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> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([x, y, z, z1], [a], M, verbose);
    The following coordinates have been protected:
                [x, y, z, z1, a]
    The following vector fields have been defined and protected:
                [D_x, D_y, D_z, D_z1, D_a]
    The following differential 1-forms have been defined and protected:
                [dx, dy, dz, dz1, da]
                frame name: M
(1)
> Der := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(Der(op(i, x)), i = 1
.. nops(x)) elif
    (type(x, '*' ) = true) then expand( $\frac{x}{y} \cdot Der(y) + y \cdot Der\left(\frac{x}{y}\right)$ ) elif
    (type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · Der(y) elif
    ( (type(x, function) = true) or (type(x, symbol) = true) ) then S(x) · W[1]
+ Tau(x) · W[2] + L(x) · W[3] + L#(x) · W[4]
    else 0 fi end proc:
derivation := proc(x) : collect( Der(x), [W[1], W[2], W[3], W[4]]) : end proc:
> L := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(L(op(i, x)), i = 1
.. nops(x)) elif
    (type(x, '*' ) = true) then expand( $L(y) \cdot \frac{x}{y} + y \cdot L\left(\frac{x}{y}\right)$ ) elif
    (type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · L(y) elif
    (type(x, function) = true) then 'L'(x) elif
    (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:
> L# := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(L#(op(i, x)), i = 1
.. nops(x)) elif
    (type(x, '*' ) = true) then expand( $L^{\#}(y) \cdot \frac{x}{y} + y \cdot L^{\#}\left(\frac{x}{y}\right)$ ) elif
    (type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · L#(y) elif
    (type(x, function) = true) then 'L#(x)' elif
    (type(x, symbol) = true) then 'L#(x)' else 0 fi end proc:
> Tau := proc(x) I · (L(L#(x)) - L#(L(x))) end proc:
M > S := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(S(op(i, x)), i = 1
.. nops(x)) elif
    (type(x, '*' ) = true) then expand( $S(y) \cdot \frac{x}{y} + y \cdot S\left(\frac{x}{y}\right)$ ) elif
    (type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · S(y) elif
    (type(x, function) = true) then 'S'(x) elif
    (type(x, symbol) = true) then 'S'(x) else 0 fi end proc:

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On a noté j la fonction D_0 c'est à dire la fonction telle que la normalisation de d s'ecrit $d = a * j$

Le nouveau coframe initial (tilde, désigné par X), s'exprime en fonction du précédent (chech, désigné par U) par la relation $X:=p.U$.
où p est la matrice:

M > $p := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [j, 0, 1, 0], [jI, 0, 0, 1]])$:

M > $pinv := MatrixInverse(p)$:

M > $U := pinv.X$:

On note, comme aux etapes precedentes, f et h les fonctions C_0 et B_0 .

Le nouveau coframe initial (chech, désigné par U), s'exprime en fonction du précédent (chapeau, désigné par V) par la relation $U:=n.V$.

où n est la matrice:

> $n := Matrix([[1, 0, 0, 0], [f, 1, 0, 0], [0, h, 1, 0], [hI \cdot A^\# \cdot B^{\frac{1}{2}}, hI, 0, 1]])$:

> $ninv := MatrixInverse(n)$:

M > $V := ninv.Vector([U[1], U[2], U[3], U[4]])$:

Le coframe chapeau, désigné par V, s'exprime en fonction du coframe initial, W, par la relation $V:=m.W$.

> $m := Matrix([[[B^{-\frac{1}{2}}, 0, 0, 0], [0, 1, 0, 0], [0, 0, B^{-\frac{1}{2}}, 0], [0, 0, 0, B^{\frac{1}{2}}]]])$:

> $minv := MatrixInverse(m)$:

> $W := minv.Vector([V[1], V[2], V[3], V[4]])$:

On donne ensuite la matrice de groupe:

On donne ensuite la matrice de groupe:

M > $g := Matrix([[a^3, 0, 0, 0], [0, a^2, 0, 0], [0, 0, a, 0], [0, 0, 0, a]])$;

$$g := \begin{bmatrix} a^3 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

(2)

