

N > restart;
conj := **proc**(x) **local** y ; $y := op(1, x) : \mathbf{if} (type(x, '+') = true) \mathbf{then} add(conj(op(i, x)), i = 1 .. nops(x)) \mathbf{elif}$
 $(type(x, '*') = true) \mathbf{then} simplify\left(conj(y) \cdot conj\left(\frac{x}{y}\right)\right) \mathbf{elif}$
 $(type(x, '^') = true) \mathbf{then} conj(y)^{op(2, x)} \mathbf{elif} (type(x, 'complex') = true) \mathbf{then} conjugate(x) \mathbf{elif}$
 $x = Q \mathbf{then} Q^\# \mathbf{elif} x = P \mathbf{then} P^\# \mathbf{elif} x = B \mathbf{then} B^\# \mathbf{elif} x = A \mathbf{then} A^\#$
 $\mathbf{elif} x = Q^\# \mathbf{then} Q \mathbf{elif} x = P^\# \mathbf{then} P \mathbf{elif} x = A^\# \mathbf{then} A \mathbf{elif} x = B^\#$
 $\mathbf{then} B \mathbf{elif} x = a \mathbf{then} a \mathbf{elif} x = d \mathbf{then} d1 \mathbf{elif} x = d1 \mathbf{then} d \mathbf{elif} (type(x, 'function') = true) \mathbf{then} \mathbf{if} op(0, x) = L \mathbf{then} (L^\#(conj(y))) \mathbf{elif} op(0, x) = L^\#$
 $\mathbf{then} (L(conj(y))) \mathbf{elif} op(0, x) = S \mathbf{then} B \cdot S(conj(y)) + A \cdot \text{Tau}(conj(y)) \mathbf{end if} \mathbf{end if} ; \mathbf{end proc} :$

N > **conjugue** := **proc**(x) ; **expand**(**Sub**(**conj**(x))) **end proc** :

> **substitution** := **proc**(s) ; **subs**($\sqrt{\frac{1}{B}} = \frac{1}{\sqrt{B}}$, s) **end proc** :

> **Sub** := **proc**(s) ; **if** ($type(s, '+') = true$) **then** **add**(**Sub**(**op**(i, s)), $i = 1 .. nops(s)$)
 $\mathbf{elif} (type(s, '^') \mathbf{and} op(2, s) < 0) = true$
 $\mathbf{then} \frac{1}{substitution(op(1, s)^{-op(2, s)})} \mathbf{elif} (type(s, '*') = true) \mathbf{then} Sub(op(1, s))$
 $\cdot Sub\left(\frac{s}{op(1, s)}\right) \mathbf{else} substitution(s) \mathbf{fi} \mathbf{end proc} :$

> **L** := **proc**(x) **local** y ; $y := op(1, x) : \mathbf{if} (type(x, '+') = true) \mathbf{then} add(L(op(i, x)), i = 1 .. nops(x)) \mathbf{elif}$
 $(type(x, '*') = true) \mathbf{then} expand\left(L(y) \cdot \frac{x}{y} + y \cdot L\left(\frac{x}{y}\right)\right) \mathbf{elif}$
 $(type(x, '^') = true) \mathbf{then} op(2, x) \cdot y^{(op(2, x) - 1)} \cdot L(y) \mathbf{elif}$
 $(type(x, 'function') = true) \mathbf{then} 'L'(x) \mathbf{elif}$
 $(type(x, 'symbol') = true) \mathbf{then} 'L'(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end proc} :$

> $L^\#$:= **proc**(x) **local** y ; $y := op(1, x) : \mathbf{if} (type(x, '+') = true) \mathbf{then} add(L^\#(op(i, x)), i = 1 .. nops(x)) \mathbf{elif}$
 $(type(x, '*') = true) \mathbf{then} expand\left(L^\#(y) \cdot \frac{x}{y} + y \cdot L^\#\left(\frac{x}{y}\right)\right) \mathbf{elif}$
 $(type(x, '^') = true) \mathbf{then} op(2, x) \cdot y^{(op(2, x) - 1)} \cdot L^\#(y) \mathbf{elif}$
 $(type(x, 'function') = true) \mathbf{then} 'L^\#(x)' \mathbf{elif}$
 $(type(x, 'symbol') = true) \mathbf{then} 'L^\#(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end proc} :$

