 = J1^{\frac{1}{3}}\right\}, \left(\frac{e\cdot Der(J1)}{3\cdot J1^{\frac{1}{3}}\cdot J^{\frac{1}{3}}}\ \&wedge\ \mathrm{kappa}\right) + \left(\frac{1}{3}\frac{Der(J)}{(J)} - \frac{1}{3}\frac{Der(J1)}{J1}\right)\&wedge\ \mathrm{zeta} + Z[3]\right)\right)\right):$

N > $Y[3] := subs\left(\left\{e = \frac{e}{J^{\frac{-1}{3}}}, e1 = \frac{e1}{J1^{\frac{-1}{3}}}\right\}, Q[3]\right):$

N > $Co2 := Torsion(Y[3], 3, 4);$

(8)

$$Co2 := -\frac{J^{1/3}\,e1\,L^{\#}(k^{\#})}{J1^{1/3}\,L(k^{\#})} - \frac{L^{\#}(L^{\#}(k))}{J1^{1/3}\,L^{\#}(k)} + e - \frac{1}{3}\,\frac{J^{1/3}\,K^{\#}(J1)\,e1}{J1^{4/3}\,L(k^{\#})} + \frac{1}{3}\,\frac{L^{\#}(J1)}{J1^{4/3}} + \frac{1}{3}\,\frac{K^{\#}(J)\,e1}{J^{2/3}\,J1^{1/3}\,L(k^{\#})} - \frac{1}{3}\,\frac{L^{\#}(J)}{J\,J1^{1/3}} \quad \textbf{(8)}$$

N > $expr := expand(simplify(-Co1 + Co2));$

$$expr := 2\,e - \frac{4}{3}\,\frac{L^{\#}(L^{\#}(k))}{J1^{1/3}\,L^{\#}(k)} - \frac{2}{3}\,\frac{P1}{J1^{1/3}} - \frac{2\,J^{1/3}\,e1\,L^{\#}(k^{\#})}{J1^{1/3}\,L(k^{\#})} - \frac{2}{3}\,\frac{J^{1/3}\,K^{\#}(J1)\,e1}{J1^{4/3}\,L(k^{\#})} + \frac{2}{3}\,\frac{L^{\#}(J1)}{J1^{4/3}} \quad \textbf{(9)}$$

N > $expand\left(subs\left(K^{\#}(J1) = -3\cdot J1\cdot L^{\#}(k^{\#}),\ \frac{expr}{2}\right)\right);$

$$e - \frac{2}{3}\,\frac{L^{\#}(L^{\#}(k))}{J1^{1/3}\,L^{\#}(k)} - \frac{1}{3}\,\frac{P1}{J1^{1/3}} + \frac{1}{3}\,\frac{L^{\#}(J1)}{J1^{4/3}} \quad \textbf{(10)}$$