M > $ginv := MatrixInverse(g)$:

M > $Mat := map(evalDG, (ExteriorDerivative(g).ginv))$:

Il y a une unique forme de MC:

M > $t[1] := \frac{da}{a}$:

On donne maintenant la forme de MC modifiée:

$$\begin{aligned} \text{N} > \text{expr1} := & \frac{\frac{1}{2} \text{IL}(L^\#(B))}{B a^2} - \frac{\frac{1}{4} \text{IL}^\#(B) Q}{B a^2} + \frac{\frac{1}{4} \text{IL}(B) Q}{a^2} - \frac{\frac{1}{2} \text{IL}(B) A}{B a^2} - \frac{\frac{1}{4} \text{IL}(B)^2}{B a^2} \\ & - \frac{\frac{3}{4} \text{IL}^\#(B) L(B)}{B^2 a^2} + \frac{\text{IL}(A)}{a^2} - \frac{\frac{1}{2} \text{IL}^\#(Q)}{a^2} + \frac{\frac{1}{2} \text{IL}(L(B))}{a^2} + \frac{\frac{1}{2} \text{IL}(L(Q))}{a^2} : \end{aligned}$$

$$\begin{aligned} \text{N} > \text{expr2} := & -\frac{\frac{1}{4} \text{IL}^\#(B) L(B) Q}{B^{3/2} a^3} - \frac{\frac{1}{2} \text{I}\sqrt{B} L(A) Q}{a^3} + \frac{\frac{1}{4} \text{IL}^\#(L(B)) Q}{\sqrt{B} a^3} \\ & - \frac{\frac{1}{4} \text{IL}(B)^2 L^\#(B)}{B^{5/2} a^3} - \frac{\frac{1}{4} \text{IL}^\#(L(B)) L(B)}{B^{3/2} a^3} - \frac{\frac{1}{4} \text{IL}^\#(Q) L(B)}{\sqrt{B} a^3} \\ & - \frac{\frac{1}{4} \text{I}\sqrt{B} L^\#(Q) Q}{a^3} + \frac{\frac{1}{4} \text{I}\sqrt{B} A Q^2}{a^3} + \frac{\text{I}\sqrt{B} A P}{a^3} + \frac{\frac{1}{4} \text{IAL}(B)^2}{B^{3/2} a^3} \\ & - \frac{\frac{1}{2} \text{IAL}(L(B))}{\sqrt{B} a^3} - \frac{\frac{1}{2} \text{I}\sqrt{B} A L(Q)}{a^3} + \frac{\frac{1}{4} \text{I}QL(B)^2}{\sqrt{B} a^3} + \frac{\frac{1}{4} \text{I}\sqrt{B} Q^2 L(B)}{a^3} \\ & + \frac{\frac{1}{4} \text{I}\sqrt{B} QL(L(B))}{a^3} + \frac{\frac{1}{4} \text{IB}^{3/2} QL(Q)}{a^3} + \frac{\frac{1}{4} \text{IL}(B) L(L(B))}{\sqrt{B} a^3} \\ & + \frac{\frac{1}{4} \text{I}\sqrt{B} L(B) L(Q)}{a^3} + \frac{2 \text{I}\sqrt{B} L(B) P}{a^3} + \frac{\frac{1}{2} \text{IL}(A) L(B)}{\sqrt{B} a^3} + \frac{\frac{1}{2} \text{IL}(L^\#(B)) L(B)}{B^{3/2} a^3} \\ & + \frac{\text{I}\sqrt{B} L(L(A))}{a^3} - \frac{\text{I}\sqrt{B} L^\#(P)}{a^3} + \frac{\text{IB}^{3/2} L(P)}{a^3} - \frac{\frac{1}{2} \text{IL}(L^\#(L(B)))}{\sqrt{B} a^3} \\ & - \frac{\frac{1}{2} \text{I}\sqrt{B} L(L^\#(Q))}{a^3} + \frac{\frac{1}{2} \text{IL}^\#(L(L(B)))}{\sqrt{B} a^3} + \frac{\frac{1}{2} \text{I}\sqrt{B} L^\#(L(Q))}{a^3} : \end{aligned}$$

