

```

> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([w, x, y, z, z1], [a, d, e, g, h, k], frame1, verbose);
    The following coordinates have been protected:
                [w, x, y, z, z1, a, d, e, g, h, k]
    The following vector fields have been defined and protected:
                [D_w, D_x, D_y, D_z, D_z1, D_a, D_d, D_e, D_g, D_h, D_k]
    The following differential 1-forms have been defined and protected:
                [dw, dx, dy, dz, dz1, da, dd, de, dg, dh, dk]
                frame name: frame1

```

(1)

Une procédure de dérivation:

```

> Der := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Der(op(i, x)), i = 1
.. nops(x)) elif
                (type(x, `*`) = true) then expand( $\frac{x}{y} \cdot Der(y) + y \cdot Der\left(\frac{x}{y}\right)$ ) elif
                (type(x, `^`) = true) then op(2, x) · y(op(2, x) - 1) · Der(y) elif
                ((type(x, function) = true) or (type(x, symbol) = true)) then R(x) · W[1]
+ S(x) · W[2] + Tau(x) · W[3] + L(x) · W[4] + L#(x) · W[5]
                else 0 fi end proc:

```

Les fonctions L et L[#]:

```

> L := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
.. nops(x)) elif
                (type(x, `*`) = true) then expand( $L(y) \cdot \frac{x}{y} + y \cdot L\left(\frac{x}{y}\right)$ ) elif
                (type(x, `^`) = true) then op(2, x) · y(op(2, x) - 1) · L(y) elif
                (type(x, function) = true) then 'L'(x) elif
                (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:

```

```

> L# := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L#(op(i, x)), i = 1
.. nops(x)) elif
                (type(x, `*`) = true) then expand( $L^{\#}(y) \cdot \frac{x}{y} + y \cdot L^{\#}\left(\frac{x}{y}\right)$ ) elif
                (type(x, `^`) = true) then op(2, x) · y(op(2, x) - 1) · L#(y) elif
                (type(x, function) = true) then 'L#(x)' elif
                (type(x, symbol) = true) then 'L#(x)' else 0 fi end proc:

```

On définit les fonctions intervenant dans la normalisation des paramètres f, c et b:

```
frame1 > f1 := f1 :
```

```
frame2 > f2 := f2 :
```

```
frame2 > f3 := f3 :
```

```
frame2 > f11 := f11 :
```

Le nouveau coframe initial (chech, désigné par U), s'exprime en fonction du précédent (chapeau,

désigné par V) par la relation $U:=n.V$.
où n est la matrice:

```
frame1 > n := Matrix([[1, 0, 0, 0, 0], [f3, 1, 0, 0, 0], [0, f2, 1, 0, 0], [0, 0, f1, 1, 0], [0, 0, f11, 0, 1]]) :
```

```
frame1 > ninv := MatrixInverse(n) :
```

```
frame1 > V := ninv.Vector([U[1], U[2], U[3], U[4], U[5]]) :
```

Le coframe chapeau, désigné par V, s'exprime en fonction du coframe initial, W, par la relation $V:=m.W$.

```
frame1 > m := Matrix([[ [B^-1, 0, 0, 0, 0], [0, B^-1/2, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 0, B^-1/2], [0, 0, 0, 0, B^1/2] ]]) :
```

```
frame1 > minv := MatrixInverse(m) :
```

```
frame1 > W := minv.Vector([V[1], V[2], V[3], V[4], V[5]]) :
```

On donne ensuite la matrice de groupe:

```
frame2 > Ma := Matrix([[a^4, 0, 0, 0, 0], [0, a^3, 0, 0, 0], [g, 0, a^2, 0, 0], [h, d, 0, a, 0], [0, a, 0], [k, e, 0, 0, a]]) :
```

$$Ma := \begin{bmatrix} a^4 & 0 & 0 & 0 & 0 \\ 0 & a^3 & 0 & 0 & 0 \\ g & 0 & a^2 & 0 & 0 \\ h & d & 0 & a & 0 \\ k & e & 0 & 0 & a \end{bmatrix} \quad (2)$$

```
frame1 > MaInv := MatrixInverse(Ma) :
```

On obtient la liste des formes de Maurer Cartan:

```
frame2 > t[1] := da/a
```

$$t_1 := \frac{da}{a} \quad (3)$$

```
frame1 > t[2] := -d da/a^4 + dd/a^3 :
```

```
frame1 > t[3] := -e da/a^4 + de/a^3 :
```

```
frame1 > t[4] := -2g da/a^5 + dg/a^4 :
```

```
frame1 > t[5] := -h da/a^5 + dh/a^4 :
```

```
frame1 > t[6] := -k da/a^5 + dk/a^4 :
```

```
frame1 > FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], dw, dx, dy, dz, dz1], frame2) :
```

```
frame1 > DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3], alpha[4], alpha[5],
```

alpha[6], tau, sigma, rho, zeta, $\zeta^\#$], verbose) :

The following coordinates have been protected:

[w, x, y, z, z1, a, d, e, g, h, k]

The following vector fields have been defined and protected:

[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11]

The following differential 1-forms have been defined and protected:

[$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \tau, \sigma, \rho, \zeta, \zeta^\#$]

(4)

Le coframe 'relevé' est noté Y. Il est relié au coframe de base U par la relation $Y:=Ma.U$.

frame2 > $Y := Vector([\tau, \sigma, \rho, \zeta, \zeta^\#]) :$

frame2 > $U := MaInv.Y :$

Les equations de courbure sur les $W[i]$ sont connues:

frame2 > $dW[1] := T \cdot (W[1] \wedge W[2]) + Q \cdot (W[1] \wedge W[3]) + K \cdot (W[1] \wedge W[5]) + G \cdot (W[1] \wedge W[4]) + N \cdot (W[2] \wedge W[3]) + B \cdot (W[2] \wedge W[5]) + (W[2] \wedge W[4]) :$

frame2 > $dW[2] := S \cdot (W[1] \wedge W[2]) + P \cdot (W[1] \wedge W[3]) + J \cdot (W[1] \wedge W[5]) + F \cdot (W[1] \wedge W[4]) + M \cdot (W[2] \wedge W[3]) + (L(B) + A) \cdot (W[2] \wedge W[5]) + B \cdot (W[3] \wedge W[5]) + (W[3] \wedge W[4]) :$

frame2 > $dW[3] := R \cdot (W[1] \wedge W[2]) + O \cdot (W[1] \wedge W[3]) + H \cdot (W[1] \wedge W[5]) + E \cdot (W[1] \wedge W[4]) + LL \cdot (W[2] \wedge W[3]) + L(A) \cdot (W[2] \wedge W[5]) + A \cdot (W[3] \wedge W[5]) + I \cdot (W[4] \wedge W[5]) :$

frame2 >

On en a déjà déduit les équations de courbure du coframe initial chapeau (V) à l'étape précédente:

frame2 > $dV[1] := evalDG(B^{-1} \cdot dW[1] + (Der(B^{-1}) \wedge W[1])) :$

frame2 > $dV[2] := evalDG\left(B^{-\frac{1}{2}} \cdot dW[2] + \left(Der\left(B^{-\frac{1}{2}}\right) \wedge W[2]\right)\right) :$

frame2 > $dV[3] := dW[3] :$

frame2 > $dV[4] := Der\left(B^{-\frac{1}{2}}\right) \wedge W[4] :$

frame2 > $dV[5] := Der\left(B^{\frac{1}{2}}\right) \wedge W[5] :$

Les équations de courbures du coframe U s'en déduisent:

frame2 > $dU[1] := evalDG(dV[1]) :$

frame2 > $dU[2] := evalDG(dV[2] + (Der(f3) \wedge V[1]) + f3 \cdot dV[1]) :$

frame2 > $dU[3] := evalDG(dV[3] + (Der(f2) \wedge V[2]) + f2 \cdot dV[2]) :$

frame2 > $dU[4] := evalDG(dV[4] + (Der(f1) \wedge V[3]) + f1 \cdot dV[3]) :$

frame2 > $dU[5] := evalDG(dV[5] + (Der(f11) \wedge V[3]) + f11 \cdot dV[3]) :$

On peut maintenant calculer les équations de courbure du coframe 'relevé':

