```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([w, x, y, z, z1], M, verbose);
```

The following coordinates have been protected:

$[w, x, y, z, z1]$

The following vector fields have been defined and protected:

$[D\_w, D\_x, D\_y, D\_z, D\_z1]$

The following differential 1-forms have been defined and protected:

$[dw, dx, dy, dz, dz1]$

frame name: M **(1)**

Une procédure de dérivation:

```
> Der := proc(x) evalDG( R(x)&wedge W[1] + S(x)&wedge W[2] + Tau(x)&wedge W[3]
      + L(x)&wedge W[4] + L#(x) &wedge W[5]); end proc:

> Tau := proc(x) I·(L(L#(x)) - L#(L(x))) end proc:
M > S := proc(x) simplify(L(Tau(x)) - Tau(L(x))) end proc:
> R := proc(x) simplify(L(S(x)) - S(L(x))) end proc:
> L := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
      .. nops(x)) elif
                (type(x, `*`) = true) then expand(L(y)·x/y + y·L(x/y)) elif
                (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·L(y) elif
                (type(x, function) = true) then 'L'(x) elif
                (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:

> L# := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L#(op(i, x)), i = 1
      .. nops(x)) elif
                (type(x, `*`) = true) then expand(L#(y)·x/y + y·L#(x/y)) elif
                (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·L#(y) elif
                (type(x, function) = true) then 'L#(x)' elif
                (type(x, symbol) = true) then 'L#'(x) else 0 fi end proc:

M > W := Vector([dw, dx, dy, dz, dz1]) :
M > dW[1] := evalDG(Tr·(W[1]&wedge W[2]) + Qr·(W[1]&wedge W[3]) + K·(W[1]
      &wedge W[5]) + G·(W[1] &wedge W[4]) + Nr·(W[2] &wedge W[3]) + B
      ·(W[2] &wedge W[5]) + (W[2] &wedge W[4]));
```

$dW_1 := Tr\ dw \wedge dx + Qr\ dw \wedge dy + G\ dw \wedge dz + K\ dw \wedge dz1 + Nr\ dx \wedge dy + dx \wedge dz + B\ dx \wedge dz1$ **(2)**

```
M > dW[2] := evalDG(Sr·(W[1]&wedge W[2]) + Pr·(W[1]&wedge W[3]) + J·(W[1]
      &wedge W[5]) + F·(W[1] &wedge W[4]) + Mr·(W[2] &wedge W[3])
      + (L(B) + A)·(W[2] &wedge W[5]) + B·(W[3] &wedge W[5]) + (W[3]
      &wedge W[4]));
```

(3)

$$dW_2 := Sr\,dw \wedge dx + Pr\,dw \wedge dy + F\,dw \wedge dz + J\,dw \wedge dz1 + Mr\,dx \wedge dy + (L(B) + A)\,dx \wedge dz1 + dy \wedge dz + B\,dy \wedge dz1 \quad (3)$$

```
> dW[3] := evalDG(Rr·(W[1]&wedge W[2]) + OR·(W[1]&wedge W[3]) + H·(W[1]
    &wedge W[5]) + E·(W[1] &wedge W[4]) + Lr·(W[2] &wedge W[3]) + L(A)
    ·(W[2] &wedge W[5]) + A·(W[3] &wedge W[5]) + I·(W[4] &wedge W[5]));
```

$$dW_3 := Rr\,dw \wedge dx + OR\,dw \wedge dy + E\,dw \wedge dz + H\,dw \wedge dz1 + Lr\,dx \wedge dy + L(A)\,dx \wedge dz1 + A\,dy \wedge dz1 + \mathrm{I}\,dz \wedge dz1 \quad (4)$$

```
> dW[4] := evalDG(0 &wedge dx &wedge dy);
```

$$dW_4 := 0\,dw \wedge dx \quad (5)$$

```
> dW[5] := evalDG(0 &wedge dx &wedge dy);
```

