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> restart :
[> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([w, x, y, z1, z], [a, a1, b, b1, c, c1, d, d1, e, e1], frame1, verbose);
    The following coordinates have been protected:
        [w, x, y, z1, z, a, a1, b, b1, c, c1, d, d1, e, e1]
    The following vector fields have been defined and protected:
[D_w, D_x, D_y, D_z1, D_z, D_a, D_a1, D_b, D_b1, D_c, D_c1, D_d, D_d1, D_e, D_e1]
    The following differential 1-forms have been defined and protected:
        [dw, dx, dy, dz1, dz, da, da1, db, db1, dc, dc1, dd, dd1, de, de1]
        frame name: frame1

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> On introduit la matrice de groupe:
> Ma := Matrix([[a*a1^2, 0, 0, 0, 0], [0, a^2*a1, 0, 0, 0], [c1, c, a*a1, 0, 0], [e1, d, b1, a1, 0],
[d1, e, b, 0, a]]);

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$$Ma := \begin{bmatrix} a a1^2 & 0 & 0 & 0 & 0 \\ 0 & a^2 a1 & 0 & 0 & 0 \\ c1 & c & a a1 & 0 & 0 \\ e1 & d & b1 & a1 & 0 \\ d1 & e & b & 0 & a \end{bmatrix} \quad (2)$$

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M > MaInv := MatrixInverse(Ma) :
M > On determine les formes de Maurer Cartan:
M > Mat := map(evalDG, (ExteriorDerivative(Ma).MaInv));

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$$Mat := \left[\left[\frac{da}{a} + \frac{2 da1}{a1}, 0 dw, 0 dw, 0 dw, 0 dw \right], \right. \\ \left[0 dw, \frac{2 da}{a} + \frac{da1}{a1}, 0 dw, 0 dw, 0 dw \right], \\ \left[-\frac{c1 da}{a1^2 a^2} - \frac{c1 da1}{a1^3 a} + \frac{dc1}{a a1^2}, -\frac{c da}{a1 a^3} - \frac{c da1}{a1^2 a^2} + \frac{dc}{a^2 a1}, \frac{da}{a} + \frac{da1}{a1}, 0 dw, 0 dw \right], \\ \left[-\frac{(e1 a a1 - b1 c1) da1}{a^2 a1^4} - \frac{c1 db1}{a^2 a1^3} + \frac{de1}{a a1^2}, -\frac{(d a a1 - b1 c) da1}{a^3 a1^3} - \frac{c db1}{a^3 a1^2} \right. \\ \left. + \frac{dd}{a^2 a1}, -\frac{b1 da1}{a a1^2} + \frac{db1}{a a1}, \frac{da1}{a1}, 0 dw \right], \\ \left[-\frac{(d1 a a1 - b c1) da}{a^3 a1^3} - \frac{c1 db}{a^2 a1^3} + \frac{dd1}{a a1^2}, -\frac{(e a a1 - b c) da}{a^4 a1^2} - \frac{c db}{a^3 a1^2} + \frac{de}{a^2 a1}, \right. \\ \left. -\frac{b da}{a^2 a1} + \frac{db}{a a1}, 0 dw, \frac{da}{a} \right] \left. \right] \quad (3)$$

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> t[1] := da/a :

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frame1 > t[2] := -c da/a1 a^3 - c da1/a1^2 a^2 + dc/a^2 a1 :

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frame1 > t[3] := - (e a a1 - b c) da / (a^4 a1^2) - c db / (a^3 a1^2) + de / (a^2 a1) :
frame1 > t[4] := - (d a a1 - b1 c) da1 / (a^3 a1^3) - c db1 / (a^3 a1^2) + dd / (a^2 a1) :
frame1 > t[5] := - b da / (a^2 a1) + db / (a a1) :
frame1 > t[6] := da1 / a1 :
frame1 > t[7] := - c1 da / (a1^2 a^2) - c1 da1 / (a1^3 a) + dc1 / (a a1^2) :
frame1 > t[8] := - (e1 a a1 - b1 c1) da1 / (a^2 a1^4) - c1 db1 / (a^2 a1^3) + de1 / (a a1^2) :
frame1 > t[9] := - (d1 a a1 - b c1) da / (a^3 a1^3) - c1 db / (a^2 a1^3) + dd1 / (a a1^2) :
frame1 > t[10] := - b1 da1 / (a a1^2) + db1 / (a a1) :
M > FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], t[7], t[8], t[9], t[10], dw, dx, dy, dz1, dz], frame2) :
M > DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], alpha#[1], alpha#[2], alpha#[3], alpha#[4], alpha#[5], sigma#, sigma, rho, zeta#, zeta], verbose);
    The following coordinates have been protected:
    [w, x, y, z1, z, a, a1, b, b1, c, c1, d, d1, e, e1]
    The following vector fields have been defined and protected:
    [E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13, E14, E15]
    The following differential 1-forms have been defined and protected:
    [alpha_1, alpha_2, alpha_3, alpha_4, alpha_5, alpha_1#, alpha_2#, alpha_3#, alpha_4#, alpha_5#, sigma#, sigma, rho, zeta#, zeta]
    frame name: frame2