frame2 > $\Omega := map(evalDG, Ma.Vector([dU[1], dU[2], dU[3], dU[4], dU[5]])) :$

frame2 > $Mat := map(evalDG, (ExteriorDerivative(Ma).MaInv)) :$

frame2 > $Mat2 := Mat \&MatrixWedge Y :$

frame2 > $SE := map(evalDG, (Mat2 \&MatrixPlus Omega)) :$

frame2 > $List := GenerateForms([\alpha[1], \alpha[2], \alpha[3], \alpha[4], \alpha[5], \alpha[6], \tau, \sigma, \rho, \zeta, \zeta^\#], 2) :$

frame2 > $Torsion := \mathbf{proc}(S, i, j) \mathbf{local} k, X; k := 11 \cdot (i - 1) - \frac{i \cdot (i - 1)}{2} + j - i; X := GetComponents(S, List); \mathbf{expand}(X[k]); \mathbf{end} \mathbf{proc} :$

frame2 > $result := \mathbf{proc}(l) \mathbf{local} k, t, X; X := 0 : t := \mathbf{expand}(GetComponents(l, List)); \mathbf{for} k \mathbf{from} 1 \mathbf{to} 55 \mathbf{do} X := X + t[k] \cdot List[k] \mathbf{od}; X; \mathbf{end} \mathbf{proc} :$

frame2 > $result(SE[1]);$

$$\begin{aligned}
& 4 \alpha_1 \wedge \tau + \left(-\frac{L^\#(B) e}{a^4 B^{3/2}} + \frac{L^\#(B) f11 f2}{a^3 B^{3/2}} - \frac{L(B) d}{a^4 \sqrt{B}} + \frac{L(B) f1 f2}{a^3 \sqrt{B}} - \frac{T(B) f2}{a^3 B} + \frac{S(B)}{a^3 \sqrt{B}} \right. \\
& + \frac{h}{a^4} - \frac{f1 g}{a^5} + \frac{f3 d}{a^4} + \frac{k}{a^4} - \frac{f11 g}{a^5} + \frac{f3 e}{a^4} + \frac{N g}{a^5 \sqrt{B}} - \frac{\sqrt{B} G d}{a^4} + \frac{\sqrt{B} G f1 f2}{a^3} \\
& \left. - \frac{K e}{a^4 \sqrt{B}} + \frac{K f11 f2}{a^3 \sqrt{B}} - \frac{Q f2}{a^3} + \frac{\sqrt{B} T}{a^3} \right) \tau \wedge \sigma + \left(-\frac{L^\#(B) f11}{a^2 B^{3/2}} - \frac{L(B) f1}{a^2 \sqrt{B}} \right. \\
& + \frac{T(B)}{a^2 B} + \frac{f3 f1}{a^2} + \frac{f3 f11}{a^2} - \frac{N f3}{a^2 \sqrt{B}} - \frac{\sqrt{B} G f1}{a^2} - \frac{K f11}{a^2 \sqrt{B}} + \frac{Q}{a^2} \left. \right) \tau \wedge \rho \\
& + \left(\frac{L(B)}{a \sqrt{B}} - \frac{f3}{a} + \frac{\sqrt{B} G}{a} \right) \tau \wedge \zeta + \left(\frac{L^\#(B)}{a B^{3/2}} - \frac{f3}{a} + \frac{K}{a \sqrt{B}} \right) \tau \wedge \zeta^\# + \left(-\frac{f1}{a} \right. \\
& \left. - \frac{f11}{a} + \frac{N}{a \sqrt{B}} \right) \sigma \wedge \rho + \sigma \wedge \zeta + \sigma \wedge \zeta^\#
\end{aligned} \tag{5}$$

frame2 > $result(SE[2]);$

$$\begin{aligned}
& 3 \alpha_1 \wedge \sigma + \left(\frac{1}{2} \frac{T(B) g}{a^6 B} - \frac{f2 h}{a^5} + \frac{\sqrt{B} L(f3) d}{a^5} + \frac{g e}{a^7} + \frac{1}{2} \frac{L^\#(B) k}{a^5 B^{3/2}} - \frac{J e}{a^5} - \frac{B F d}{a^5} \right. \\
& + \frac{M g}{a^6} + \frac{A k}{a^5 \sqrt{B}} + \frac{L(B) k}{a^5 \sqrt{B}} - \frac{f2 k}{a^5} + \frac{f3 S(B)}{a^4 \sqrt{B}} + \frac{\sqrt{B} f3 T}{a^4} + \frac{g d}{a^7} - \frac{A f11 g}{a^6 \sqrt{B}} \\
& + \frac{L(B) f3 e}{a^5 \sqrt{B}} - \frac{L(B) f11 g}{a^6 \sqrt{B}} - \frac{f2 f3 e}{a^5} - \frac{f2 f3 d}{a^5} - \frac{1}{2} \frac{L(B) f1 g}{a^6 \sqrt{B}} - \frac{1}{2} \frac{f3 L(B) d}{a^5 \sqrt{B}} \\
& - \frac{L^\#(f3) f11 f2}{a^4 \sqrt{B}} + \frac{f3 K f11 f2}{a^4 \sqrt{B}} + \frac{f3 L^\#(B) f11 f2}{a^4 B^{3/2}} + \frac{\sqrt{B} f3 G f1 f2}{a^4} + \frac{f3 h}{a^5} + \frac{B S}{a^4} \\
& - \frac{\sqrt{B} S(f3)}{a^4} - \frac{1}{2} \frac{R(B)}{a^4} - \frac{f3 Q f2}{a^4} - \frac{1}{2} \frac{L^\#(B) f11 g}{a^6 B^{3/2}} - \frac{1}{2} \frac{f3 L^\#(B) e}{a^5 B^{3/2}} + \frac{J f11 f2}{a^4} \\
& + \frac{B F f1 f2}{a^4} + \frac{A f3 e}{a^5 \sqrt{B}} + \frac{f3 N g}{a^6 \sqrt{B}} + \frac{f3 k}{a^5} + \frac{f3 L(B) f1 f2}{a^4 \sqrt{B}} - \frac{f3 f11 g}{a^6} - \frac{f3 f1 g}{a^6} \\
& \left. - \frac{f3 T(B) f2}{a^4 B} - \frac{\sqrt{B} P f2}{a^4} + \frac{f3^2 d}{a^5} - \frac{\sqrt{B} f3 G d}{a^5} + \frac{f3^2 e}{a^5} - \frac{f3 K e}{a^5 \sqrt{B}} \right)
\end{aligned} \tag{6}$$