> $B^\# := \frac{1}{B} : A^\# := -B^\# \cdot A : \text{Tau} := \mathbf{proc}(x) I \cdot (L(L^\#(x)) - L^\#(L(x))) \mathbf{end proc} :$

> **expand**(**solve**($L(B) + B \cdot Q + A - B \cdot L^\#(B^\#) - Q^\# - A^\# \cdot B, A$)) :

> $Q^\# := \mathbf{expand}(solve(L(B) + B \cdot Q + A - B \cdot L^\#(B^\#) - Q^\# - A^\# \cdot B, Q^\#)) :$

> $eq2 := -P^\# - A \cdot Q^\# + L^\#(A) + A^2 + B \cdot L(A) + B^2 \cdot P :$

> $P^\# := \mathbf{expand}(solve(eq2, P^\#)) :$

> $S := \mathbf{proc}(x) \mathbf{local} y; y := op(1, x) : \mathbf{if} (type(x, '+') = true) \mathbf{then} add(S(op(i, x)), i = 1 .. nops(x)) \mathbf{elif}$

$(type(x, '*') = true) \mathbf{then} expand\left(S(y) \cdot \frac{x}{y} + y \cdot S\left(\frac{x}{y}\right)\right) \mathbf{elif}$

$(type(x, '^') = true) \mathbf{then} op(2, x) \cdot y^{(op(2, x) - 1)} \cdot S(y) \mathbf{elif}$

$(type(x, function) = true) \mathbf{then} 'S'(x) \mathbf{elif}$

$(type(x, symbol) = true) \mathbf{then} 'S'(x) \mathbf{else} 0 \mathbf{fi} \mathbf{end} \mathbf{proc}:$

