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> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([w, x, y, z1, z], [a, a1, b, b1, d, d1, e, e1], frame1, verbose);
    The following coordinates have been protected:
        [w, x, y, z1, z, a, a1, b, b1, d, d1, e, e1]
    The following vector fields have been defined and protected:
[D_w, D_x, D_y, D_z1, D_z, D_a, D_a1, D_b, D_b1, D_d, D_d1, D_e, D_e1]
    The following differential 1-forms have been defined and protected:
        [dw, dx, dy, dz1, dz, da, da1, db, db1, dd, dd1, de, de1]
        frame name: frame1

```

(1)

Par rapport à la session précédente, $e := e - b \cdot B1$, $d := d - b1 \cdot B$. On a normalisé $c := a \cdot a1 \cdot B1$.

```

frame1 > Der := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true)
    then add(Der(op(i, x)), i = 1 .. nops(x)) elif
        (type(x, `*`) = true) then expand( $\frac{x}{y} \cdot Der(y) + y$ 
        ·Der( $\frac{x}{y}$ )) elif
        (type(x, `^`) = true) then op(2, x) · y(op(2, x) - 1)
        ·Der(y) elif
        ((type(x, function) = true) or (type(x, symbol)
        = true)) then S#(x) · W[1] + S(x) · W[2] + Tau(x) · W[3] + L#(x)
        · W[4] + L(x) · W[5]
    else 0 fi end proc:

```

On exprime le nouveau coframe initial (chapeau) en fonction du premier.

W represente le premier coframe initial et V represente le nouveau coframe initial (chapeau). On a la relation $V := m \cdot W$.

```

frame1 > m := Matrix([[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [B, B1, 1, 0, 0], [0, 0, 0, 1,
    0], [0, 0, 0, 0, 1]]):

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> minv := MatrixInverse(m) :

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M > W := minv.Vector([V[1], V[2], V[3], V[4], V[5]]):

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On donne ensuite la nouvelle matrice de groupe:

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frame1 > Ma := Matrix([[a·a12, 0, 0, 0, 0], [0, a2·a1, 0, 0, 0], [0, 0, a·a1, 0, 0],
    [e1, d, b1, a1, 0], [d1, e, b, 0, a]])

```

:

$$Ma := \begin{bmatrix} a a1^2 & 0 & 0 & 0 & 0 \\ 0 & a^2 a1 & 0 & 0 & 0 \\ 0 & 0 & a a1 & 0 & 0 \\ e1 & d & b1 & a1 & 0 \\ d1 & e & b & 0 & a \end{bmatrix}$$

(2)

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frame1 > MaInv := MatrixInverse(Ma) :

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M > On determine les formes de Maurer Cartan:

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M > $Mat := map(evalDG, (ExteriorDerivative(Ma).MaInv));$

frame1 >