$$\begin{aligned}
& - \frac{\sqrt{B} L(f_3) f_1 f_2}{a^4} + \frac{T(f_3) f_2}{a^4} + \frac{L^\#(f_3) e}{a^5 \sqrt{B}} + \frac{1}{2} \frac{L(B) h}{a^5 \sqrt{B}} \Big) \tau \wedge \sigma + \left(\right. \\
& - \frac{1}{2} \frac{f_3 L^\#(B) f_{11}}{a^3 B^{3/2}} - \frac{1}{2} \frac{f_3 L(B) f_1}{a^3 \sqrt{B}} + \frac{1}{2} \frac{f_3 T(B)}{a^3 B} + \frac{f_3^2 f_1}{a^3} + \frac{f_3^2 f_{11}}{a^3} - \frac{f_3^2 N}{a^3 \sqrt{B}} \\
& - \frac{\sqrt{B} f_3 G f_1}{a^3} - \frac{f_3 K f_{11}}{a^3 \sqrt{B}} + \frac{f_3 Q}{a^3} + \frac{L^\#(f_3) f_{11}}{a^3 \sqrt{B}} + \frac{\sqrt{B} L(f_3) f_1}{a^3} - \frac{T(f_3)}{a^3} + \frac{h}{a^4} \\
& + \frac{k}{a^4} + \frac{f_3 f_{11} L(B)}{a^3 \sqrt{B}} + \frac{f_3 f_{11} A}{a^3 \sqrt{B}} - \frac{M f_3}{a^3} - \frac{B F f_1}{a^3} - \frac{J f_{11}}{a^3} + \frac{\sqrt{B} P}{a^3} \Big) \tau \wedge \rho \\
& + \left(\frac{1}{2} \frac{L(B) f_3}{a^2 \sqrt{B}} - \frac{f_3^2}{a^2} + \frac{\sqrt{B} f_3 G}{a^2} - \frac{\sqrt{B} L(f_3)}{a^2} - \frac{g}{a^4} + \frac{f_3 f_2}{a^2} + \frac{B F}{a^2} \right) \tau \wedge \zeta \\
& + \left(\frac{1}{2} \frac{L^\#(B) f_3}{a^2 B^{3/2}} - \frac{f_3^2}{a^2} + \frac{f_3 K}{a^2 \sqrt{B}} - \frac{L^\#(f_3)}{a^2 \sqrt{B}} - \frac{g}{a^4} + \frac{f_3 f_2}{a^2} - \frac{L(B) f_3}{a^2 \sqrt{B}} - \frac{f_3 A}{a^2 \sqrt{B}} \right. \\
& \left. + \frac{J}{a^2} \right) \tau \wedge \zeta^\# + \left(- \frac{f_3 f_1}{a^2} - \frac{f_3 f_{11}}{a^2} + \frac{N f_3}{a^2 \sqrt{B}} - \frac{1}{2} \frac{L^\#(B) f_{11}}{a^2 B^{3/2}} - \frac{1}{2} \frac{L(B) f_1}{a^2 \sqrt{B}} \right. \\
& \left. + \frac{1}{2} \frac{T(B)}{a^2 B} + \frac{d}{a^3} + \frac{e}{a^3} - \frac{f_{11} L(B)}{a^2 \sqrt{B}} - \frac{f_{11} A}{a^2 \sqrt{B}} + \frac{M}{a^2} \right) \sigma \wedge \rho + \left(\frac{f_3}{a} + \frac{1}{2} \frac{L(B)}{a \sqrt{B}} \right. \\
& \left. - \frac{f_2}{a} \right) \sigma \wedge \zeta + \left(\frac{f_3}{a} + \frac{1}{2} \frac{L^\#(B)}{a B^{3/2}} - \frac{f_2}{a} + \frac{L(B)}{a \sqrt{B}} + \frac{A}{a \sqrt{B}} \right) \sigma \wedge \zeta^\# + \rho \wedge \zeta \\
& + \rho \wedge \zeta^\#
\end{aligned}$$

frame2 > result(SE[3]);

$$\begin{aligned}
& 2 \alpha_1 \wedge \rho + \alpha_4 \wedge \tau + \left(\frac{1}{2} \frac{f_2 L^\#(B) f_{11} g}{a^7 B^{3/2}} + \frac{1}{2} \frac{f_2 f_3 L^\#(B) e}{a^6 B^{3/2}} - \frac{f_2 A f_{11} g}{a^7 \sqrt{B}} + \frac{B^{3/2} R}{a^5} \right. \\
& + \frac{B R(f_2)}{a^5} + \frac{1}{2} \frac{f_2 L^\#(B) k}{a^6 B^{3/2}} - \frac{g L^\#(B) e}{a^8 B^{3/2}} + \frac{A g e}{a^8 \sqrt{B}} + \frac{B^{3/2} E f_1 f_2}{a^5} + \frac{\sqrt{B} H f_{11} f_2}{a^5} \\
& - \frac{\sqrt{B} g G d}{a^8} - \frac{g K e}{a^8 \sqrt{B}} + \frac{f_2 g d}{a^8} + \frac{f_2 g e}{a^8} - \frac{\sqrt{B} f_3 L(f_2) d}{a^6} + \frac{\sqrt{B} L(f_2) f_1 g}{a^7} \\
& - \frac{1}{2} \frac{f_2 T(B) g}{a^7 B} + \frac{1}{2} \frac{f_2 L(B) h}{a^6 \sqrt{B}} - \frac{f_2 J e}{a^6} - \frac{B f_2 F d}{a^6} + \frac{f_2 M g}{a^7} + \frac{f_2 L(B) k}{a^6 \sqrt{B}} \\
& + \frac{J f_{11} f_2^2}{a^5} + \frac{B F f_1 f_2^2}{a^5} - \frac{f_2^2 f_3 e}{a^6} - \frac{f_2^2 f_3 d}{a^6} - \frac{f_3 L^\#(f_2) e}{a^6 \sqrt{B}} + \frac{L^\#(f_2) f_{11} g}{a^7 \sqrt{B}} \\
& - \frac{L(A) f_{11} g}{a^7} + \frac{L(A) f_3 e}{a^6} - \frac{g L(B) d}{a^8 \sqrt{B}} + \frac{g f_3 d}{a^8} + \frac{g f_3 e}{a^8} - \frac{g Q f_2}{a^7} + \frac{I d f_{11} g}{a^8} \\
& - \frac{I f_1 g e}{a^8} + \frac{I f_1 f_2 k}{a^6} - \frac{I h f_{11} f_2}{a^6} - \frac{\sqrt{B} P f_2^2}{a^5} + \frac{\sqrt{B} L L g}{a^7} - \frac{B^{3/2} E d}{a^6} - \frac{\sqrt{B} H e}{a^6}
\end{aligned} \tag{7}$$