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(4)

Le coframe relevé Y est relié au coframe de base W par la relation $Y = Ma \cdot W$:

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N > Y := Vector([sigma#, sigma, rho, zeta#, zeta]) :
    W := MaInv.Y :

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Les équations de courbure du coframe initial sont connues:

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frame2 > dW[1] := -K1 \cdot (W[1] \wedge W[2]) + F1 \cdot (W[1] \wedge W[3]) + Q1
    \cdot (W[1] \wedge W[4]) + B1 \cdot (W[1] \wedge W[5]) + G \cdot (W[2]
    \wedge W[3]) + B1 \cdot (W[2] \wedge W[4]) + R \cdot (W[2]
    \wedge W[5]) + (W[3] \wedge W[4]) :
frame2 > dW[2] := K \cdot (W[1] \wedge W[2]) + G1 \cdot (W[1] \wedge W[3]) + R1
    \cdot (W[1] \wedge W[4]) + B \cdot (W[1] \wedge W[5]) + F \cdot (W[2]
    \wedge W[3]) + B \cdot (W[2] \wedge W[4]) + Q \cdot (W[2]
    \wedge W[5]) + (W[3] \wedge W[5]) :
frame2 > dW[3] := I \cdot J \cdot (W[1] \wedge W[2]) + E# \cdot (W[1] \wedge W[3]) + P1
    \cdot (W[1] \wedge W[4]) + A \cdot (W[1] \wedge W[5]) + E \cdot (W[2]
    \wedge W[3]) + A \cdot (W[2] \wedge W[4]) + P \cdot (W[2]

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&wedge W[5]) - I · (W[4] &wedge W[5]) :

N > $dW[4] := 0 : dW[5] := 0 :$

frame2 > $dW := \text{Vector}([dW[1], dW[2], dW[3], dW[4], dW[5]]) :$

frame2 > $\text{Omega} := \text{map}(\text{evalDG}, \text{Ma}.dW) :$

frame2 > $\text{Mat} := \text{map}(\text{evalDG}, (\text{ExteriorDerivative}(\text{Ma}).\text{MaInv})) :$

frame2 > $\text{Mat2} := \text{Mat} \&\text{MatrixWedge } Y :$

frame2 > $\text{SE} := \text{map}(\text{evalDG}, (\text{Mat2} \&\text{MatrixPlus } \text{Omega})) :$

frame2 > $\text{List} := \text{GenerateForms}([\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \text{alpha}[4],$
 $\text{alpha}[5], \alpha^\# [1], \alpha^\# [2], \alpha^\# [3], \alpha^\# [4], \alpha^\# [5], \sigma^\#, \text{sigma}, \rho, \zeta^\#, \text{zeta}],$
 $2) :$

frame2 > $\text{result} := \text{proc}(l) \text{ local } k, t, X; X := 0 : t := \text{expand}(\text{GetComponents}(l,$
 $\text{List})) : \text{for } k \text{ from } 1 \text{ to } 105 \text{ do } X := X + t[k] \cdot \text{List}[k] \text{ od}; X; \text{end}$
proc:

frame2 > $\text{result}(\text{SE}[1]);$

$$\begin{aligned} & \alpha_1 \wedge \sigma^\# + 2 \alpha_1^\# \wedge \sigma^\# + \left(-\frac{c e l}{a^3 a l^3} + \frac{c l d}{a^3 a l^3} + \frac{R d l}{a^3 a l} - \frac{R b c l}{a^4 a l^2} + \frac{B l e l}{a^2 a l^2} - \frac{B l b l c l}{a^3 a l^3} \right. \\ & + \frac{G c l}{a^3 a l^2} - \frac{B l e}{a^3 a l} + \frac{B l b c}{a^4 a l^2} - \frac{Q l d}{a^2 a l^2} + \frac{Q l b l c}{a^3 a l^3} - \frac{F l c}{a^3 a l^2} - \frac{K l}{a^2 a l} \left. \right) \sigma^\# \wedge \sigma \\ & + \left(\frac{e l}{a l^2 a} - \frac{B l b}{a l a^2} - \frac{Q l b l}{a l^2 a} + \frac{F l}{a l a} \right) \sigma^\# \wedge \rho + \left(-\frac{c l}{a l^2 a} + \frac{Q l}{a l} \right) \sigma^\# \wedge \zeta^\# \\ & + \frac{B l \sigma^\# \wedge \zeta}{a} + \left(\frac{d}{a^2 a l} - \frac{R b}{a^3} - \frac{B l b l}{a^2 a l} + \frac{G}{a^2} \right) \sigma \wedge \rho + \left(-\frac{c}{a^2 a l} + \frac{B l}{a} \right) \sigma \wedge \zeta^\# \\ & + \frac{a l R \sigma \wedge \zeta}{a^2} + \rho \wedge \zeta^\# \end{aligned} \quad (5)$$

frame2 > $\text{result}(\text{SE}[3]);$

$$\begin{aligned} & \alpha_1 \wedge \rho + \alpha_2 \wedge \sigma + \alpha_1^\# \wedge \rho + \alpha_2^\# \wedge \sigma^\# + \left(\frac{I e l b c}{a^4 a l^4} + \frac{I b l c l e}{a^4 a l^4} - \frac{I d b c l}{a^4 a l^4} - \frac{I b l c d l}{a^4 a l^4} \right. \\ & - \frac{c Q b c l}{a^5 a l^4} - \frac{c B b l c l}{a^4 a l^5} + \frac{c l B l b c}{a^5 a l^4} + \frac{c l Q l b l c}{a^4 a l^5} + \frac{c Q d l}{a^4 a l^3} + \frac{c B e l}{a^3 a l^4} + \frac{c F c l}{a^4 a l^4} \\ & - \frac{c B e}{a^4 a l^3} - \frac{c R l d}{a^3 a l^4} + \frac{c l R d l}{a^4 a l^3} + \frac{c l B l e l}{a^3 a l^4} - \frac{c l B l e}{a^4 a l^3} - \frac{c l Q l d}{a^3 a l^4} - \frac{c l F l c}{a^4 a l^4} \\ & - \frac{P b c l}{a^4 a l^3} - \frac{A b l c l}{a^3 a l^4} + \frac{A b c}{a^4 a l^3} + \frac{P l b l c}{a^3 a l^4} + \frac{I d d l}{a^3 a l^3} - \frac{I e l e}{a^3 a l^3} + \frac{c c l e}{a^5 a l^4} + \frac{B b c^2}{a^5 a l^4} \\ & + \frac{R l b l c^2}{a^4 a l^5} - \frac{c l c e l}{a^4 a l^5} - \frac{R b c l^2}{a^5 a l^4} - \frac{B l b l c l^2}{a^4 a l^5} - \frac{c^2 d l}{a^5 a l^4} + \frac{c l^2 d}{a^4 a l^5} - \frac{G l c^2}{a^4 a l^4} \\ & + \frac{c K}{a^3 a l^3} + \frac{G c l^2}{a^4 a l^4} - \frac{c l K l}{a^3 a l^3} + \frac{P d l}{a^3 a l^2} + \frac{A e l}{a^2 a l^3} + \frac{E c l}{a^3 a l^3} - \frac{A e}{a^3 a l^2} - \frac{P l d}{a^2 a l^3} \\ & \left. - \frac{E^\# c}{a^3 a l^3} + \frac{I J}{a^2 a l^2} \right) \sigma^\# \wedge \sigma + \left(\frac{I b l d l}{a^2 a l^3} - \frac{I b e l}{a^2 a l^3} - \frac{b A}{a^2 a l^2} - \frac{b l P l}{a a l^3} + \frac{E^\#}{a a l^2} \right. \end{aligned} \quad (6)$$