>
$$\begin{aligned} expl := & \frac{5}{18} \frac{AL(A) Q}{a^4} - \frac{1}{6} \frac{APL(B)}{a^4} + \frac{5}{9} \frac{QL(A) L(B)}{a^4} - \frac{5}{18} \frac{BQPL(B)}{a^4} \\ & - \frac{2}{27} \frac{AL^\#(B) Q^2}{Ba^4} - \frac{1}{2} \frac{AL^\#(Q) L(B)}{Ba^4} - \frac{1}{6} \frac{AL^\#(Q) Q}{a^4} - \frac{AL^\#(L(B)) L(B)}{B^2 a^4} \\ & - \frac{1}{6} \frac{AL^\#(L(B)) Q}{Ba^4} - \frac{1}{2} \frac{AL^\#(B) L(B)^2}{B^3 a^4} - \frac{1}{18} \frac{BAPQ}{a^4} + \frac{17}{216} \frac{AQ^2 L(B)}{a^4} \\ & + \frac{7}{24} \frac{AQL(B)^2}{Ba^4} + \frac{1}{6} \frac{L^\#(B) L(A) L(B)}{B^2 a^4} + \frac{1}{18} \frac{L^\#(B) L(A) Q}{Ba^4} \\ & + \frac{1}{6} \frac{L^\#(B) PL(B)}{Ba^4} + \frac{1}{18} \frac{L^\#(B) PQ}{a^4} + \frac{5}{6} \frac{AL(A) L(B)}{Ba^4} - \frac{\frac{1}{18} Idl A Q}{a^5} \\ & + \frac{AL(L^\#(B)) L(B)}{B^2 a^4} + \frac{1}{6} \frac{AL(L^\#(B)) Q}{Ba^4} + \frac{17}{36} \frac{AL(B) L(Q)}{a^4} + \frac{1}{12} \frac{AQL(L(B))}{a^4} \\ & + \frac{1}{6} \frac{AL^\#(B) L(L(B))}{B^2 a^4} + \frac{5}{12} \frac{AL(L(B)) L(B)}{Ba^4} + \frac{1}{18} \frac{AL^\#(B) L(Q)}{Ba^4} \\ & + \frac{5}{36} \frac{BAL(Q) Q}{a^4} + \frac{1}{6} \frac{L^\#(A) L(B) Q}{Ba^4} - \frac{\frac{1}{2} IL(B) S(B)}{Ba^4} - \frac{\frac{1}{6} IB dL(Q)}{a^5} \\ & - \frac{\frac{1}{36} IB dl Q^2}{a^5} - \frac{\frac{5}{12} Idl L(B)^2}{Ba^5} + \frac{\frac{1}{4} IdL(B)^2}{Ba^5} - \frac{\frac{3}{2} Idl L^\#(L(B))}{Ba^5} \\ & - \frac{\frac{1}{36} IB d Q^2}{a^5} + \frac{IB dl L(Q)}{a^5} + \frac{IL(L^\#(B)) dl}{Ba^5} + \frac{\frac{5}{9} Idl QL(B)}{a^5} \\ & - \frac{\frac{1}{6} IdQL(B)}{a^5} - \frac{\frac{1}{6} IdQA}{a^5} - \frac{1}{3} \frac{AL^\#(B) QL(B)}{B^2 a^4} + \frac{1}{54} \frac{A^2 Q^2}{a^4} - \frac{1}{216} \frac{BAQ^3}{a^4} \\ & - \frac{1}{8} \frac{AL(B)^3}{B^2 a^4} + \frac{1}{6} \frac{BQ^2 L(A)}{a^4} + \frac{1}{6} \frac{L(B)^2 L(A)}{Ba^4} - \frac{5}{6} \frac{L(B)^2 P}{a^4} \\ & - \frac{1}{6} \frac{BQL(L(A))}{a^4} + \frac{1}{6} \frac{BQL^\#(P)}{a^4} - \frac{1}{6} \frac{B^2 QL(P)}{a^4} - \frac{1}{2} \frac{L(B) L(L(A))}{a^4} \\ & + \frac{1}{2} \frac{L(B) L^\#(P)}{a^4} - \frac{1}{2} \frac{BL(B) L(P)}{a^4} + \frac{dl d}{a^6} + \frac{dl^2}{a^6} + \frac{\frac{1}{2} IS(L(B))}{a^4} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{AL^\#(L(L(B)))}{Ba^4} - \frac{\frac{4}{9} \text{Idl } L^\#(B) Q}{Ba^5} - \frac{\frac{1}{2} \text{Id } AL(B)}{Ba^5} - \frac{\frac{1}{6} \text{Idl } L^\#(B) L(B)}{B^2 a^5} \\
& - \frac{\frac{5}{6} \text{Idl } AL(B)}{Ba^5} + \frac{1}{6} \frac{AL^\#(L(Q))}{a^4} - \frac{1}{2} \frac{AL(L^\#(L(B)))}{Ba^4} - \frac{1}{6} \frac{AL(L^\#(Q))}{a^4} \\
& - \frac{1}{6} \frac{A^2 L(L(B))}{Ba^4} - \frac{1}{18} \frac{A^2 L(Q)}{a^4} + \frac{1}{4} \frac{L^\#(A) L(B)^2}{B^2 a^4} + \frac{1}{36} \frac{L^\#(A) Q^2}{a^4} \\
& - \frac{1}{2} \frac{L(L^\#(A)) L(B)}{Ba^4} - \frac{1}{6} \frac{L(L^\#(A)) Q}{a^4} + \frac{1}{2} \frac{L^\#(L(A)) L(B)}{Ba^4} + \frac{1}{6} \frac{L^\#(L(A)) Q}{a^4} \\
& - \frac{\frac{7}{6} \text{Idl } L^\#(Q)}{a^5} - \frac{\frac{1}{2} \text{Id } L(L(B))}{a^5} + \frac{\text{Idl } L(L(B))}{a^5} + \frac{2 \text{Idl } L(A)}{a^5} \\
& + \frac{\frac{1}{6} \text{IBS}(Q)}{a^4} :
\end{aligned}$$