$$\text{N} > \text{expr3} := \frac{1}{6} \frac{\sqrt{B} Q}{a} :$$

$$\text{N} > \text{expr4} := \frac{1}{6} \frac{L^\#(B) + 2 A B + B^2 Q + L(B) B}{a B^{3/2}} :$$

$$\text{N} > v := \text{evalDG} \left(\frac{da}{a} + \frac{\text{expr2}}{2} \cdot dx - \frac{\text{expr1}}{3} \cdot dy - \text{expr3} \cdot dz - \text{expr4} \cdot dz1 \right);$$

$$v := -\frac{1}{B^{5/2} a^3} \left(\frac{1}{8} \text{I}(-4 B^3 L(L(A)) - 2 B^3 L^\#(L(Q)) + 4 B^3 L^\#(P) - 4 B^4 L(P) \right. \tag{3}$$

$$\left. + L(B)^2 L^\#(B) - 2 L^\#(L(L(B))) B^2 + 2 B^3 L(L^\#(Q)) + 2 L(L^\#(L(B))) B^2 \right.$$

$$\left. + L^\#(B) L(B) Q B - B^4 Q L(Q) - A L(B)^2 B - 2 L(L^\#(B)) L(B) B + L^\#(L(B)) L(B) B \right.$$

$$\left. - B^3 A Q^2 - B^3 Q^2 L(B) - B^3 Q L(L(B)) - B^3 L(B) L(Q) - 4 B^3 A P - L^\#(L(B)) Q B^2 \right)$$

$$\begin{aligned}
& -QL(B)^2 B^2 - L(B) L(L(B)) B^2 - 2L(A) L(B) B^2 + 2B^3 L(A) Q + 2B^3 AL(Q) \\
& + L^\#(Q) L(B) B^2 + B^3 L^\#(Q) Q + 2AL(L(B)) B^2 - 8B^3 L(B) P) dx \\
& + \frac{1}{B^2 a^2} \left(\frac{1}{12} I(-2L(L^\#(B)) B + L^\#(B) QB - L(B) QB^2 + 2L(B) AB + L(B)^2 B \right. \\
& \left. + 3L^\#(B) L(B) - 4L(A) B^2 + 2L^\#(Q) B^2 - 2L(L(B)) B^2 - 2B^3 L(Q) \right) dy \\
& - \frac{1}{6} \frac{\sqrt{B} Q dz}{a} - \frac{1}{6} \frac{(L^\#(B) + 2AB + B^2 Q + L(B) B) dzl}{a B^{3/2}} + \frac{da}{a}
\end{aligned}$$

> FD := FrameData([v, dx, dy, dz, dzl], N) :

DGsetup(FD, [E], [beta[1], sigma, rho, zeta, ζ[#]], verbose) :

The following coordinates have been protected:

[x, y, z, zl, a]

The following vector fields have been defined and protected:

[E1, E2, E3, E4, E5]

The following differential 1-forms have been defined and protected:

[β₁, σ, ρ, ζ, ζ[#]]

(4)

Le coframe 'relevé' est noté Y. Il est relié au coframe de base X par la relation Y:=g.X.

N > Y := Vector([sigma, rho, zeta, ζ[#]]) :

N > X := ginv.Y :

Les equations de courbure de coframe W sont connues:

M > dW[1] := H·(W[1]&wedge W[2]) + F·(W[1]&wedge W[4]) + Q·(W[1] &wedge W[3]) + B·(W[2] &wedge W[4]) + (W[2] &wedge W[3]) :

M > dW[2] := G·(W[1]&wedge W[2]) + E·(W[1]&wedge W[4]) + P·(W[1] &wedge W[3]) + A·(W[2]&wedge W[4]) + I·(W[3]&wedge W[4]) :

Les equations de courbures du coframe V ont été calculées dans le fichier second step:

N > dV[1] := evalDG(B^{-1/2}·dW[1] + (Der(B^{-1/2}) &wedge W[1])) :