$$\begin{aligned}
& + \frac{\sqrt{B} g T}{a^7} - \frac{f^2 k}{a^6} - \frac{f^2 h}{a^6} + \frac{B f_2 S}{a^5} - \frac{T(f_2) g}{a^7} - \frac{L^\#(f_2) k}{a^6 \sqrt{B}} + \frac{L(A) k}{a^6} - \frac{B O f_2}{a^5} \\
& - \frac{\sqrt{B} L(f_2) h}{a^6} + \frac{g S(B)}{a^7 \sqrt{B}} + \frac{g h}{a^8} + \frac{g k}{a^8} + \frac{N g^2}{a^9 \sqrt{B}} - \frac{f_{11} g^2}{a^9} - \frac{f_1 g^2}{a^9} - \frac{1}{2} \frac{f_2 R(B)}{a^5} \\
& - \frac{I d k}{a^7} + \frac{I h e}{a^7} + \frac{\sqrt{B} g G f_1 f_2}{a^7} + \frac{g K f_{11} f_2}{a^7 \sqrt{B}} + \frac{1}{2} \frac{f_2 f_3 L(B) d}{a^6 \sqrt{B}} + \frac{1}{2} \frac{f_2 L(B) f_1 g}{a^7 \sqrt{B}} \\
& + \left. \frac{f_2 L(B) f_3 e}{a^6 \sqrt{B}} - \frac{f_2 L(B) f_{11} g}{a^7 \sqrt{B}} - \frac{I d f_{11} f_2 f_3}{a^6} + \frac{I f_1 f_2 f_3 e}{a^6} \right) \tau \wedge \sigma + \left(\frac{T(B) g}{a^6 B} \right. \\
& + \frac{f_2 h}{a^5} + \frac{B O}{a^4} + \frac{A k}{a^5 \sqrt{B}} + \frac{f_2 k}{a^5} - \frac{L(B) f_1 g}{a^6 \sqrt{B}} + \frac{1}{2} \frac{f_3 L^\#(B) f_{11} f_2}{a^4 B^{3/2}} - \frac{L^\#(B) f_{11} g}{a^6 B^{3/2}} \\
& - \frac{J f_{11} f_2}{a^4} - \frac{B F f_1 f_2}{a^4} - \frac{f_3 N g}{a^6 \sqrt{B}} + \frac{1}{2} \frac{f_3 L(B) f_1 f_2}{a^4 \sqrt{B}} + \frac{f_3 f_{11} g}{a^6} + \frac{f_3 f_1 g}{a^6} \\
& - \frac{1}{2} \frac{f_3 T(B) f_2}{a^4 B} + \frac{\sqrt{B} P f_2}{a^4} + \frac{f_{11} L(A) f_3}{a^4} - \frac{f_2 M f_3}{a^4} - \frac{f_3 L^\#(f_2) f_{11}}{\sqrt{B} a^4} \\
& - \frac{\sqrt{B} f_3 L(f_2) f_1}{a^4} - \frac{\sqrt{B} g G f_1}{a^6} - \frac{g K f_{11}}{\sqrt{B} a^6} - \frac{\sqrt{B} f_{11} H}{a^4} + \frac{g Q}{a^6} - \frac{B^{3/2} f_1 E}{a^4} \\
& - \left. \frac{\sqrt{B} L L f_3}{a^4} + \frac{f_3 T(f_2)}{a^4} + \frac{I f_{11} h}{a^5} - \frac{I f_1 k}{a^5} + \frac{f_{11} A f_2 f_3}{\sqrt{B} a^4} + \frac{f_2 f_3 f_{11} L(B)}{\sqrt{B} a^4} \right) \tau \wedge \rho \\
& + \left(-\frac{1}{2} \frac{f_2 L(B) f_3}{a^3 \sqrt{B}} - \frac{f_2 g}{a^5} + \frac{f_3 f_2^2}{a^3} + \frac{B f_2 F}{a^3} + \frac{\sqrt{B} L(f_2) f_3}{a^3} + \frac{I k}{a^4} - \frac{I f_{11} g}{a^5} \right. \\
& + \left. \frac{I f_{11} f_2 f_3}{a^3} + \frac{B^{3/2} E}{a^3} + \frac{g L(B)}{a^5 \sqrt{B}} - \frac{g f_3}{a^5} + \frac{\sqrt{B} g G}{a^5} \right) \tau \wedge \zeta + \left(-\frac{1}{2} \frac{f_2 L^\#(B) f_3}{a^3 B^{3/2}} \right. \\
& - \frac{f_2 g}{a^5} + \frac{f_3 f_2^2}{a^3} - \frac{f_2 L(B) f_3}{a^3 \sqrt{B}} + \frac{f_2 J}{a^3} + \frac{L^\#(f_2) f_3}{a^3 \sqrt{B}} - \frac{I h}{a^4} + \frac{I f_1 g}{a^5} - \frac{I f_1 f_2 f_3}{a^3} \\
& - \left. \frac{A g}{a^5 \sqrt{B}} - \frac{L(A) f_3}{a^3} + \frac{\sqrt{B} H}{a^3} + \frac{g L^\#(B)}{a^5 B^{3/2}} - \frac{g f_3}{a^5} + \frac{g K}{a^5 \sqrt{B}} \right) \tau \wedge \zeta^\# + \left(\right. \\
& - \frac{1}{2} \frac{L^\#(B) f_{11} f_2}{a^3 B^{3/2}} - \frac{1}{2} \frac{L(B) f_1 f_2}{a^3 \sqrt{B}} + \frac{1}{2} \frac{T(B) f_2}{a^3 B} + \frac{d f_2}{a^4} + \frac{f_2 e}{a^4} - \frac{f_2 f_{11} L(B)}{a^3 \sqrt{B}} \\
& - \frac{f_2 f_{11} A}{a^3 \sqrt{B}} + \frac{f_2 M}{a^3} + \frac{L^\#(f_2) f_{11}}{a^3 \sqrt{B}} + \frac{\sqrt{B} L(f_2) f_1}{a^3} - \frac{T(f_2)}{a^3} - \frac{I f_1 e}{a^4} + \frac{I f_{11} d}{a^4} \\
& + \left. \frac{A e}{a^4 \sqrt{B}} - \frac{L(A) f_{11}}{a^3} + \frac{\sqrt{B} L L}{a^3} - \frac{f_1 g}{a^5} - \frac{f_{11} g}{a^5} + \frac{N g}{a^5 \sqrt{B}} \right) \sigma \wedge \rho \\
& + \left(\frac{1}{2} \frac{f_2 L(B)}{a^2 \sqrt{B}} - \frac{f_2^2}{a^2} - \frac{\sqrt{B} L(f_2)}{a^2} + \frac{I e}{a^3} - \frac{I f_{11} f_2}{a^2} + \frac{g}{a^4} \right) \sigma \wedge \zeta
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{1}{2} \frac{f_2 L^\#(B)}{a^2 B^{3/2}} - \frac{f_2^2}{a^2} + \frac{f_2 L(B)}{a^2 \sqrt{B}} - \frac{L^\#(f_2)}{a^2 \sqrt{B}} - \frac{Id}{a^3} + \frac{If_1 f_2}{a^2} + \frac{L(A)}{a^2} \right. \\
& \left. + \frac{g}{a^4} \right) \sigma \wedge \zeta^\# + \left(\frac{f_2}{a} + \frac{If_1}{a} \right) \rho \wedge \zeta + \left(\frac{f_2}{a} - \frac{If_1}{a} + \frac{A}{a \sqrt{B}} \right) \rho \wedge \zeta^\# + I \zeta \wedge \zeta^\#
\end{aligned}$$

frame2 > result(SE[4]);

$$\begin{aligned}
& \alpha_1 \wedge \zeta + \alpha_2 \wedge \sigma + \alpha_5 \wedge \tau + \left(-\frac{1}{2} \frac{dL(B) f_1 g}{\sqrt{B} a^9} + \frac{B d F f_1 f_2}{a^7} + \frac{d J f_1 f_2}{a^7} - \frac{d f_3 Q f_2}{a^7} \right. \\
& - \frac{1}{2} \frac{d f_3 T(B) f_2}{B a^7} - \frac{d f_3 f_1 g}{a^9} - \frac{d f_3 f_1 f_1 g}{a^9} - \frac{\sqrt{B} d L(f_3) f_1 f_2}{a^7} + \frac{h L(B) f_1 f_2}{\sqrt{B} a^7} \\
& + \frac{1}{2} \frac{h L^\#(B) f_1 f_2}{B^{3/2} a^7} - \frac{d A f_1 g}{\sqrt{B} a^9} + \frac{d A f_3 e}{\sqrt{B} a^8} + \frac{d f_3 N g}{\sqrt{B} a^9} - \frac{d f_3 K e}{\sqrt{B} a^8} + \frac{\sqrt{B} h G f_1 f_2}{a^7} \\
& + \frac{h K f_1 f_2}{\sqrt{B} a^7} + \frac{1}{2} \frac{d f_3 L^\#(B) f_1 f_2}{B^{3/2} a^7} + \frac{\sqrt{B} d f_3 G f_1 f_2}{a^7} + \frac{d f_3 K f_1 f_2}{\sqrt{B} a^7} \\
& + \frac{d f_3 L(B) f_1 f_2}{\sqrt{B} a^7} + \frac{1}{2} \frac{L^\#(B) e f_1 f_2 f_3}{B^{3/2} a^7} - \frac{f_1 A f_2 f_3 e}{\sqrt{B} a^7} - \frac{I f_1 d f_1 f_2 f_3}{a^7} + \frac{e g d}{a^{10}} \\
& - \frac{d J e}{a^8} + \frac{1}{2} \frac{d f_3 S(B)}{\sqrt{B} a^7} + \frac{2 d f_3 h}{a^8} - \frac{1}{2} \frac{f_3 L(B) d^2}{\sqrt{B} a^8} - \frac{f_2 f_3 d^2}{a^8} - \frac{1}{2} \frac{h T(B) f_2}{B a^7} \\
& - \frac{h f_1 g}{a^9} - \frac{h f_1 f_1 g}{a^9} - \frac{h Q f_2}{a^7} - \frac{\sqrt{B} d P f_2}{a^7} + \frac{d A k}{\sqrt{B} a^8} - \frac{\sqrt{B} f_3 G d^2}{a^8} + \frac{\sqrt{B} d f_3 T}{a^7} \\
& + \frac{h N g}{\sqrt{B} a^9} - \frac{\sqrt{B} h G d}{a^8} - \frac{h K e}{\sqrt{B} a^8} - \frac{1}{2} \frac{d L(B) h}{\sqrt{B} a^8} + \frac{d L^\#(f_3) e}{\sqrt{B} a^8} + \frac{d T(f_3) f_2}{a^7} \\
& - \frac{d f_2 h}{a^8} - \frac{d f_2 k}{a^8} + \frac{d M g}{a^9} - \frac{1}{2} \frac{h L^\#(B) e}{B^{3/2} a^8} + \frac{\sqrt{B} f_1 L L g}{a^8} - \frac{B^{3/2} f_1 E d}{a^7} \\
& + \frac{B^{3/2} f_1^2 E f_2}{a^6} - \frac{\sqrt{B} f_1 H e}{a^7} - \frac{1}{2} \frac{R(B) f_1 f_2}{a^6} + \frac{1}{2} \frac{S(B) f_1 g}{\sqrt{B} a^8} + \frac{f_1 L(A) k}{a^7} \\
& - \frac{B f_1 O f_2}{a^6} + \frac{\sqrt{B} f_2 L(f_1) h}{a^7} + \frac{f_2 L^\#(f_1) k}{\sqrt{B} a^7} - \frac{\sqrt{B} L(f_1) d g}{a^9} - \frac{L^\#(f_1) e g}{\sqrt{B} a^9} \\
& + \frac{e f_3 h}{a^8} + \frac{e f_3^2 d}{a^8} + \frac{k L(B) d}{\sqrt{B} a^8} + \frac{k f_3 d}{a^8} - \frac{I f_1^2 g e}{a^9} + \frac{I f_1^2 f_2 k}{a^7} + \frac{I f_1 h e}{a^8} \\
& - \frac{I f_1 d k}{a^8} + \frac{g d^2}{a^{10}} + \frac{k h}{a^8} + \frac{\sqrt{B} h T}{a^7} + \frac{\sqrt{B} L(f_3) d^2}{a^8} - \frac{B F d^2}{a^8} + \frac{f_3^2 d^2}{a^8} \\
& - \frac{\sqrt{B} d S(f_3)}{a^7} + \frac{B d S}{a^7} + \frac{1}{2} \frac{h S(B)}{\sqrt{B} a^7} + \frac{B^{3/2} f_1 R}{a^6} - \frac{B f_2 R(f_1)}{a^6} + \frac{\sqrt{B} S(f_1) g}{a^8} \\
& - \frac{d L^\#(f_3) f_1 f_2}{\sqrt{B} a^7} - \frac{1}{2} \frac{d f_3 L^\#(B) e}{B^{3/2} a^8} + \frac{e f_3 L(B) d}{\sqrt{B} a^8} - \frac{e f_2 f_3 d}{a^8} - \frac{f_1 A f_2 k}{\sqrt{B} a^7}
\end{aligned} \tag{8}$$