$$dW_5 := 0\,dw \wedge dx \quad (6)$$

```
>
> List2 := GenerateForms([dw, dx, dy, dz, dz1], 2) : List1 := [dw, dx, dy, dz, dz1] :
> tr1(1) := 1 :
> tr1(2) := 1 :
> tr1(3) := 1 :
> tr1(4) := 1 :
> tr1(5) := 2 :
> tr1(6) := 2 :
> tr1(7) := 2 :
> tr1(8) := 3 :
> tr1(9) := 3 :
> tr1(10) := 4 :
> tr2(1) := 2 :
> tr2(2) := 3 :
> tr2(3) := 4 :
> tr2(4) := 5 :
> tr2(5) := 3 :
> tr2(6) := 4 :
> tr2(7) := 5 :
> tr2(8) := 4 :
> tr2(9) := 5 :
> tr2(10) := 5 :
> DF :=proc(omega) local T, Res; T := GetComponents(omega, List2);
    Res[1] := evalDG(add(Der(T[i])&wedge List2[i], i = 1..10));
    Res[2] := evalDG(add(T[i] &wedge dW[tr1(i)] &wedge W[tr2(i)], i = 1..10));
    Res[3] := evalDG(add(T[i] &wedge W[tr1(i)] &wedge dW[tr2(i)], i = 1..10));
    evalDG(Res[1] + Res[2] - Res[3]);
    end proc:
> DF(dW[1]);
```

$$(Nr\,OR - Qr\,Lr - Tr\,Mr + Nr\,Sr + \mathrm{I}L(L(L(L^{\#}(Nr)))) - 3\,\mathrm{I}L(L(L^{\#}(L(Nr)))) + 3\,\mathrm{I}L(L^{\#}(L(L(Nr)))) - \mathrm{I}L^{\#}(L(L(L(Nr)))) - \mathrm{I}L(L(L^{\#}(Qr))) + 2\,\mathrm{I}L(L^{\#}(L(Qr))) - \mathrm{I}L^{\#}(L(L(Qr))) + \mathrm{I}L(L^{\#}(Tr)) - \mathrm{I}L^{\#}(L(Tr)))\,dw \wedge dx \wedge dy + (Nr\,E + Sr \quad (7)$$

$$-\mathrm{I}\,L\left(L\left(L^{\#}(G)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(G))\right)-\mathrm{I}\,L^{\#}(L(L(G)))+L(Tr)\Big)\,dw\wedge dx\wedge dz+\big(Nr\,H$$
$$-Qr\,L(A)-Tr\,L(B)-Tr\,A+B\,Sr+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(B)\right)\right)\right)-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(B))\right)\right)$$
$$+3\,\mathrm{I}\,L\left(L^{\#}(L(L(B)))\right)-\mathrm{I}\,L^{\#}(L(L(L(B))))-\mathrm{I}\,L\left(L\left(L^{\#}(K)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(K))\right)$$
$$-\mathrm{I}\,L^{\#}(L(L(K)))+L^{\#}(Tr)\Big)\,dw\wedge dx\wedge dz1+\big(-Tr+Pr-Nr\,F-\mathrm{I}\,L\left(L^{\#}(G)\right)$$
$$+\mathrm{I}\,L^{\#}(L(G))+L(Qr)\Big)\,dw\wedge dy\wedge dz+\big(-Qr\,A-Tr\,B+B\,Pr-Nr\,J-\mathrm{I}\,L\left(L^{\#}(K)\right)$$
$$+\mathrm{I}\,L^{\#}(L(K))+L^{\#}(Qr)\Big)\,dw\wedge dy\wedge dz1+\big(-\mathrm{I}\,Qr+B\,F-J-L(K)$$
$$+L^{\#}(G)\big)\,dw\wedge dz\wedge dz1+(Mr+G\,Nr-Qr+L(Nr))\,dx\wedge dy\wedge dz+\big(-2\,Nr\,A$$
$$+B\,Mr-Nr\,L(B)+K\,Nr-Qr\,B-\mathrm{I}\,L\left(L^{\#}(B)\right)+\mathrm{I}\,L^{\#}(L(B))+L^{\#}(Nr)\Big)\,dx\wedge dy\wedge dz1$$
$$+(-\mathrm{I}\,Nr-2\,L(B)-A+K-G\,B)\,dx\wedge dz\wedge dz1$$