N > dV[2] := dW[2] :

N > dV[3] := Der(B^{-1/2}) &wedge W[3] :

N > dV[4] := Der(B^{1/2}) &wedge W[4] :

N >

Les équations de courbures du coframe U s'en déduisent:

$$\begin{aligned}
\mathbf{N} > dU[1] &:= dV[1]: \\
\mathbf{N} > dU[2] &:= dV[2] + (Der(f) \&wedge V[1]) + f \cdot dV[1]: \\
\mathbf{N} > dU[3] &:= dV[3] + (Der(h) \&wedge V[2]) + h \cdot dV[2]: \\
\mathbf{N} > dU[4] &:= dV[4] + (Der(h1) \&wedge V[2]) + h1 \cdot dV[2] + \left(Der \left(h1 \cdot A^\# \cdot B^{\frac{1}{2}} \right) \right. \\
&\quad \left. \&wedge V[1] \right) \\
&\quad + h1 \cdot A^\# \cdot B^{\frac{1}{2}} \cdot dV[1]:
\end{aligned}$$

Les équations de courbures du coframe X sont donc:

$$\begin{aligned}
\mathbf{N} > dX[1] &:= dU[1]: \\
\mathbf{N} > dX[2] &:= dU[2]: \\
\mathbf{N} > dX[3] &:= dU[3] + (Der(j) \&wedge V[1]) + j \cdot dV[1]: \\
\mathbf{N} > dX[4] &:= dU[4] + (Der(j1) \&wedge V[1]) + j1 \cdot dV[1]:
\end{aligned}$$

On peut maintenant calculer les équations de courbure du coframe 'relevé':

$$\begin{aligned}
\mathbf{N} > \text{Omega} &:= \text{map}(\text{evalDG}, g.\text{Vector}([dX[1], dX[2], dX[3], dX[4]])): \\
\mathbf{N} > \text{Mat} &:= \text{map}(\text{evalDG}, (\text{ExteriorDerivative}(g).\text{ginv})): \\
\mathbf{N} > \text{Mat2} &:= \text{Mat} \&\text{MatrixWedge } Y: \\
\mathbf{N} > \text{SE} &:= \text{map}(\text{evalDG}, (\text{Mat2} \&\text{MatrixPlus } \text{Omega})): \\
\mathbf{N} > \text{List} &:= \text{GenerateForms}([\text{beta}[1], \text{sigma}, \text{rho}, \text{zeta}, \zeta^\#], 2): \\
\mathbf{N} > \text{result} &:= \text{proc}(l) \text{ local } k, t, X; X := 0 : t := \text{expand}(\text{GetComponents}(l, \text{List})) : \text{for } k \\
&\quad \text{from } 1 \text{ to } 10 \text{ do } X := X + t[k] \cdot \text{List}[k] \text{ od}; X; \text{end proc}: \\
&\quad \text{Torsion} := \text{proc}(S, i, j) \text{ local } k, X; k := 5 \cdot (i - 1) - \frac{i \cdot (i - 1)}{2} + j - i; X \\
&\quad := \text{map}(\text{expand}, \text{GetComponents}(S, \text{List})); X[k]; \text{end proc}:
\end{aligned}$$

On entre les valeurs des fonctions inconnues introduites:

$$\begin{aligned}
\mathbf{N} > B^\# &:= \frac{1}{B}: \\
\mathbf{N} > A^\# &:= -B^\# \cdot A: \\
\mathbf{N} > F &:= L(B) + B \cdot Q + A: \\
\mathbf{N} > h1 &:= \frac{I}{2} \cdot \left(\frac{L(B)}{B^2} + \frac{1}{3} \cdot B^{\frac{1}{2}} \cdot Q \right) : f := \frac{1}{2} \left(\frac{L(B)}{B^2} + B^{\frac{1}{2}} \cdot Q \right) : \\
\mathbf{N} > h &:= -\frac{I}{2} \cdot \left(L^\#(B^\#) \cdot B^{\frac{1}{2}} + \frac{1}{3} \cdot \frac{Q^\#}{B^2} \right) : h := \\
&\quad -\frac{\frac{1}{6} I \cdot (-2 L^\#(B) + 2 A B + B^2 Q + L(B) B)}{B^{3/2}}: \\
\mathbf{N} > E &:= L(A) + B \cdot P:
\end{aligned}$$