$$Mat := \begin{bmatrix} \frac{da}{a} + \frac{2 da l}{a l} & 0 dw & 0 dw & 0 dw & 0 dw \\ 0 dw & \frac{2 da}{a} + \frac{da l}{a l} & 0 dw & 0 dw & 0 dw \\ 0 dw & 0 dw & \frac{da}{a} + \frac{da l}{a l} & 0 dw & 0 dw \\ -\frac{e l da l}{a a l^3} + \frac{de l}{a a l^2} & -\frac{d da l}{a^2 a l^2} + \frac{dd}{a^2 a l} & -\frac{b l da l}{a a l^2} + \frac{db l}{a a l} & \frac{da l}{a l} & 0 dw \\ -\frac{d l da}{a^2 a l^2} + \frac{d d l}{a a l^2} & -\frac{e da}{a^3 a l} + \frac{de}{a^2 a l} & -\frac{b da}{a^2 a l} + \frac{db}{a a l} & 0 dw & \frac{da}{a} \end{bmatrix} \quad (3)$$

frame1 > $t[1] := \frac{da}{a} :$

frame1 > $t[2] := -\frac{b da}{a^2 a l} + \frac{db}{a a l} :$

frame1 > $t[3] := -\frac{d da l}{a^2 a l^2} + \frac{dd}{a^2 a l} :$

frame1 > $t[4] := -\frac{e da}{a^3 a l} + \frac{de}{a^2 a l} :$

frame1 > $t[5] := \frac{da l}{a l} :$

frame1 > $t[6] := -\frac{b l da l}{a a l^2} + \frac{db l}{a a l} :$

frame1 > $t[7] := -\frac{d l da}{a^2 a l^2} + \frac{d d l}{a a l^2} :$

frame1 > $t[8] := -\frac{e l da l}{a a l^3} + \frac{de l}{a a l^2} :$

M > $FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], t[7], t[8], dw, dx, dy, dz1, dz], frame2) :$

M > $DGsetup(FD, [E], [\alpha[1], \alpha[2], \alpha[3], \alpha[4], \alpha^\#[1], \alpha^\#[2], \alpha^\#[3], \alpha^\#[4], \sigma^\#, \sigma, \rho, \zeta^\#, \zeta], verbose);$

The following coordinates have been protected:

$[w, x, y, z1, z, a, al, b, bl, d, dl, e, el]$

The following vector fields have been defined and protected:

$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13]$

The following differential 1-forms have been defined and protected:

$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha^\#[1], \alpha^\#[2], \alpha^\#[3], \alpha^\#[4], \sigma^\#, \sigma, \rho, \zeta^\#, \zeta]$

frame name: frame2

frame2 > $Y := Vector([\sigma^\#, \sigma, \rho, \zeta^\#, \zeta]) :$

(4)

frame2 > $V := MaInv.Y :$

Les equations de courbure sur les $W[i]$ sont connues :

frame2 > $dW[1] := -K1 \cdot (W[1] \wedge W[2]) + F1 \cdot (W[1] \wedge W[3]) + Q1 \cdot (W[1] \wedge W[4]) + B1 \cdot (W[1] \wedge W[5]) + G \cdot (W[2] \wedge W[3]) + B1 \cdot (W[2] \wedge W[4]) + R \cdot (W[2] \wedge W[5]) + (W[3] \wedge W[4]) :$

frame2 > $dW[2] := K \cdot (W[1] \wedge W[2]) + G1 \cdot (W[1] \wedge W[3]) + R1 \cdot (W[1] \wedge W[4]) + B \cdot (W[1] \wedge W[5]) + F \cdot (W[2] \wedge W[3]) + B \cdot (W[2] \wedge W[4]) + Q \cdot (W[2] \wedge W[5]) + (W[3] \wedge W[5]) :$

frame2 > $dW[3] := I \cdot J \cdot (W[1] \wedge W[2]) + E^\# \cdot (W[1] \wedge W[3]) + P1 \cdot (W[1] \wedge W[4]) + A \cdot (W[1] \wedge W[5]) + E \cdot (W[2] \wedge W[3]) + A \cdot (W[2] \wedge W[4]) + P \cdot (W[2] \wedge W[5]) - I \cdot (W[4] \wedge W[5]) :$