M > $DF(dW[2]);$

$$\left(Mr\,OR-Pr\,Lr+Pr\,Tr-Sr\,Qr+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(Mr)\right)\right)\right)-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(Mr))\right)\right)\right. \tag{8}$$
$$+3\,\mathrm{I}\,L\left(L^{\#}(L(L(Mr)))\right)-\mathrm{I}\,L^{\#}(L(L(L(Mr))))-\mathrm{I}\,L\left(L\left(L^{\#}(Pr)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(Pr))\right)$$
$$-\mathrm{I}\,L^{\#}(L(L(Pr)))+\mathrm{I}\,L\left(L^{\#}(Sr)\right)-\mathrm{I}\,L^{\#}(L(Sr))\Big)\,dw\wedge dx\wedge dy+\big(Mr\,E+Rr+F\,Tr$$
$$-Sr\,G-\mathrm{I}\,L\left(L\left(L^{\#}(F)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(F))\right)-\mathrm{I}\,L^{\#}(L(L(F)))+L(Sr)\Big)\,dw\wedge dx\wedge dz$$
$$+\big(Mr\,H-Pr\,L(A)+B\,Rr+J\,Tr-Sr\,K+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(A)\right)\right)\right)-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(A))\right)\right)$$
$$+3\,\mathrm{I}\,L\left(L^{\#}(L(L(L(B))))\right)-\mathrm{I}\,L^{\#}(L(L(J)))-\mathrm{I}\,L\left(L\left(L^{\#}(J)\right)\right)+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(L(B))\right)\right)\right)$$
$$-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(L(B)))\right)\right)+3\,\mathrm{I}\,L\left(L^{\#}(L(L(A)))\right)-\mathrm{I}\,L^{\#}(L(L(L(A))))$$
$$+2\,\mathrm{I}\,L\left(L^{\#}(L(J))\right)-\mathrm{I}\,L^{\#}(L(L(L(L(B)))))+L^{\#}(Sr)\Big)\,dw\wedge dx\wedge dz1+\big(-Sr+OR$$
$$-Mr\,F+F\,Qr-Pr\,G-\mathrm{I}\,L\left(L^{\#}(F)\right)+\mathrm{I}\,L^{\#}(L(F))+L(Pr)\Big)\,dw\wedge dy\wedge dz+\big(-B\,Sr$$
$$+B\,OR+Pr\,L(B)-Mr\,J+J\,Qr-Pr\,K+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(B)\right)\right)\right)-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(B))\right)\right)$$
$$+3\,\mathrm{I}\,L\left(L^{\#}(L(L(B)))\right)-\mathrm{I}\,L^{\#}(L(L(L(B))))-\mathrm{I}\,L\left(L^{\#}(J)\right)+\mathrm{I}\,L^{\#}(L(J))+L^{\#}(Pr)\Big)$$
$$dw\wedge dy\wedge dz1+\big(-\mathrm{I}\,Pr+B\,E-H+F\,L(B)+F\,A+J\,G-F\,K-L(J)$$
$$+L^{\#}(F)\big)\,dw\wedge dz\wedge dz1+(Lr+Nr\,F-Pr+L(Mr))\,dx\wedge dy\wedge dz+\big(-Mr\,A+B\,Lr$$
$$+Nr\,J-B\,Pr+\mathrm{I}\,L\left(L\left(L^{\#}(B)\right)\right)-3\,\mathrm{I}\,L\left(L^{\#}(L(B))\right)+2\,\mathrm{I}\,L^{\#}(L(L(B)))-\mathrm{I}\,L\left(L^{\#}(A)\right)$$
$$+\mathrm{I}\,L^{\#}(L(A))+L^{\#}(Mr)\Big)\,dx\wedge dy\wedge dz1+(-\mathrm{I}\,Mr-2\,L(A)+J-B\,F$$
$$\left.-L(L(B)))\,dx\wedge dz\wedge dz1\right.$$