$$\begin{aligned}
& + \frac{cdl}{a^3 al^3} - \frac{Bbc}{a^3 al^3} - \frac{Rlblc}{a^2 al^4} + \frac{Gl c}{a^2 al^3} + \frac{el cl}{a^2 al^4} - \frac{cl Bl b}{a^3 al^3} - \frac{cl Ql bl}{a^2 al^4} \\
& + \frac{cl Fl}{a^2 al^3} \Big) \sigma^\# \wedge \rho + \left(-\frac{Idl}{a al^2} + \frac{Ib cl}{a^2 al^3} + \frac{Pl}{al^2} + \frac{cRl}{a al^3} - \frac{cl^2}{a^2 al^4} \right. \\
& + \frac{cl Ql}{a al^3} \Big) \sigma^\# \wedge \zeta^\# + \left(\frac{Iel}{a al^2} - \frac{Ib l cl}{a^2 al^3} + \frac{A}{a al} - \frac{c cl}{a^3 al^3} + \frac{cB}{a^2 al^2} \right. \\
& + \frac{cl Bl}{a^2 al^2} \Big) \sigma^\# \wedge \zeta + \left(\frac{Ibl e}{a^3 al^2} - \frac{Ib d}{a^3 al^2} - \frac{bP}{a^3 al} - \frac{bl A}{a^2 al^2} + \frac{E}{a^2 al} + \frac{ec}{a^4 al^2} \right. \\
& - \frac{cQb}{a^4 al^2} - \frac{cBbl}{a^3 al^3} + \frac{cF}{a^3 al^2} + \frac{cl d}{a^3 al^3} - \frac{Rb cl}{a^4 al^2} - \frac{Bl bl cl}{a^3 al^3} + \frac{Gcl}{a^3 al^2} \Big) \sigma \wedge \rho + \left(\right. \\
& - \frac{Ie}{a^2 al} + \frac{Ibc}{a^3 al^2} + \frac{A}{a al} - \frac{c cl}{a^3 al^3} + \frac{cB}{a^2 al^2} + \frac{cl Bl}{a^2 al^2} \Big) \sigma \wedge \zeta^\# + \left(\frac{Id}{a^2 al} - \frac{Ibl c}{a^3 al^2} \right. \\
& + \frac{P}{a^2} - \frac{c^2}{a^4 al^2} + \frac{cQ}{a^3 al} + \frac{cl R}{a^3 al} \Big) \sigma \wedge \zeta + \left(-\frac{Ib}{a al} + \frac{cl}{al^2 a} \right) \rho \wedge \zeta^\# + \left(\frac{Ibl}{a al} \right. \\
& \left. + \frac{c}{a^2 al} \right) \rho \wedge \zeta - I \zeta^\# \wedge \zeta
\end{aligned}$$

frame2 > result(SE[2]);

$$\begin{aligned}
& 2 \alpha_1 \wedge \sigma + \alpha_1^\# \wedge \sigma + \left(-\frac{cdl}{a^3 al^3} + \frac{cl e}{a^3 al^3} + \frac{Qdl}{a^2 al^2} - \frac{Qb cl}{a^3 al^3} + \frac{Bel}{a al^3} - \frac{Bbl cl}{a^2 al^4} \right. \\
& + \frac{Fcl}{a^2 al^3} - \frac{Be}{a^2 al^2} + \frac{Bbc}{a^3 al^3} - \frac{Rld}{a al^3} + \frac{Rl bl c}{a^2 al^4} - \frac{Gl c}{a^2 al^3} + \frac{K}{a al^2} \Big) \sigma^\# \wedge \sigma \\
& + \left(\frac{dl}{al^2 a} - \frac{Bb}{al^2 a} - \frac{Rl bl}{al^3} + \frac{Gl}{al^2} \right) \sigma^\# \wedge \rho + \frac{aRl \sigma^\# \wedge \zeta^\#}{al^2} + \left(-\frac{cl}{al^2 a} \right. \\
& + \frac{B}{al} \Big) \sigma^\# \wedge \zeta + \left(\frac{e}{al a^2} - \frac{Qb}{al a^2} - \frac{Bbl}{al^2 a} + \frac{F}{al a} \right) \sigma \wedge \rho + \frac{B \sigma \wedge \zeta^\#}{al} + \left(-\frac{c}{a^2 al} \right. \\
& \left. + \frac{Q}{a} \right) \sigma \wedge \zeta + \rho \wedge \zeta
\end{aligned} \tag{7}$$

[>