N > $dW[4] := 0 : dW[5] := 0 :$

frame2 > $dW := Vector([dW[1], dW[2], dW[3], dW[4], dW[5]]) :$

On en deduit les nouvelles courbures initiales:

frame2 > $dV[1] := dW[1] :$

frame2 > $dV[2] := dW[2] :$

frame2 > $dV[3] := evalDG(dW[3] + (Der(B) \wedge W[1]) + B \cdot dW[1] + (Der(B1) \wedge W[2]) + B1 \cdot dW[2]) :$

frame2 > $dV[4] := dW[4] :$

frame2 > $dV[5] := dW[5] :$

frame2 > $dV := Vector([dV[1], dV[2], dV[3], dV[4], dV[5]]) :$

frame2 > $\Omega := map(evalDG, Ma.dV) :$

frame2 > $Mat := map(evalDG, (ExteriorDerivative(Ma).MaInv)) ;$

$$Mat := \begin{bmatrix} \alpha_1 + 2 \alpha_1^\# & 0 \alpha_1 & 0 \alpha_1 & 0 \alpha_1 & 0 \alpha_1 \\ 0 \alpha_1 & 2 \alpha_1 + \alpha_1^\# & 0 \alpha_1 & 0 \alpha_1 & 0 \alpha_1 \\ 0 \alpha_1 & 0 \alpha_1 & \alpha_1 + \alpha_1^\# & 0 \alpha_1 & 0 \alpha_1 \\ \alpha_4^\# & \alpha_3 & \alpha_2^\# & \alpha_1^\# & 0 \alpha_1 \\ \alpha_3^\# & \alpha_4 & \alpha_2 & 0 \alpha_1 & \alpha_1 \end{bmatrix}$$

(5)

frame2 > $Mat2 := Mat \&MatrixWedge Y :$

frame2 > $SE := map(evalDG, (Mat2 \&MatrixPlus \Omega)) :$

frame2 > $List := GenerateForms([\alpha[1], \alpha[2], \alpha[3], \alpha[4], \alpha^\#[1], \alpha^\#[2], \alpha^\#[3], \alpha^\#[4], \sigma^\#, \sigma, \rho, \zeta^\#, \zeta], 2) :$

frame2 > $Torsion := \mathbf{proc}(S, i, j) \mathbf{local} k, X; k := 13 \cdot (i - 1) - \frac{i \cdot (i - 1)}{2} + j - i; X := GetComponents(S, List); \mathbf{expand}(X[k]); \mathbf{end proc} :$

frame2 > $\mathbf{result} := \mathbf{proc}(l) \mathbf{local} k, t, X; X := 0 : t := \mathbf{expand}(GetComponents(l, List)); \mathbf{for} k \mathbf{from} 1 \mathbf{to} 78 \mathbf{do} X := X + t[k] \cdot List[k] \mathbf{od}; X; \mathbf{end proc} :$

frame2 > result(SE[1]);

$$\begin{aligned} & \alpha_1 \wedge \sigma^\# + 2 \alpha_1^\# \wedge \sigma^\# + \left(\frac{Bd}{a^2 al^2} + \frac{Rdl}{a^3 al} + \frac{GB}{a^2 al} - \frac{Bl e}{a^3 al} - \frac{Qld}{a^2 al^2} - \frac{Fl Bl}{a^2 al} \right. \\ & \quad \left. - \frac{Kl}{a^2 al} \right) \sigma^\# \wedge \sigma + \left(\frac{el}{a al^2} + \frac{Bbl}{a al^2} - \frac{Bl b}{a^2 al} - \frac{Ql bl}{a al^2} + \frac{Fl}{a al} \right) \sigma^\# \wedge \rho + \left(-\frac{B}{al} \right. \\ & \quad \left. + \frac{Ql}{al} \right) \sigma^\# \wedge \zeta^\# + \frac{Bl \sigma^\# \wedge \zeta}{a} + \left(\frac{d}{a^2 al} - \frac{Rb}{a^3} + \frac{G}{a^2} \right) \sigma \wedge \rho + \frac{al R \sigma \wedge \zeta}{a^2} \\ & \quad + \rho \wedge \zeta^\# \end{aligned} \tag{6}$$

frame2 > result(SE[2]);