M > $DF(dW[3]);$

$$\left(-Rr\,Mr+Lr\,Sr+OR\,Tr-Rr\,Qr+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(Lr)\right)\right)\right)-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(Lr))\right)\right)\right. \tag{9}$$
$$+3\,\mathrm{I}\,L\left(L^{\#}(L(L(Lr)))\right)-\mathrm{I}\,L^{\#}(L(L(L(Lr))))-\mathrm{I}\,L\left(L\left(L^{\#}(OR)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(OR))\right)$$
$$-\mathrm{I}\,L^{\#}(L(L(OR)))+\mathrm{I}\,L\left(L^{\#}(Rr)\right)-\mathrm{I}\,L^{\#}(L(Rr))\Big)\,dw\wedge dx\wedge dy+\big(Lr\,E+E\,Tr-Rr\,G$$
$$-\mathrm{I}\,L\left(L\left(L^{\#}(E)\right)\right)+2\,\mathrm{I}\,L\left(L^{\#}(L(E))\right)-\mathrm{I}\,L^{\#}(L(L(E)))+L(Rr)\Big)\,dw\wedge dx\wedge dz+\big(Lr\,H$$
$$-OR\,L(A)-Rr\,L(B)+L(A)\,Sr+H\,Tr-Rr\,K+2\,\mathrm{I}\,L\left(L^{\#}(L(H))\right)-\mathrm{I}\,L^{\#}(L(L(H)))$$
$$-3\,\mathrm{I}\,L\left(L\left(L^{\#}(L(L(A)))\right)\right)+3\,\mathrm{I}\,L\left(L^{\#}(L(L(L(A))))\right)-\mathrm{I}\,L\left(L\left(L^{\#}(H)\right)\right)$$
$$+\mathrm{I}\,L\left(L\left(L\left(L^{\#}(L(A))\right)\right)\right)-\mathrm{I}\,L^{\#}(L(L(L(L(A)))))+L^{\#}(Rr)\Big)\,dw\wedge dx\wedge dz1+\big(-Rr$$
$$-Lr\,F+E\,Qr-OR\,G-\mathrm{I}\,L\left(L^{\#}(E)\right)+\mathrm{I}\,L^{\#}(L(E))+L(OR)\Big)\,dw\wedge dy\wedge dz+\big(-B\,Rr$$

$$+Pr\,L(A)-Lr\,J+H\,Qr-OR\,K+\mathrm{I}\,L(L(L(L^{\#}(A))))-3\,\mathrm{I}\,L(L(L^{\#}(L(A))))$$
$$+3\,\mathrm{I}\,L(L^{\#}(L(L(A))))-\mathrm{I}\,L^{\#}(L(L(L(A))))-\mathrm{I}\,L(L^{\#}(H))+\mathrm{I}\,L^{\#}(L(H))+L^{\#}(OR))$$
$$dw\wedge dy\wedge dz1+(-\mathrm{I}\,OR+A\,E+L(A)\,F+H\,G-E\,K-L(H)$$
$$+L^{\#}(E))\,dw\wedge dz\wedge dz1+(Nr\,E-OR+L(Lr))\,dx\wedge dy\wedge dz+(-A\,Lr+L(A)\,Mr$$
$$-Lr\,L(B)+Nr\,H-B\,OR+\mathrm{I}\,L(L(L^{\#}(A)))-3\,\mathrm{I}\,L(L^{\#}(L(A)))+2\,\mathrm{I}\,L^{\#}(L(L(A)))$$
$$+L^{\#}(Lr))\,dx\wedge dy\wedge dz1+(-\mathrm{I}\,Lr+H-B\,E-L(L(A)))\,dx\wedge dz\wedge dz1$$

```
> BI := proc(omega) local R, i; R := GetComponents(DF(omega), List3); for i from 1
      to 10 do print(R[i] = 0); od; end proc:
```

```
> List3 := GenerateForms([dw, dx, dy, dz, dz1], 3);
```

$$List3 := [dw\wedge dx\wedge dy, dw\wedge dx\wedge dz, dw\wedge dx\wedge dz1, dw\wedge dy\wedge dz, dw\wedge dy\wedge dz1,$$
$$dw\wedge dz\wedge dz1, dx\wedge dy\wedge dz, dx\wedge dy\wedge dz1, dx\wedge dz\wedge dz1, dy\wedge dz\wedge dz1] \quad (10)$$

```
> BI(dW[1]);
```