$$\begin{aligned}
& + \frac{\sqrt{B} fl H fl f2}{a^6} + \frac{fl A g e}{\sqrt{B} a^9} + \frac{\sqrt{B} L(fl) d f2 f3}{a^7} - \frac{fl L(A) fl f1 g}{a^8} + \frac{fl L(A) f3 e}{a^7} \\
& + \frac{L^\#(fl) e f2 f3}{\sqrt{B} a^7} - \frac{L(B) d fl f1 g}{\sqrt{B} a^9} + \frac{1}{2} \frac{L^\#(B) k fl f2}{B^{3/2} a^7} - \frac{1}{2} \frac{L^\#(B) e fl g}{B^{3/2} a^9} \\
& - \frac{fl h fl f1 f2}{a^7} + \frac{fl d fl f1 g}{a^9} + \frac{fl^2 f2 f3 e}{a^7} + \frac{h^2}{a^8} \Big) \tau \wedge \sigma + \left(-\frac{1}{2} \frac{h L^\#(B) fl f1}{B^{3/2} a^6} \right. \\
& - \frac{1}{2} \frac{fl L^\#(B) k}{B^{3/2} a^6} + \frac{fl A k}{\sqrt{B} a^6} - \frac{\sqrt{B} fl LL f3}{a^5} - \frac{\sqrt{B} fl H fl f1}{a^5} - \frac{1}{2} \frac{fl S(B) f3}{\sqrt{B} a^5} \\
& + \frac{fl f1 h}{a^6} + \frac{h f3 fl}{a^6} + \frac{h f3 fl f1}{a^6} - \frac{d M f3}{a^6} - \frac{B d F fl}{a^6} - \frac{d J fl f1}{a^6} - \frac{h L(B) fl}{\sqrt{B} a^6} \\
& - \frac{d f3^2 N}{\sqrt{B} a^6} - \frac{h N f3}{\sqrt{B} a^6} - \frac{\sqrt{B} h G fl}{a^6} - \frac{h K fl f1}{\sqrt{B} a^6} + \frac{1}{2} \frac{d f3 T(B)}{B a^6} + \frac{d f3^2 fl}{a^6} \\
& + \frac{d f3^2 fl f1}{a^6} + \frac{d f3 Q}{a^6} + \frac{d L^\#(f3) fl f1}{\sqrt{B} a^6} + \frac{\sqrt{B} d L(f3) fl}{a^6} + \frac{\sqrt{B} d P}{a^6} - \frac{d T(f3)}{a^6} \\
& + \frac{1}{2} \frac{h T(B)}{B a^6} + \frac{h Q}{a^6} + \frac{d h}{a^7} + \frac{d k}{a^7} - \frac{B^{3/2} fl^2 E}{a^5} + \frac{B fl O}{a^5} - \frac{L^\#(fl) k}{\sqrt{B} a^6} \\
& - \frac{\sqrt{B} L(fl) h}{a^6} - \frac{\sqrt{B} S(fl) f3}{a^5} + \frac{1}{2} \frac{fl R(B)}{a^5} - \frac{fl^2 k}{a^6} - \frac{1}{2} \frac{d f3 L^\#(B) fl f1}{B^{3/2} a^6} \\
& + \frac{fl L(A) f3 fl f1}{a^5} - \frac{\sqrt{B} d f3 G fl}{a^6} - \frac{d f3 K fl f1}{\sqrt{B} a^6} + \frac{d f3 fl f1 A}{\sqrt{B} a^6} - \frac{1}{2} \frac{d f3 L(B) fl}{\sqrt{B} a^6} \\
& + \frac{d f3 fl f1 L(B)}{\sqrt{B} a^6} + \frac{B R(fl)}{a^5} \Big) \tau \wedge \rho + \left(\frac{fl f1 f1 f2 f3}{a^4} + \frac{fl k}{a^5} - \frac{fl f1 f1 g}{a^6} \right. \\
& + \frac{B^{3/2} fl E}{a^4} + \frac{\sqrt{B} L(fl) g}{a^6} - \frac{\sqrt{B} L(fl) f2 f3}{a^4} + \frac{1}{2} \frac{L^\#(B) k}{a^5 B^{3/2}} - \frac{1}{2} \frac{L^\#(B) fl f1 g}{a^6 B^{3/2}} \\
& + \frac{1}{2} \frac{f3 L^\#(B) fl f1 f2}{a^4 B^{3/2}} + \frac{1}{2} \frac{T(B) g}{a^6 B} - \frac{1}{2} \frac{f3 T(B) f2}{a^4 B} + \frac{1}{2} \frac{f3 S(B)}{a^4 \sqrt{B}} - \frac{1}{2} \frac{R(B)}{a^4} \\
& + \frac{1}{2} \frac{f3 L(B) d}{a^5 \sqrt{B}} - \frac{f3^2 d}{a^5} + \frac{\sqrt{B} f3 G d}{a^5} - \frac{\sqrt{B} L(f3) d}{a^5} - \frac{g d}{a^7} + \frac{f2 f3 d}{a^5} + \frac{B F d}{a^5} \\
& + \frac{L(B) h}{a^5 \sqrt{B}} - \frac{f3 h}{a^5} + \frac{\sqrt{B} h G}{a^5} \Big) \tau \wedge \zeta + \left(-\frac{fl h}{a^5} + \frac{fl^2 g}{a^6} - \frac{fl^2 f2 f3}{a^4} - \frac{fl A g}{\sqrt{B} a^6} \right. \\
& + \frac{fl A f2 f3}{\sqrt{B} a^4} - \frac{fl L(A) f3}{a^4} + \frac{\sqrt{B} fl H}{a^4} + \frac{L^\#(fl) g}{\sqrt{B} a^6} - \frac{L^\#(fl) f2 f3}{\sqrt{B} a^4} + \frac{1}{2} \frac{h L^\#(B)}{B^{3/2} a^5} \\
& + \frac{1}{2} \frac{L^\#(B) fl g}{B^{3/2} a^6} - \frac{1}{2} \frac{L^\#(B) fl f2 f3}{B^{3/2} a^4} + \frac{1}{2} \frac{d L^\#(B) f3}{B^{3/2} a^5} - \frac{f3^2 d}{a^5} + \frac{d f3 K}{\sqrt{B} a^5}
\end{aligned}$$