$$\mathbf{N} > j1 := \frac{1}{2} \frac{I \cdot L(B)^2}{B} + \frac{1}{3} I \cdot QL(B) + \frac{1}{3} I \cdot BQ^2 - \frac{1}{2} I \cdot L(L(B)) - \frac{1}{2} I \cdot BL(Q) \\ + \frac{1}{2} \frac{I \cdot AL(B)}{B} + \frac{1}{6} I \cdot AQ + I \cdot BP;$$

$$\mathbf{N} > j := -\frac{2}{3} I \cdot L(B) Q - \frac{1}{6} \frac{I \cdot L(B) A}{B} - \frac{1}{6} I \cdot AQ + \frac{1}{6} \frac{I \cdot L^\#(B) Q}{B} - \frac{1}{3} \frac{I \cdot L(B)^2}{B} \\ - \frac{1}{3} I \cdot BQ^2 - I \cdot L(A) - \frac{1}{3} \frac{I \cdot L^\#(B) L(B)}{B^2} + \frac{1}{2} \frac{I \cdot L^\#(L(B))}{B} + \frac{1}{2} I \cdot L^\#(Q) \\ - I \cdot BP;$$

$$H := \text{expand}(I \cdot L(L(B)) + I \cdot Q \cdot L(B) + I \cdot B \cdot L(Q) + 2 \cdot I \cdot L(A) - I \cdot L^\#(Q));$$

$$G := I \cdot L(L(A)) + 2 \cdot I \cdot P \cdot L(B) - I \cdot L^\#(P) - I \cdot Q \cdot L(A) + I \cdot B \cdot L(P);$$

$$H := IL(L(B)) + IQL(B) + IBL(Q) + 2IL(A) - IL^\#(Q)$$

$$G := IL(L(A)) + 2IPL(B) - IL^\#(P) - IQL(A) + IBL(P) \quad (5)$$

> result(SE[1]);

$$3 \beta_1 \wedge \sigma + \rho \wedge \zeta + \rho \wedge \zeta^\# \quad (6)$$

$\mathbf{N} > \text{result(SE[2]);}$

$$2 \beta_1 \wedge \rho + I \zeta \wedge \zeta^\# \quad (7)$$

$\mathbf{N} > \text{result(SE[3]);}$

$$\beta_1 \wedge \zeta + \left(\frac{1}{6} \frac{BAQP}{a^4} + \frac{1}{12} \frac{AQL^\#(L(B))}{Ba^4} - \frac{1}{12} \frac{AQL^\#(Q)}{a^4} + \frac{1}{6} \frac{L^\#(B)QL^\#(Q)}{Ba^4} \right) \quad (8) \\ + \frac{1}{24} \frac{L^\#(B)L(B)^2Q}{B^2a^4} + \frac{1}{6} \frac{L^\#(B)L(B)P}{Ba^4} + \frac{5}{12} \frac{QL^\#(L(B))L(B)}{Ba^4} \\ + \frac{1}{4} \frac{AL^\#(Q)L(B)}{Ba^4} - \frac{1}{8} \frac{Q^2L(B)A}{a^4} - \frac{1}{2} \frac{L(A)L(B)A}{Ba^4} + \frac{L(A)L(B)Q}{a^4} \\ + \frac{1}{12} \frac{L^\#(B)L(L(B))Q}{Ba^4} - \frac{1}{6} \frac{QL(L(B))A}{a^4} + \frac{1}{4} \frac{QL(L(B))L(B)}{a^4} \\ + \frac{1}{6} \frac{AL(Q)L^\#(B)}{Ba^4} - \frac{1}{6} \frac{BAL(Q)Q}{a^4} - \frac{1}{6} \frac{AL(Q)L(B)}{a^4} - \frac{1}{6} \frac{PL(B)A}{a^4} \\ + \frac{1}{2} \frac{BPL(B)Q}{a^4} + \frac{5}{12} \frac{L(B)L(Q)L^\#(B)}{Ba^4} + \frac{1}{4} \frac{BL(B)L(Q)Q}{a^4}$$