$$\begin{aligned} & 2 \alpha_1 \wedge \sigma + \alpha_1^\# \wedge \sigma + \left(-\frac{Bl dl}{a^2 al^2} + \frac{Qdl}{a^2 al^2} + \frac{Bel}{a al^3} + \frac{FB}{a al^2} - \frac{Rld}{a al^3} - \frac{Gl Bl}{a al^2} \right. \\ & \quad \left. + \frac{K}{a al^2} \right) \sigma^\# \wedge \sigma + \left(\frac{dl}{a al^2} - \frac{Rl bl}{al^3} + \frac{Gl}{al^2} \right) \sigma^\# \wedge \rho + \frac{a Rl \sigma^\# \wedge \zeta^\#}{al^2} + \left(\frac{e}{a^2 al} \right. \\ & \quad \left. + \frac{Bl b}{a^2 al} - \frac{Qb}{a^2 al} - \frac{Bbl}{a al^2} + \frac{F}{a al} \right) \sigma \wedge \rho + \frac{B \sigma \wedge \zeta^\#}{al} + \left(-\frac{Bl}{a} + \frac{Q}{a} \right) \sigma \wedge \zeta \\ & \quad + \rho \wedge \zeta \end{aligned} \tag{7}$$

frame2 > result(SE[3]);

$$\begin{aligned} & \alpha_1 \wedge \rho + \alpha_1^\# \wedge \rho + \left(\frac{Pdl}{a^3 al^2} + \frac{Ael}{a^2 al^3} + \frac{EB}{a^2 al^2} - \frac{BKl}{a^2 al^2} + \frac{GB^2}{a^2 al^2} + \frac{T(B) Bl}{a^2 al^2} \right. \\ & \quad \left. + \frac{L^\#(B) d}{a^2 al^3} - \frac{L(B) dl}{a^3 al^2} - \frac{Ae}{a^3 al^2} + \frac{B^2 d}{a^2 al^3} - \frac{Bl^2 dl}{a^3 al^2} + \frac{Bl K}{a^2 al^2} - \frac{Gl Bl^2}{a^2 al^2} \right. \\ & \quad \left. + \frac{IJ}{a^2 al^2} - \frac{Pl d}{a^2 al^3} + \frac{L(B) e}{a^3 al^2} - \frac{L^\#(B) el}{a^2 al^3} - \frac{E^\# Bl}{a^2 al^2} - \frac{T(B) B}{a^2 al^2} - \frac{B Bl e}{a^3 al^2} \right. \\ & \quad \left. + \frac{BRdl}{a^3 al^2} - \frac{BF Bl}{a^2 al^2} - \frac{BQld}{a^2 al^3} + \frac{Bl FB}{a^2 al^2} - \frac{S(B)}{a^2 al^2} + \frac{S^\#(Bl)}{a^2 al^2} - \frac{Iel e}{a^3 al^3} + \frac{Id dl}{a^3 al^3} \right. \\ & \quad \left. - \frac{Bl Rld}{a^2 al^3} + \frac{Bl Bel}{a^2 al^3} + \frac{Bl Qdl}{a^3 al^2} \right) \sigma^\# \wedge \sigma + \left(\frac{Bl dl}{a^2 al^2} - \frac{Bl Rl bl}{a al^3} + \frac{Gl Bl}{a al^2} \right. \\ & \quad \left. + \frac{Bel}{a al^3} + \frac{B^2 bl}{a al^3} - \frac{B Bl b}{a^2 al^2} - \frac{BQl bl}{a al^3} + \frac{BF l}{a al^2} + \frac{L(B) b}{a^2 al^2} + \frac{L^\#(B) bl}{a al^3} - \frac{T(B)}{a al^2} \right. \\ & \quad \left. + \frac{Ibl dl}{a^2 al^3} - \frac{Iel b}{a^2 al^3} - \frac{Ab}{a^2 al^2} - \frac{Pl bl}{a al^3} + \frac{E^\#}{a al^2} \right) \sigma^\# \wedge \rho + \left(\frac{Bl Rl}{al^2} - \frac{B^2}{al^2} \right. \\ & \quad \left. + \frac{BQl}{al^2} - \frac{L^\#(B)}{al^2} - \frac{Idl}{a al^2} + \frac{Pl}{al^2} \right) \sigma^\# \wedge \zeta^\# + \left(\frac{B Bl}{a al} - \frac{L(B)}{a al} + \frac{Iel}{a al^2} \right. \\ & \quad \left. + \frac{A}{a al} \right) \sigma^\# \wedge \zeta + \left(\frac{Bl e}{a^3 al} + \frac{Bl^2 b}{a^3 al} - \frac{Bl Qb}{a^3 al} - \frac{Bl B bl}{a^2 al^2} + \frac{Bl F}{a^2 al} + \frac{L(B) b}{a^3 al} \right. \\ & \quad \left. + \frac{L^\#(Bl) bl}{a^2 al^2} - \frac{T(Bl)}{a^2 al} + \frac{Bd}{a^2 al^2} - \frac{BRb}{a^3 al} + \frac{GB}{a^2 al} + \frac{Ibl e}{a^3 al^2} - \frac{Id b}{a^3 al^2} - \frac{Pb}{a^3 al} \right) \end{aligned} \tag{8}$$