$$Nr\,OR-Qr\,Lr-Tr\,Mr+Nr\,Sr+\mathrm{I}\,L(L(L(L^{\#}(Nr))))-3\,\mathrm{I}\,L(L(L^{\#}(L(Nr))))$$
$$+3\,\mathrm{I}\,L(L^{\#}(L(L(Nr))))-\mathrm{I}\,L^{\#}(L(L(L(Nr))))-\mathrm{I}\,L(L(L^{\#}(Qr)))+2\,\mathrm{I}\,L(L^{\#}(L(Qr)))$$
$$-\mathrm{I}\,L^{\#}(L(L(Qr)))+\mathrm{I}\,L(L^{\#}(Tr))-\mathrm{I}\,L^{\#}(L(Tr))=0$$
$$Nr\,E+Sr-\mathrm{I}\,L(L(L^{\#}(G)))+2\,\mathrm{I}\,L(L^{\#}(L(G)))-\mathrm{I}\,L^{\#}(L(L(G)))+L(Tr)=0$$
$$Nr\,H-Qr\,L(A)-Tr\,L(B)-Tr\,A+B\,Sr+\mathrm{I}\,L(L(L(L^{\#}(B))))-3\,\mathrm{I}\,L(L(L^{\#}(L(B))))$$
$$+3\,\mathrm{I}\,L(L^{\#}(L(L(B))))-\mathrm{I}\,L^{\#}(L(L(L(B))))-\mathrm{I}\,L(L(L^{\#}(K)))+2\,\mathrm{I}\,L(L^{\#}(L(K)))$$
$$-\mathrm{I}\,L^{\#}(L(L(K)))+L^{\#}(Tr)=0$$
$$-Tr+Pr-Nr\,F-\mathrm{I}\,L(L^{\#}(G))+\mathrm{I}\,L^{\#}(L(G))+L(Qr)=0$$
$$-Qr\,A-Tr\,B+B\,Pr-Nr\,J-\mathrm{I}\,L(L^{\#}(K))+\mathrm{I}\,L^{\#}(L(K))+L^{\#}(Qr)=0$$
$$-\mathrm{I}\,Qr+B\,F-J-L(K)+L^{\#}(G)=0$$
$$Mr+G\,Nr-Qr+L(Nr)=0$$
$$-2\,Nr\,A+B\,Mr-Nr\,L(B)+K\,Nr-Qr\,B-\mathrm{I}\,L(L^{\#}(B))+\mathrm{I}\,L^{\#}(L(B))+L^{\#}(Nr)=0$$
$$-\mathrm{I}\,Nr-2\,L(B)-A+K-G\,B=0$$
$$0=0 \quad (11)$$

```
> BI(dW[2]);
```

$$Mr\,OR-Pr\,Lr+Pr\,Tr-Sr\,Qr+\mathrm{I}\,L(L(L(L^{\#}(Mr))))-3\,\mathrm{I}\,L(L(L^{\#}(L(Mr))))$$
$$+3\,\mathrm{I}\,L(L^{\#}(L(L(Mr))))-\mathrm{I}\,L^{\#}(L(L(L(Mr))))-\mathrm{I}\,L(L(L^{\#}(Pr)))+2\,\mathrm{I}\,L(L^{\#}(L(Pr)))$$
$$-\mathrm{I}\,L^{\#}(L(L(Pr)))+\mathrm{I}\,L(L^{\#}(Sr))-\mathrm{I}\,L^{\#}(L(Sr))=0$$
$$Mr\,E+Rr+F\,Tr-Sr\,G-\mathrm{I}\,L(L(L^{\#}(F)))+2\,\mathrm{I}\,L(L^{\#}(L(F)))-\mathrm{I}\,L^{\#}(L(L(F)))+L(Sr)=0$$
$$Mr\,H-Pr\,L(A)+B\,Rr+J\,Tr-Sr\,K+\mathrm{I}\,L(L(L(L^{\#}(A))))-3\,\mathrm{I}\,L(L(L^{\#}(L(A))))$$
$$+3\,\mathrm{I}\,L(L^{\#}(L(L(L(B)))))-\mathrm{I}\,L^{\#}(L(L(J)))-\mathrm{I}\,L(L(L^{\#}(J)))+\mathrm{I}\,L(L(L(L^{\#}(L(B)))))$$
$$-3\,\mathrm{I}\,L(L(L^{\#}(L(L(B)))))+3\,\mathrm{I}\,L(L^{\#}(L(L(A))))-\mathrm{I}\,L^{\#}(L(L(L(A))))$$
$$+2\,\mathrm{I}\,L(L^{\#}(L(J)))-\mathrm{I}\,L^{\#}(L(L(L(L(B)))))+L^{\#}(Sr)=0$$
$$-Sr+OR-Mr\,F+F\,Qr-Pr\,G-\mathrm{I}\,L(L^{\#}(F))+\mathrm{I}\,L^{\#}(L(F))+L(Pr)=0$$
$$-B\,Sr+B\,OR+Pr\,L(B)-Mr\,J+J\,Qr-Pr\,K+\mathrm{I}\,L(L(L(L^{\#}(B))))-3\,\mathrm{I}\,L(L(L^{\#}(L(B))))$$