$$\begin{aligned}
> \text{exp2} := & - \frac{7}{18} \frac{AL(A) Q}{a^4} + \frac{11}{18} \frac{APL(B)}{a^4} - \frac{5}{18} \frac{QL(A) L(B)}{a^4} + \frac{5}{18} \frac{BQPL(B)}{a^4} \\
& - \frac{2}{9} \frac{PL^\#(B) A}{Ba^4} + \frac{2}{27} \frac{AL^\#(B)^2 Q}{B^3 a^4} - \frac{1}{6} \frac{AL^\#(L^\#(B)) L(B)}{B^3 a^4} - \frac{1}{18} \frac{AL^\#(L^\#(B)) Q}{B^2 a^4} \\
& + \frac{1}{6} \frac{AL^\#(A) L(B)}{B^2 a^4} + \frac{1}{18} \frac{AL^\#(A) Q}{Ba^4} + \frac{1}{18} \frac{A^2 L^\#(B) L(B)}{B^3 a^4} + \frac{1}{54} \frac{A^2 L^\#(B) Q}{B^2 a^4} \\
& + \frac{1}{108} \frac{AL^\#(B) Q^2}{Ba^4} + \frac{1}{12} \frac{AL^\#(Q) L(B)}{Ba^4} + \frac{1}{36} \frac{AL^\#(Q) Q}{a^4} \\
& + \frac{1}{12} \frac{AL^\#(L(B)) L(B)}{B^2 a^4} + \frac{1}{36} \frac{AL^\#(L(B)) Q}{Ba^4} - \frac{1}{18} \frac{AL^\#(B) L(B)^2}{B^3 a^4} \\
& + \frac{2}{9} \frac{AL^\#(B)^2 L(B)}{B^4 a^4} + \frac{1}{18} \frac{BAPQ}{a^4} - \frac{1}{27} \frac{A^2 QL(B)}{Ba^4} + \frac{5}{216} \frac{AQ^2 L(B)}{a^4} \\
& + \frac{7}{216} \frac{AQL(B)^2}{Ba^4} + \frac{1}{6} \frac{L^\#(B) L(A) L(B)}{B^2 a^4} + \frac{7}{18} \frac{L^\#(B) L(A) Q}{Ba^4} \\
& - \frac{11}{18} \frac{L^\#(B) PL(B)}{Ba^4} - \frac{1}{18} \frac{L^\#(B) PQ}{a^4} - \frac{1}{6} \frac{AL(A) L(B)}{Ba^4} - \frac{\frac{1}{3} \text{IS}(B) L^\#(B)}{B^2 a^4} \\
& + \frac{\frac{4}{9} \text{Idl } L^\#(B)^2}{B^3 a^5} + \frac{\frac{7}{6} \text{IBdL}(Q)}{a^5} + \frac{\frac{1}{36} \text{IBdl } Q^2}{a^5} + \frac{\frac{1}{36} \text{Idl } L(B)^2}{Ba^5} \\
& - \frac{\frac{5}{12} \text{Id } L(B)^2}{Ba^5} - \frac{\frac{1}{3} \text{Idl } L^\#(L^\#(B))}{B^2 a^5} + \frac{\frac{1}{3} \text{Idl } L^\#(A)}{Ba^5} + \frac{\frac{1}{6} \text{Idl } L^\#(L(B))}{Ba^5} \\
& - \frac{\frac{1}{3} \text{Id } L(L^\#(B))}{Ba^5} + \frac{\frac{1}{36} \text{IBd } Q^2}{a^5} - \frac{\frac{2}{9} \text{Idl } A^2}{Ba^5} + \frac{\frac{1}{18} \text{Idl } QL(B)}{a^5}
\end{aligned}$$