$$\begin{aligned}
& -\frac{Ab l}{a^2 a l^2} + \frac{E}{a^2 a l} \Big) \sigma \wedge \rho + \left(\frac{B B l}{a a l} - \frac{L^\#(B l)}{a a l} - \frac{I e}{a^2 a l} + \frac{A}{a a l} \right) \sigma \wedge \zeta^\# + \left(-\frac{B l^2}{a^2} \right. \\
& + \frac{B l Q}{a^2} - \frac{L(B l)}{a^2} + \frac{B R}{a^2} + \frac{I d}{a^2 a l} + \frac{P}{a^2} \Big) \sigma \wedge \zeta + \left(\frac{B}{a l} - \frac{I b}{a a l} \right) \rho \wedge \zeta^\# + \left(\frac{B l}{a} \right. \\
& \left. + \frac{I b l}{a a l} \right) \rho \wedge \zeta - I \zeta^\# \wedge \zeta
\end{aligned}$$

frame2 > result(SE[5]);

$$\begin{aligned}
& \alpha_1 \wedge \zeta + \alpha_2 \wedge \rho + \alpha_4 \wedge \sigma + \alpha_3^\# \wedge \sigma^\# + \left(\frac{e K}{a^3 a l^3} - \frac{d l K l}{a^3 a l^3} + \frac{b S^\#(B l)}{a^3 a l^3} - \frac{b S(B)}{a^3 a l^3} \right. \\
& + \frac{R d l^2}{a^4 a l^3} - \frac{b B F l B l}{a^3 a l^3} + \frac{b B l F B}{a^3 a l^3} + \frac{b B R d l}{a^4 a l^3} - \frac{b B B l e}{a^4 a l^3} - \frac{b B Q l d}{a^3 a l^4} \\
& + \frac{b B l Q d l}{a^4 a l^3} + \frac{b B l B e l}{a^3 a l^4} - \frac{b B l R l d}{a^3 a l^4} + \frac{I b d d l}{a^4 a l^4} - \frac{I b e l e}{a^4 a l^4} + \frac{e F B}{a^3 a l^3} - \frac{e G l B l}{a^3 a l^3} \\
& + \frac{d l G B}{a^3 a l^3} - \frac{d l F l B l}{a^3 a l^3} - \frac{b E^\# B l}{a^3 a l^3} + \frac{b G B^2}{a^3 a l^3} - \frac{b B K l}{a^3 a l^3} + \frac{b E B}{a^3 a l^3} - \frac{b G l B l^2}{a^3 a l^3} \\
& + \frac{b B l K}{a^3 a l^3} - \frac{b T(B l) B}{a^3 a l^3} + \frac{I b J}{a^3 a l^3} + \frac{b T(B) B l}{a^3 a l^3} - \frac{2 e B l d l}{a^4 a l^3} + \frac{e Q d l}{a^4 a l^3} + \frac{e B e l}{a^3 a l^4} \\
& - \frac{e R l d}{a^3 a l^4} + \frac{d l B d}{a^3 a l^4} - \frac{d l Q l d}{a^3 a l^4} - \frac{b P l d}{a^3 a l^4} + \frac{b B^2 d}{a^3 a l^4} - \frac{b A e}{a^4 a l^3} + \frac{b A e l}{a^3 a l^4} + \frac{b P d l}{a^4 a l^3} \\
& - \frac{b B l^2 d l}{a^4 a l^3} - \frac{b L(B l) d l}{a^4 a l^3} - \frac{b L^\#(B l) e l}{a^3 a l^4} + \frac{b L^\#(B) d}{a^3 a l^4} + \frac{b L(B) e}{a^4 a l^3} \Big) \sigma^\# \wedge \sigma + \left(\right. \\