$$+3\,\mathrm{I}L\left(L^{\#}(L(L(B)))\right)-\mathrm{I}L^{\#}(L(L(L(B))))-\mathrm{I}L\left(L^{\#}(J)\right)+\mathrm{I}L^{\#}(L(J))+L^{\#}(Pr)=0$$

$$-\mathrm{I}Pr+B\,E-H+F\,L(B)+F\,A+J\,G-F\,K-L(J)+L^{\#}(F)=0$$

$$Lr+Nr\,F-Pr+L(Mr)=0$$

$$-Mr\,A+B\,Lr+Nr\,J-B\,Pr+\mathrm{I}L\left(L\left(L^{\#}(B)\right)\right)-3\,\mathrm{I}L\left(L^{\#}(L(B))\right)+2\,\mathrm{I}L^{\#}(L(L(B)))$$
$$-\mathrm{I}L\left(L^{\#}(A)\right)+\mathrm{I}L^{\#}(L(A))+L^{\#}(Mr)=0$$

$$-\mathrm{I}Mr-2\,L(A)+J-B\,F-L(L(B))=0$$

$$0=0 \qquad (12)$$

```
> BI(dW[3]);
```

$$-Rr\,Mr+Lr\,Sr+OR\,Tr-Rr\,Qr+\mathrm{I}L\left(L\left(L\left(L^{\#}(Lr)\right)\right)\right)-3\,\mathrm{I}L\left(L\left(L^{\#}(L(Lr))\right)\right)$$
$$+3\,\mathrm{I}L\left(L^{\#}(L(L(Lr)))\right)-\mathrm{I}L^{\#}(L(L(L(Lr))))-\mathrm{I}L\left(L\left(L^{\#}(OR)\right)\right)+2\,\mathrm{I}L\left(L^{\#}(L(OR))\right)$$
$$-\mathrm{I}L^{\#}(L(L(OR)))+\mathrm{I}L\left(L^{\#}(Rr)\right)-\mathrm{I}L^{\#}(L(Rr))=0$$

$$Lr\,E+E\,Tr-Rr\,G-\mathrm{I}L\left(L\left(L^{\#}(E)\right)\right)+2\,\mathrm{I}L\left(L^{\#}(L(E))\right)-\mathrm{I}L^{\#}(L(L(E)))+L(Rr)=0$$

$$Lr\,H-OR\,L(A)-Rr\,L(B)+L(A)\,Sr+H\,Tr-Rr\,K+2\,\mathrm{I}L\left(L^{\#}(L(H))\right)-\mathrm{I}L^{\#}(L(L(H)))$$
$$-3\,\mathrm{I}L\left(L\left(L^{\#}(L(L(A)))\right)\right)+3\,\mathrm{I}L\left(L^{\#}(L(L(L(A))))\right)-\mathrm{I}L\left(L\left(L^{\#}(H)\right)\right)$$
$$+\mathrm{I}L\left(L\left(L\left(L^{\#}(L(A))\right)\right)\right)-\mathrm{I}L^{\#}(L(L(L(L(A)))))+L^{\#}(Rr)=0$$

$$-Rr-Lr\,F+E\,Qr-OR\,G-\mathrm{I}L\left(L^{\#}(E)\right)+\mathrm{I}L^{\#}(L(E))+L(OR)=0$$

$$-B\,Rr+Pr\,L(A)-Lr\,J+H\,Qr-OR\,K+\mathrm{I}L\left(L\left(L\left(L^{\#}(A)\right)\right)\right)-3\,\mathrm{I}L\left(L\left(L^{\#}(L(A))\right)\right)$$
$$+3\,\mathrm{I}L\left(L^{\#}(L(L(A)))\right)-\mathrm{I}L^{\#}(L(L(L(A))))-\mathrm{I}L\left(L^{\#}(H)\right)+\mathrm{I}L^{\#}(L(H))+L^{\#}(OR)=0$$

$$-\mathrm{I}OR+A\,E+L(A)\,F+H\,G-E\,K-L(H)+L^{\#}(E)=0$$

$$Nr\,E-OR+L(Lr)=0$$

$$-A\,Lr+L(A)\,Mr-Lr\,L(B)+Nr\,H-B\,OR+\mathrm{I}L\left(L\left(L^{\#}(A)\right)\right)-3\,\mathrm{I}L\left(L^{\#}(L(A))\right)$$
$$+2\,\mathrm{I}L^{\#}(L(L(A)))+L^{\#}(Lr)=0$$