$$\begin{aligned}
& - \frac{dL^\#(f3)}{\sqrt{B} a^5} - \frac{g d}{a^7} + \frac{f2 f3 d}{a^5} - \frac{f3 L(B) d}{a^5 \sqrt{B}} - \frac{d f3 A}{\sqrt{B} a^5} + \frac{d J}{a^5} - \frac{f3 h}{a^5} + \frac{h K}{\sqrt{B} a^5} \Big) \\
\tau \wedge \zeta^\# & + \left(\frac{d f3 N}{\sqrt{B} a^5} - \frac{d f11 A}{\sqrt{B} a^5} - \frac{d f3 f1}{a^5} - \frac{d f3 f11}{a^5} - \frac{1}{2} \frac{d L(B) f1}{\sqrt{B} a^5} - \frac{f11 L(B) d}{\sqrt{B} a^5} \right. \\
& + \frac{d^2}{a^6} + \frac{\sqrt{B} S(f1)}{a^4} + \frac{e d}{a^6} + \frac{h N}{\sqrt{B} a^5} - \frac{1 f1^2 e}{a^5} + \frac{d M}{a^5} - \frac{f1 h}{a^5} - \frac{h f11}{a^5} \\
& + \frac{\sqrt{B} f1 L L}{a^4} - \frac{L^\#(f1) e}{\sqrt{B} a^5} - \frac{\sqrt{B} L(f1) d}{a^5} - \frac{1}{2} \frac{f1 L^\#(B) e}{B^{3/2} a^5} + \frac{1}{2} \frac{f1 S(B)}{\sqrt{B} a^4} \\
& + \frac{1 f1 f11 d}{a^5} + \frac{f1 A e}{\sqrt{B} a^5} - \frac{f1 L(A) f11}{a^4} \Big) \sigma \wedge \rho + \left(\frac{1 f1 e}{a^4} - \frac{1 f1 f11 f2}{a^3} \right. \\
& + \frac{\sqrt{B} L(f1) f2}{a^3} + \frac{1}{2} \frac{L^\#(B) e}{a^4 B^{3/2}} - \frac{1}{2} \frac{L^\#(B) f11 f2}{a^3 B^{3/2}} + \frac{1}{2} \frac{T(B) f2}{a^3 B} - \frac{1}{2} \frac{S(B)}{a^3 \sqrt{B}} \\
& + \frac{f3 d}{a^4} + \frac{1}{2} \frac{L(B) d}{a^4 \sqrt{B}} - \frac{d f2}{a^4} + \frac{h}{a^4} \Big) \sigma \wedge \zeta + \left(- \frac{1 f1 d}{a^4} + \frac{1 f1^2 f2}{a^3} - \frac{f1 A f2}{\sqrt{B} a^3} \right. \\
& + \frac{f1 L(A)}{a^3} + \frac{L^\#(f1) f2}{\sqrt{B} a^3} + \frac{1}{2} \frac{L^\#(B) f1 f2}{B^{3/2} a^3} + \frac{f3 d}{a^4} - \frac{d f2}{a^4} + \frac{L(B) d}{a^4 \sqrt{B}} + \frac{d A}{\sqrt{B} a^4} \\
& + \frac{h}{a^4} \Big) \sigma \wedge \zeta^\# + \left(\frac{1 f11 f1}{a^2} - \frac{\sqrt{B} L(f1)}{a^2} + \frac{1}{2} \frac{L^\#(B) f11}{a^2 B^{3/2}} - \frac{1}{2} \frac{T(B)}{a^2 B} + \frac{d}{a^3} \right) \rho \wedge \zeta \\
& + \left(- \frac{1 f1^2}{a^2} + \frac{f1 A}{a^2 \sqrt{B}} - \frac{L^\#(f1)}{a^2 \sqrt{B}} - \frac{1}{2} \frac{L^\#(B) f1}{a^2 B^{3/2}} + \frac{d}{a^3} \right) \rho \wedge \zeta^\# + \left(\frac{1 f1}{a} \right. \\
& \left. + \frac{1}{2} \frac{L^\#(B)}{a B^{3/2}} \right) \zeta \wedge \zeta^\#
\end{aligned}$$

frame2 > result(SE[5]);

$$\begin{aligned}
\alpha_1 \wedge \zeta^\# + \alpha_3 \wedge \sigma + \alpha_6 \wedge \tau & + \left(\frac{e f3 N g}{\sqrt{B} a^9} - \frac{\sqrt{B} e f3 G d}{a^8} + \frac{\sqrt{B} k G f1 f2}{a^7} + \frac{k K f11 f2}{\sqrt{B} a^7} \right. \\
& - \frac{e L^\#(f3) f11 f2}{\sqrt{B} a^7} - \frac{e L(B) f1 g}{\sqrt{B} a^9} - \frac{e L(B) f11 g}{\sqrt{B} a^9} + \frac{B e F f1 f2}{a^7} + \frac{e J f11 f2}{a^7} \\
& - \frac{e f3 Q f2}{a^7} - \frac{3}{2} \frac{e f3 T(B) f2}{B a^7} - \frac{e f3 f1 g}{a^9} - \frac{e f3 f11 g}{a^9} - \frac{\sqrt{B} e L(f3) f1 f2}{a^7} \\
& + \frac{3}{2} \frac{k L(B) f1 f2}{\sqrt{B} a^7} - \frac{f11 A f2 k}{\sqrt{B} a^7} + \frac{B^{3/2} f11 E f1 f2}{a^6} + \frac{f11 L(A) f3 e}{a^7} + \frac{L^\#(f11) e f2 f3}{\sqrt{B} a^7} \\
& - \frac{1}{2} \frac{L(B) h f11 f2}{\sqrt{B} a^7} + \frac{\sqrt{B} L(f11) d f2 f3}{a^7} - \frac{1}{2} \frac{e L^\#(B) f11 g}{B^{3/2} a^9} + \frac{k L^\#(B) f11 f2}{B^{3/2} a^7} \\
& + \frac{1 f11 f1 f2 k}{a^7} - \frac{1 f11 f1 g e}{a^9} - \frac{1 f11^2 d f2 f3}{a^7} + \frac{\sqrt{B} k T}{a^7} + \frac{k^2}{a^8} - \frac{f11 A f2 f3 e}{\sqrt{B} a^7}
\end{aligned}$$

(9)