& - \frac{b B l R l b l}{a^2 a l^4} + \frac{b G l B l}{a^2 a l^3} + \frac{b B e l}{a^2 a l^4} + \frac{b B^2 b l}{a^2 a l^4} - \frac{B B l b^2}{a^3 a l^3} - \frac{b B Q l b l}{a^2 a l^4} + \frac{b B F l}{a^2 a l^3} \\
& + \frac{L(B) b^2}{a^3 a l^3} + \frac{b L^\#(B) b l}{a^2 a l^4} - \frac{b T(B)}{a^2 a l^3} + \frac{I b b l d l}{a^3 a l^4} - \frac{I e l b^2}{a^3 a l^4} - \frac{A b^2}{a^3 a l^3} - \frac{b P l b l}{a^2 a l^4} \\
& + \frac{b E^\#}{a^2 a l^3} + \frac{e d l}{a^3 a l^3} - \frac{e R l b l}{a^2 a l^4} + \frac{e G l}{a^2 a l^3} + \frac{d l e l}{a^2 a l^4} + \frac{d l B b l}{a^2 a l^4} - \frac{d l Q l b l}{a^2 a l^4} \\
& \left. + \frac{d l F l}{a^2 a l^3} \right) \sigma^\# \wedge \rho + \left(\frac{b B l R l}{a a l^3} - \frac{b B^2}{a a l^3} + \frac{b B Q l}{a a l^3} - \frac{b L^\#(B)}{a a l^3} - \frac{I b d l}{a^2 a l^3} + \frac{b P l}{a a l^3} \right. \\
& + \frac{e R l}{a a l^3} - \frac{d l B}{a a l^3} + \frac{d l Q l}{a a l^3} \Big) \sigma^\# \wedge \zeta^\# + \left(\frac{B B l b}{a^2 a l^2} - \frac{L(B) b}{a^2 a l^2} + \frac{I e l b}{a^2 a l^3} + \frac{A b}{a^2 a l^2} \right. \\
& + \frac{B l d l}{a^2 a l^2} \Big) \sigma^\# \wedge \zeta + \left(\frac{2 e B l b}{a^4 a l^2} + \frac{B l^2 b^2}{a^4 a l^2} - \frac{B l Q b^2}{a^4 a l^2} - \frac{b B l B b l}{a^3 a l^3} + \frac{b B l F}{a^3 a l^2} \right. \\
& + \frac{L(B l) b^2}{a^4 a l^2} + \frac{b L^\#(B l) b l}{a^3 a l^3} - \frac{b T(B l)}{a^3 a l^2} + \frac{b B d}{a^3 a l^3} - \frac{B R b^2}{a^4 a l^2} + \frac{b G B}{a^3 a l^2} + \frac{I b b l e}{a^4 a l^3} \\
& - \frac{I d b^2}{a^4 a l^3} - \frac{P b^2}{a^4 a l^2} - \frac{b A b l}{a^3 a l^3} + \frac{b E}{a^3 a l^2} + \frac{e^2}{a^4 a l^2} - \frac{e Q b}{a^4 a l^2} - \frac{e B b l}{a^3 a l^3} + \frac{e F}{a^3 a l^2}
\end{aligned} \tag{9}$$

$$\begin{aligned}
& + \frac{d dl}{a^3 a l^3} - \frac{d l R b}{a^4 a l^2} + \frac{d l G}{a^3 a l^2} \Big) \sigma \wedge \rho + \left(\frac{B B l b}{a^2 a l^2} - \frac{b L^\#(B l)}{a^2 a l^2} - \frac{I b e}{a^3 a l^2} + \frac{A b}{a^2 a l^2} \right. \\
& + \left. \frac{B e}{a^2 a l^2} \right) \sigma \wedge \zeta^\# + \left(-\frac{B l^2 b}{a^3 a l} + \frac{B l Q b}{a^3 a l} - \frac{L(B l) b}{a^3 a l} + \frac{B R b}{a^3 a l} + \frac{I d b}{a^3 a l^2} + \frac{P b}{a^3 a l} \right. \\
& - \left. \frac{B l e}{a^3 a l} + \frac{e Q}{a^3 a l} + \frac{R d l}{a^3 a l} \right) \sigma \wedge \zeta + \left(\frac{b B}{a a l^2} - \frac{I b^2}{a^2 a l^2} + \frac{d l}{a a l^2} \right) \rho \wedge \zeta^\# + \left(\frac{B l b}{a^2 a l} \right. \\
& + \left. \frac{I b b l}{a^2 a l^2} + \frac{e}{a^2 a l} \right) \rho \wedge \zeta - \frac{I b \zeta^\# \wedge \zeta}{a a l}
\end{aligned}$$

frame2 >