$$-\mathrm{I}Lr+H-B\,E-L(L(A))=0$$

$$0=0 \qquad (13)$$

```
> R1 := GetComponents(DF(dW[1]), List3):
> R2 := GetComponents(DF(dW[2]), List3):
> R3 := GetComponents(DF(dW[3]), List3):
> Nr := solve(R1[9], Nr);
```

$$Nr := \mathrm{I}\,(2\,L(B)+A-K+G\,B) \qquad (14)$$

```
> Mr := solve(R2[9], Mr);
```

$$Mr := \mathrm{I}\,(2\,L(A)-J+B\,F+L(L(B))) \qquad (15)$$

```
> Lr := solve(R3[9], Lr);
```

$$Lr := \mathrm{I}\,(-H+B\,E+L(L(A))) \qquad (16)$$

```
> Qr := solve(R1[6], Qr);
```

$$Qr := -\mathrm{I}\left(B\,F-J-L(K)+L^{\#}(G)\right) \qquad (17)$$

```
> Pr := solve(R2[6], Pr);
```

$$Pr := -\mathrm{I}\left(B\,E-H+F\,L(B)+F\,A+J\,G-F\,K-L(J)+L^{\#}(F)\right) \qquad (18)$$

```
> OR := solve(R3[6], OR);
```

$$OR := -\mathrm{I}\left(A\,E+L(A)\,F+H\,G-E\,K-L(H)+L^{\#}(E)\right) \qquad (19)$$

```
> for i from 7 to 8 do print(expand(R1[i]) = 0); print(expand(R2[i]) = 0);
      print(expand(R3[i]) = 0); od;
```

$$3\,I L(L(B)) + 3\,I L(A) + I L(G)\,B + 3\,I G L(B) - 2\,I J + I G A - I G K + I G^2 B - 2\,I L(K) + 2\,I B F + I L^{\#}(G) = 0$$

$$2\,I B E + 3\,I L(L(A)) + I L(L(L(B))) + I F G B + I J G + I L^{\#}(F) - 2\,I H + 2\,I F A + 4\,I F L(B) - 2\,I L(J) - 2\,I F K + I B L(F) = 0$$

$$3\,I L(B)\,E + 2\,I E A - 2\,I E K + I E G B + I L(A)\,F + I H G - 2\,I L(H) + I L^{\#}(E) + I B L(E) + I L(L(L(A))) = 0$$

$$2\,I L^{\#}(G)\,B - I K^2 + I L^{\#}(A) - 2\,I L(B)^2 + 3\,I K A - I L\left(L^{\#}(B)\right) - 2\,I A G B - I L(B)\,G B + I K G B + I G L^{\#}(B) - 2\,I A^2 + I B L(L(B)) + 3\,I L^{\#}(L(B)) - I L^{\#}(K) + 2\,I B L(A) - 2\,I B J - I B L(K) - 5\,I L(B)\,A + 2\,I B^2 F + 3\,I L(B)\,K = 0$$

$$-2\,I B H - I B F K - I L^{\#}(J) + 3\,I L^{\#}(L(A)) - 3\,I L\left(L^{\#}(L(B))\right) - 2\,I A L(A) + I L\left(L\left(L^{\#}(B)\right)\right) - I B L(J) + 2\,I A J + 2\,I B^2 E + I B L(L(A)) - I A L(L(B)) + 2\,I J G B + 2\,I J L(B) - I J K - I L\left(L^{\#}(A)\right) + 3\,I L^{\#}(L(L(B))) + I B F L(B) + I L^{\#}(B)\,F + 2\,I B L^{\#}(F) = 0$$

$$-I L^{\#}(H) - I B E K + 2\,I L(A)\,B F + 3\,I L(B)\,H + I L(A)\,L(L(B)) - 3\,I L\left(L^{\#}(L(A))\right) - I L(B)\,B E + 3\,I L^{\#}(L(L(A))) - I A L(L(A)) + 2\,I A H + 2\,I H G B + 2\,I L(A)^2 + I L^{\#}(B)\,E - I B L(H) - I H K - I L(B)\,L(L(A)) + I L\left(L\left(L^{\#}(A)\right)\right) - I L(A)\,J + 2\,I B L^{\#}(E) = 0 \tag{20}$$

```
> solve(expand(R1[7]), J);
```