$$\begin{aligned}
& + \frac{7}{9} \frac{\text{Id} QL(B)}{a^5} + \frac{1}{18} \frac{\text{Id} QA}{a^5} - \frac{1}{18} \frac{\text{Id} l A Q}{a^5} + \frac{1}{108} \frac{AL^\#(B) QL(B)}{B^2 a^4} \\
& + \frac{1}{18} \frac{\text{Id} l L^\#(B) Q}{Ba^5} - \frac{5}{6} \frac{\text{Id} AL(B)}{Ba^5} - \frac{1}{9} \frac{\text{Id} l L^\#(B) L(B)}{B^2 a^5} - \frac{1}{18} \frac{\text{Id} l AL(B)}{Ba^5} \\
& - \frac{7}{18} \frac{\text{Id} QL^\#(B)}{Ba^5} + \frac{1}{9} \frac{\text{Id} l L^\#(B) A}{B^2 a^5} + \frac{1}{6} \frac{\text{Id} L^\#(B) L(B)}{B^2 a^5} - \frac{1}{27} \frac{A^3 Q}{Ba^4} \\
& - \frac{1}{108} \frac{A^2 Q^2}{a^4} + \frac{1}{216} \frac{BAQ^3}{a^4} + \frac{1}{9} \frac{PA^2}{a^4} - \frac{1}{9} \frac{A^3 L(B)}{B^2 a^4} - \frac{1}{36} \frac{A^2 L(B)^2}{B^2 a^4} \\
& + \frac{1}{72} \frac{AL(B)^3}{B^2 a^4} - \frac{7}{36} \frac{BQ^2 L(A)}{a^4} - \frac{1}{12} \frac{L(B)^2 L(A)}{Ba^4} + \frac{5}{18} \frac{L(B)^2 P}{a^4} \\
& + \frac{1}{9} \frac{PL^\#(B)^2}{B^2 a^4} - \frac{1}{3} \frac{L^\#(B) L(L(A))}{Ba^4} + \frac{1}{3} \frac{L^\#(B) L^\#(P)}{Ba^4} - \frac{1}{3} \frac{L^\#(B) L(P)}{a^4} \\
& + \frac{1}{3} \frac{AL(L(A))}{a^4} - \frac{1}{3} \frac{AL^\#(P)}{a^4} + \frac{1}{3} \frac{BAL(P)}{a^4} + \frac{1}{6} \frac{BQL(L(A))}{a^4} \\
& - \frac{1}{6} \frac{BQL^\#(P)}{a^4} + \frac{1}{6} \frac{B^2 QL(P)}{a^4} + \frac{1}{6} \frac{L(B) L(L(A))}{a^4} - \frac{1}{6} \frac{L(B) L^\#(P)}{a^4} \\
& + \frac{1}{6} \frac{BL(B) L(P)}{a^4} + \frac{dl d}{a^6} + \frac{7}{3} \frac{\text{Id} L(A)}{a^5} + \frac{1}{6} \frac{\text{Id} l L^\#(Q)}{a^5} - \frac{\text{Id} L^\#(Q)}{a^5} \\
& - \frac{1}{6} \frac{\text{IS}(B) Q}{a^4} + \frac{7}{6} \frac{\text{Id} L(L(B))}{a^5} - \frac{1}{6} \frac{\text{IBS}(Q)}{a^4} + \frac{1}{3} \frac{\text{IS}(L^\#(B))}{Ba^4} + \frac{d^2}{a^6} \\
& - \frac{1}{3} \frac{\text{IS}(A)}{a^4} - \frac{1}{6} \frac{\text{IS}(L(B))}{a^4} :
\end{aligned}$$

> `expand(exp1 - conjugue(exp2));`

0

(1)

$$\begin{aligned}
> \text{exp3} := & \frac{1}{2} \frac{\text{Ia} L(B)^2}{B} + \frac{1}{3} \text{Ia} L(B) Q + \frac{1}{3} \text{Ia} B Q^2 - \frac{1}{2} \text{Ia} L(L(B)) - \frac{1}{2} \text{Ia} B L(Q) \\
& + \frac{1}{2} \frac{\text{Ia} A L(B)}{B} + \frac{1}{6} \text{Ia} A Q + \text{Ia} B P :
\end{aligned}$$

$$\begin{aligned}
> \text{exp4} := & -\frac{1}{6} \frac{\text{Ia} L(B) A}{B} - \frac{2}{3} \text{Ia} L(B) Q - \frac{1}{3} \frac{\text{Ia} L^\#(B) L(B)}{B^2} + \frac{1}{6} \frac{\text{Ia} L^\#(B) Q}{B} \\
& - \frac{1}{3} \frac{\text{Ia} L(B)^2}{B} - \frac{1}{3} \text{Ia} B Q^2 + \frac{1}{2} \frac{\text{Ia} L^\#(L(B))}{B} + \frac{1}{2} \text{Ia} L^\#(Q) - \text{Ia} B P - \text{Ia} L(A)
\end{aligned}$$

$$-\frac{1}{6} \text{Ia} A Q:$$

> *expand(conjugue(exp3) - exp4);*

0

(2)

> *conjugue(B·P);*

$$L(A) - \frac{AL(B)}{B} - A Q - \frac{A^2}{B} - \frac{AL^\#(B)}{B^2} + \frac{L^\#(A)}{B} + B P$$

(3)