$$\begin{aligned}
& -\frac{1}{2} \frac{L(B) d f11 f2 f3}{\sqrt{B} a^7} + \frac{e f3 L^\#(B) f11 f2}{B^{3/2} a^7} + \frac{1 f11 f1 f2 f3 e}{a^7} - \frac{1}{2} \frac{f3 L^\#(B) e^2}{B^{3/2} a^8} \\
& -\frac{1}{2} \frac{e L^\#(B) k}{B^{3/2} a^8} + \frac{\sqrt{B} f11 L L g}{a^8} - \frac{B^{3/2} f11 E d}{a^7} - \frac{\sqrt{B} f11 H e}{a^7} + \frac{\sqrt{B} f11^2 H f2}{a^6} \\
& -\frac{L^\#(f11) e g}{\sqrt{B} a^9} - \frac{\sqrt{B} L(f11) d g}{a^9} + \frac{\sqrt{B} f2 L(f11) h}{a^7} + \frac{f2 L^\#(f11) k}{\sqrt{B} a^7} \\
& + \frac{1}{2} \frac{R(B) f11 f2}{a^6} - \frac{1}{2} \frac{S(B) f11 g}{\sqrt{B} a^8} + \frac{f11 L(A) k}{a^7} - \frac{f11^2 L(A) g}{a^8} - \frac{B f11 O f2}{a^6} \\
& + \frac{A f3 e^2}{\sqrt{B} a^8} - \frac{\sqrt{B} e P f2}{a^7} + \frac{e A k}{\sqrt{B} a^8} - \frac{f3 K e^2}{\sqrt{B} a^8} + \frac{\sqrt{B} e f3 T}{a^7} + \frac{k N g}{\sqrt{B} a^9} - \frac{\sqrt{B} k G d}{a^8} \\
& -\frac{k K e}{\sqrt{B} a^8} - \frac{f2 f3 e^2}{a^8} + \frac{L(B) f3 e^2}{\sqrt{B} a^8} + \frac{e L(B) h}{\sqrt{B} a^8} + \frac{e T(B) g}{B a^9} + \frac{e T(f3) f2}{a^7} \\
& + \frac{\sqrt{B} e L(f3) d}{a^8} - \frac{e f2 h}{a^8} - \frac{e f2 k}{a^8} + \frac{e L(B) k}{\sqrt{B} a^8} + \frac{e M g}{a^9} - \frac{B e F d}{a^8} + \frac{3}{2} \frac{e f3 S(B)}{\sqrt{B} a^7} \\
& + \frac{2 e f3 k}{a^8} - \frac{3}{2} \frac{k T(B) f2}{B a^7} - \frac{k f1 g}{a^9} - \frac{k f11 g}{a^9} - \frac{k Q f2}{a^7} - \frac{1 f11 d k}{a^8} - \frac{1 f11^2 h f2}{a^7} \\
& + \frac{1 f11 h e}{a^8} + \frac{1 f11^2 d g}{a^9} + \frac{e g d}{a^{10}} + \frac{e f3 h}{a^8} + \frac{e f3^2 d}{a^8} - \frac{3}{2} \frac{k L(B) d}{\sqrt{B} a^8} + \frac{k f3 d}{a^8} + \frac{k h}{a^8} \\
& + \frac{\sqrt{B} e f3 G f1 f2}{a^7} + \frac{e f3 K f11 f2}{\sqrt{B} a^7} + \frac{3}{2} \frac{e f3 L(B) f1 f2}{\sqrt{B} a^7} + \frac{L^\#(f3) e^2}{\sqrt{B} a^8} - \frac{J e^2}{a^8} + \frac{f3^2 e^2}{a^8} \\
& -\frac{\sqrt{B} e S(f3)}{a^7} - \frac{e R(B)}{a^7} + \frac{B e S}{a^7} + \frac{3}{2} \frac{k S(B)}{\sqrt{B} a^7} + \frac{B^{3/2} f11 R}{a^6} - \frac{B f2 R(f11)}{a^6} \\
& + \frac{\sqrt{B} S(f11) g}{a^8} + \frac{g e^2}{a^{10}} - \frac{1}{2} \frac{e f3 L(B) d}{\sqrt{B} a^8} - \frac{e f2 f3 d}{a^8} + \frac{1}{2} \frac{L(B) d f11 g}{\sqrt{B} a^9} \Big) \tau \wedge \sigma \\
& + \left(-\frac{\sqrt{B} e f3 G f1}{a^6} - \frac{e f3 K f11}{a^6 \sqrt{B}} + \frac{e f3 f11 A}{a^6 \sqrt{B}} - \frac{1}{2} \frac{e f3 L(B) f1}{a^6 \sqrt{B}} + \frac{e f3 f11 L(B)}{a^6 \sqrt{B}} \right. \\
& -\frac{1}{2} \frac{e f3 L^\#(B) f11}{a^6 B^{3/2}} + \frac{B R(f11)}{a^5} + \frac{f11 A k}{a^6 \sqrt{B}} - \frac{\sqrt{B} f11 L L f3}{a^5} - \frac{B^{3/2} f11 E f1}{a^5} \\
& + \frac{f11^2 L(A) f3}{a^5} + \frac{1}{2} \frac{f11 L(B) h}{a^6 \sqrt{B}} + \frac{1}{2} \frac{f11 S(B) f3}{a^5 \sqrt{B}} - \frac{k L^\#(B) f11}{a^6 B^{3/2}} - \frac{e f3^2 N}{a^6 \sqrt{B}} \\
& -\frac{k N f3}{a^6 \sqrt{B}} - \frac{\sqrt{B} k G f1}{a^6} - \frac{k K f11}{a^6 \sqrt{B}} + \frac{1}{2} \frac{e f3 T(B)}{a^6 B} + \frac{e f3^2 f1}{a^6} + \frac{e f3^2 f11}{a^6} + \frac{e f3 Q}{a^6} \\
& + \frac{e L^\#(f3) f11}{a^6 \sqrt{B}} + \frac{\sqrt{B} e L(f3) f1}{a^6} - \frac{e M f3}{a^6} - \frac{B e F f1}{a^6} - \frac{e J f11}{a^6} - \frac{3}{2} \frac{k L(B) f1}{a^6 \sqrt{B}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{kf_3 f_1}{a^6} + \frac{kf_3 f_{11}}{a^6} - \frac{1f_{11} f_1 k}{a^6} + \frac{\sqrt{B} e P}{a^6} - \frac{e T(f_3)}{a^6} + \frac{3}{2} \frac{k T(B)}{a^6 B} + \frac{k Q}{a^6} \\
& - \frac{\sqrt{B} f_1 l^2 H}{a^5} + \frac{B f_{11} O}{a^5} - \frac{L^\#(f_{11}) k}{a^6 \sqrt{B}} - \frac{\sqrt{B} L(f_{11}) h}{a^6} - \frac{\sqrt{B} S(f_{11}) f_3}{a^5} \\
& - \frac{1}{2} \frac{f_{11} R(B)}{a^5} + \frac{e h}{a^7} + \frac{e k}{a^7} + \frac{1f_1 l^2 h}{a^6} \Big) \tau \wedge \rho + \left(\frac{1f_{11} k}{a^5} + \frac{1f_1 l^2 f_2 f_3}{a^4} - \frac{1f_1 l^2 g}{a^6} \right. \\
& + \frac{B^{3/2} f_{11} E}{a^4} + \frac{\sqrt{B} L(f_{11}) g}{a^6} - \frac{\sqrt{B} L(f_{11}) f_2 f_3}{a^4} + \frac{3}{2} \frac{L(B) k}{a^5 \sqrt{B}} - \frac{1}{2} \frac{L(B) f_{11} g}{a^6 \sqrt{B}} \\
& + \frac{1}{2} \frac{f_2 f_3 f_{11} L(B)}{\sqrt{B} a^4} + \frac{1}{2} \frac{L(B) f_3 e}{a^5 \sqrt{B}} - \frac{f_3^2 e}{a^5} + \frac{\sqrt{B} e f_3 G}{a^5} - \frac{\sqrt{B} e L(f_3)}{a^5} - \frac{g e}{a^7} \\
& + \left. \frac{f_2 f_3 e}{a^5} + \frac{B e F}{a^5} - \frac{f_3 k}{a^5} + \frac{\sqrt{B} k G}{a^5} \right) \tau \wedge \zeta + \left(-\frac{1}{2} \frac{T(B) g}{a^6 B} - \frac{1f_{11} f_1 f_2 f_3}{a^4} \right. \\
& + \frac{k K}{a^5 \sqrt{B}} + \frac{L^\#(f_{11}) g}{a^6 \sqrt{B}} - \frac{1f_{11} h}{a^5} - \frac{L^\#(f_{11}) f_2 f_3}{a^4 \sqrt{B}} + \frac{1f_{11} f_1 g}{a^6} - \frac{g e}{a^7} + \frac{L^\#(B) k}{a^5 B^{3/2}} \\
& + \frac{J e}{a^5} - \frac{1}{2} \frac{f_3 S(B)}{a^4 \sqrt{B}} - \frac{A f_{11} g}{a^6 \sqrt{B}} - \frac{L(B) f_3 e}{a^5 \sqrt{B}} + \frac{f_2 f_3 e}{a^5} + \frac{1}{2} \frac{L(B) f_1 g}{a^6 \sqrt{B}} + \frac{1}{2} \frac{R(B)}{a^4} \\
& + \frac{1}{2} \frac{f_3 L^\#(B) e}{a^5 B^{3/2}} - \frac{A f_3 e}{a^5 \sqrt{B}} - \frac{f_3 k}{a^5} - \frac{1}{2} \frac{f_3 L(B) f_1 f_2}{a^4 \sqrt{B}} + \frac{1}{2} \frac{f_3 T(B) f_2}{a^4 B} - \frac{f_3^2 e}{a^5} \\
& + \left. \frac{f_3 K e}{a^5 \sqrt{B}} - \frac{L^\#(f_3) e}{a^5 \sqrt{B}} - \frac{f_{11} L(A) f_3}{a^4} + \frac{\sqrt{B} f_{11} H}{a^4} - \frac{1}{2} \frac{L(B) h}{a^5 \sqrt{B}} + \frac{f_{11} A f_2 f_3}{\sqrt{B} a^4} \right) \\
& \tau \wedge \zeta^\# + \left(-\frac{e f_{11} L(B)}{\sqrt{B} a^5} + \frac{e^2}{a^6} + \frac{e f_3 N}{\sqrt{B} a^5} - \frac{e f_3 f_1}{a^5} - \frac{e f_3 f_{11}}{a^5} - \frac{e L(B) f_1}{\sqrt{B} a^5} \right. \\
& + \frac{e T(B)}{B a^5} + \frac{e M}{a^5} - \frac{k f_1}{a^5} - \frac{1}{2} \frac{e L^\#(B) f_{11}}{B^{3/2} a^5} - \frac{1f_{11} f_1 e}{a^5} + \frac{1}{2} \frac{f_{11} L(B) d}{\sqrt{B} a^5} + \frac{1f_1 l^2 d}{a^5} \\
& + \frac{\sqrt{B} S(f_{11})}{a^4} + \frac{k N}{\sqrt{B} a^5} + \frac{e d}{a^6} - \frac{f_{11} k}{a^5} - \frac{f_1 l^2 L(A)}{a^4} + \frac{\sqrt{B} f_{11} L L}{a^4} - \frac{L^\#(f_{11}) e}{\sqrt{B} a^5} \\
& - \left. \frac{\sqrt{B} L(f_{11}) d}{a^5} - \frac{1}{2} \frac{f_{11} S(B)}{\sqrt{B} a^4} \right) \sigma \wedge \rho + \left(\frac{1f_{11} e}{a^4} - \frac{1f_1 l^2 f_2}{a^3} + \frac{\sqrt{B} L(f_{11}) f_2}{a^3} \right. \\
& + \frac{L(B) e}{\sqrt{B} a^4} - \frac{1}{2} \frac{f_2 f_{11} L(B)}{a^3 \sqrt{B}} + \frac{e f_3}{a^4} - \frac{f_2 e}{a^4} + \frac{k}{a^4} \Big) \sigma \wedge \zeta + \left(-\frac{1f_{11} d}{a^4} + \frac{1f_1 f_{11} f_2}{a^3} \right. \\
& - \frac{f_2 f_{11} A}{a^3 \sqrt{B}} + \frac{L(A) f_{11}}{a^3} + \frac{L^\#(f_{11}) f_2}{\sqrt{B} a^3} - \frac{1}{2} \frac{L(B) d}{a^4 \sqrt{B}} + \frac{1}{2} \frac{L(B) f_1 f_2}{a^3 \sqrt{B}} - \frac{1}{2} \frac{T(B) f_2}{a^3 B} \\
& + \left. \frac{1}{2} \frac{S(B)}{a^3 \sqrt{B}} + \frac{e f_3}{a^4} + \frac{1}{2} \frac{L^\#(B) e}{a^4 B^{3/2}} - \frac{f_2 e}{a^4} + \frac{L(B) e}{\sqrt{B} a^4} + \frac{A e}{a^4 \sqrt{B}} + \frac{k}{a^4} \right) \sigma \wedge \zeta^\#
\end{aligned}$$