$$\frac{3}{2} L(L(B)) + \frac{3}{2} L(A) + \frac{1}{2} L(G)\,B + \frac{3}{2} G L(B) + \frac{1}{2} G A - \frac{1}{2} G K + \frac{1}{2} G^2 B - L(K) + B F + \frac{1}{2} L^{\#}(G) \tag{21}$$

```
> solve(expand(R2[7]), H);
```

$$2\,F L(B) + \frac{1}{2} F G B + \frac{1}{2} L^{\#}(F) + B E - L(J) + \frac{1}{2} B L(F) - F K + \frac{3}{2} L(L(A)) + F A + \frac{1}{2} L(L(L(B))) + \frac{1}{2} J G \tag{22}$$

```
> Tr := solve(R1[4], Tr);
```

$$Tr := I L^{\#}(L(G)) - I B E + I H - 4\,I F L(B) - 2\,I F A - I J G + 2\,I F K + 2\,I L(J) - I L^{\#}(F) - I F G B - 2\,I L\left(L^{\#}(G)\right) + I L(L(K)) - I B L(F) \tag{23}$$

```
> Sr := solve(R2[4], Sr);
```

$$Sr := I E G B + I L^{\#}(L(F)) - I L(F)\,L(B) - I B L(E) + I L(F)\,K - I J L(G) - I L(F)\,A - I L(B)\,E + I G^2 J + I G L^{\#}(F) + 2\,I F L(K) - 2\,I F L(L(B)) - 4\,I L(A)\,F - 2\,I L(J)\,G - 2\,I F^2 B + 2\,I F J - I F L^{\#}(G) - I E A - 2\,I H G + I E K + I G F L(B) + I G F A - I G F K + I L(L(J)) - 2\,I L\left(L^{\#}(F)\right) + 2\,I L(H) - I L^{\#}(E) \tag{24}$$

```
> Rr := solve(R3[4], Rr);
```

$$Rr := -2\,I L(L(A))\,F - 2\,I L\left(L^{\#}(E)\right) + I G A E + 2\,I E L(K) + I L(L(H)) - I G E K \tag{25}$$

$$-\mathrm{I}\,L(A)\,L(F)-\mathrm{I}\,L(A)\,E-\mathrm{I}\,H\,L(G)+\mathrm{I}\,L^{\#}(L(E))-\mathrm{I}\,A\,L(E)-2\,\mathrm{I}\,L(H)\,G+\mathrm{I}\,F\,H$$
$$+\mathrm{I}\,G^{2}\,H+\mathrm{I}\,G\,L^{\#}(E)-2\,\mathrm{I}\,F\,B\,E+\mathrm{I}\,G\,L(A)\,F+\mathrm{I}\,E\,J+\mathrm{I}\,L(E)\,K-\mathrm{I}\,E\,L^{\#}(G)$$

frame2 > $expr1 := \dfrac{\frac{1}{10}\,\mathrm{I}\,(6\,L(B)+G\,B)}{\sqrt{B}}$:

frame2 > $expr2 := -\dfrac{\frac{1}{10}\,\mathrm{I}\cdot(-3\,L^{\#}(B)+2\,B\,A+B\,K+B\,L(B))}{B^{3/2}}$:

M > $torsion := expand\left(-\dfrac{expr2}{a}-\dfrac{expr1}{a}+\dfrac{Nr}{a\sqrt{B}}\right)$;

$$torsion := -\frac{\frac{3}{10}\,\mathrm{I}\,L^{\#}(B)}{B^{3/2}\,a}+\frac{\frac{6}{5}\,\mathrm{I}\,A}{\sqrt{B}\,a}-\frac{\frac{9}{10}\,\mathrm{I}\,K}{\sqrt{B}\,a}+\frac{\frac{3}{2}\,\mathrm{I}\,L(B)}{\sqrt{B}\,a}+\frac{\frac{9}{10}\,\mathrm{I}\sqrt{B}\,G}{a} \qquad (26)$$

> $Inv := \dfrac{1}{2}\,\dfrac{L(B)}{\sqrt{B}}+\dfrac{3}{10}\sqrt{B}\,G-\dfrac{1}{10}\,\dfrac{L^{\#}(B)}{B^{3/2}}+\dfrac{2}{5}\,\dfrac{A}{\sqrt{B}}-\dfrac{3}{10}\,\dfrac{K}{\sqrt{B}}$:

M > $expand\left(torsion-\dfrac{3\cdot I\cdot Inv}{a}\right)$;

$$0 \qquad (27)$$

M >