$$+ \left(\frac{1fl^2}{a^2} - \frac{\sqrt{B} L(fl)}{a^2} + \frac{1}{2} \frac{flL(B)}{a^2\sqrt{B}} + \frac{e}{a^3} \right) \rho \wedge \zeta + \left(-\frac{1flfl}{a^2} + \frac{flA}{a^2\sqrt{B}} \right. \\ \left. - \frac{L^\#(fl)}{\sqrt{B}a^2} - \frac{1}{2} \frac{L(B)fl}{a^2\sqrt{B}} + \frac{1}{2} \frac{\Gamma(B)}{a^2B} + \frac{e}{a^3} \right) \rho \wedge \zeta^\# + \left(\frac{1fl}{a} + \frac{1}{2} \frac{L(B)}{a\sqrt{B}} \right) \zeta \wedge \zeta^\#$$

frame2 > *Torsion*(SE[2], 7, 10);

$$\frac{1}{2} \frac{L(B)f3}{a^2\sqrt{B}} - \frac{f3^2}{a^2} + \frac{\sqrt{B}f3G}{a^2} - \frac{\sqrt{B}L(f3)}{a^2} - \frac{g}{a^4} + \frac{f3f2}{a^2} + \frac{BF}{a^2} \quad (10)$$

frame2 > *Torsion*(SE[2], 7, 11);

$$\frac{1}{2} \frac{L^\#(B)f3}{a^2B^{3/2}} - \frac{f3^2}{a^2} + \frac{f3K}{a^2\sqrt{B}} - \frac{L^\#(f3)}{a^2\sqrt{B}} - \frac{g}{a^4} + \frac{f3f2}{a^2} - \frac{L(B)f3}{a^2\sqrt{B}} - \frac{f3A}{a^2\sqrt{B}} + \frac{J}{a^2} \quad (11)$$

frame2 > *gg := expand(solve(Torsion(SE[2], 7, 10), g));*

$$gg := \frac{1}{2} \frac{a^2L(B)f3}{\sqrt{B}} - a^2f3^2 + \sqrt{B}a^2f3G - \sqrt{B}L(f3)a^2 + f2f3a^2 + BFa^2 \quad (12)$$

frame2 > *Torsion*(SE[4], 9, 11); $\frac{1}{2} \frac{a^2L(B)f3}{\sqrt{B}} - a^2f3^2 + a^2\sqrt{B}f3G$

$$- a^2\sqrt{B}L(f3) + f2f3a^2 + a^2 \\ - \frac{1fl^2}{a^2} + \frac{flA}{a^2\sqrt{B}} - \frac{L^\#(fl)}{a^2\sqrt{B}} - \frac{1}{2} \frac{L^\#(B)fl}{a^2B^{3/2}} + \frac{d}{a^3}$$

$$\frac{1}{2} \frac{a^2L(B)f3}{\sqrt{B}} - a^2f3^2 + \sqrt{B}a^2f3G - \sqrt{B}L(f3)a^2 + f2f3a^2 + a^2 \quad (13)$$

frame2 > *expand(solve(Torsion(SE[4], 9, 11), d));*

$$1afl^2 - \frac{aflA}{\sqrt{B}} + \frac{aL^\#(fl)}{\sqrt{B}} + \frac{1}{2} \frac{aL^\#(B)fl}{B^{3/2}} \quad (14)$$

frame2 > *Torsion*(SE[5], 9, 10);

$$\frac{1fl^2}{a^2} - \frac{\sqrt{B}L(fl)}{a^2} + \frac{1}{2} \frac{flL(B)}{a^2\sqrt{B}} + \frac{e}{a^3} \quad (15)$$

frame2 > *expand(solve(Torsion(SE[5], 9, 10), e));*

$$-1afl^2 + \sqrt{B}aL(fl) - \frac{1}{2} \frac{aflL(B)}{\sqrt{B}} \quad (16)$$

frame2 > *Torsion*(SE[1], 7, 10);

$$\frac{L(B)}{a\sqrt{B}} - \frac{f3}{a} + \frac{\sqrt{B}G}{a} \quad (17)$$

frame2 > *Torsion*(SE[1], 7, 11);

$$\frac{L^\#(B)}{aB^{3/2}} - \frac{f3}{a} + \frac{K}{a\sqrt{B}} \quad (18)$$

frame2 > $f3 := \frac{3}{5} \frac{(L(B) + GB)}{\sqrt{B}} ;$

frame2 > *Torsion*(SE[1], 7, 10);

$$\frac{2}{5} \frac{L(B)}{a\sqrt{B}} + \frac{2}{5} \frac{\sqrt{B} G}{a} \quad (19)$$

frame2 > $\text{conjugue}\left(f = \frac{3}{5} \frac{a^3 (L(B) + GB)}{\sqrt{B}}\right);$

$$\text{conjugue}\left(f = \frac{3}{5} \frac{a^3 (L(B) + GB)}{\sqrt{B}}\right) \quad (20)$$

frame2 >