$$\begin{aligned}
& - \frac{1}{4} \frac{L(L^\#(B)) QL(B)}{B a^4} - \frac{1}{6} \frac{AL(L(B)) L(B)}{B a^4} - \frac{1}{6} \frac{L(A) QA}{a^4} - \frac{5}{24} \frac{L(B)^2 A Q}{B a^4} \\
& - \frac{1}{12} \frac{L^\#(B) A Q^2}{B a^4} - \frac{1}{3} \frac{L^\#(B) A P}{B a^4} - \frac{1}{6} \frac{L^\#(B) Q P}{a^4} + \frac{1}{12} \frac{AL^\#(L(B)) L(B)}{B^2 a^4} \\
& + \frac{1}{6} \frac{L^\#(B) L^\#(L(B)) L(B)}{B^3 a^4} + \frac{1}{3} \frac{L(B) L(L(B)) L^\#(B)}{B^2 a^4} + \frac{2}{3} \frac{L(B) L^\#(B) L(L^\#(B))}{B^3 a^4} \\
& + \frac{1}{12} \frac{L^\#(L^\#(B)) QL(B)}{B^2 a^4} - \frac{1}{6} \frac{QL^\#(B) L(L^\#(B))}{B^2 a^4} - \frac{1}{6} \frac{QL(B) L^\#(B)^2}{B^3 a^4} \\
& + \frac{1}{2} \frac{L(A) L^\#(B) L(B)}{B^2 a^4} - \frac{1}{6} \frac{AL^\#(B) L(B)^2}{B^3 a^4} - \frac{1}{6} \frac{L^\#(B) L^\#(Q) L(B)}{B^2 a^4} \\
& + \frac{1}{3} \frac{L^\#(B) QL^\#(L(B))}{B^2 a^4} - \frac{1}{6} \frac{L(B) QL^\#(Q)}{a^4} + \frac{1}{6} \frac{L^\#(B) QL(Q)}{a^4} \\
& - \frac{1}{3} \frac{IS(B) L^\#(B)}{B^2 a^4} + \frac{1}{4} \frac{L^\#(L(B))^2}{B^2 a^4} + \frac{BL^\#(L(P))}{a^4} + \frac{L^\#(L(L(A)))}{a^4} \\
& - \frac{1}{2} \frac{L^\#(L(L^\#(Q)))}{a^4} - \frac{1}{24} \frac{L(B)^4}{B^2 a^4} + \frac{4}{3} \frac{L(A)^2}{a^4} + \frac{1}{4} \frac{L^\#(Q)^2}{a^4} \\
& + \frac{1}{2} \frac{L(L^\#(L^\#(L(B))))}{B a^4} - \frac{L(L^\#(L(A)))}{a^4} + \frac{1}{2} \frac{L(L^\#(L^\#(Q)))}{a^4} - \frac{BL(L^\#(P))}{a^4} \\
& - \frac{1}{2} \frac{L^\#(L(L^\#(L(B))))}{B a^4} - \frac{1}{3} \frac{IS(A)}{a^4} - \frac{1}{6} \frac{IS(L(B))}{a^4} - \frac{7}{12} \frac{L(L^\#(Q)) L(B)}{a^4} \\
& - \frac{4}{3} \frac{L(L^\#(B)) P}{a^4} - \frac{7}{12} \frac{QL(L^\#(L(B)))}{a^4} - \frac{1}{6} \frac{L(L^\#(A)) Q}{a^4} - \frac{5}{12} \frac{L(L^\#(B)) Q^2}{a^4} \\
& - \frac{7}{12} \frac{BQL(L^\#(Q))}{a^4} - \frac{7}{12} \frac{L(B) L(L^\#(L(B)))}{B a^4} - \frac{1}{3} \frac{L(L^\#(L^\#(B))) L(B)}{B^2 a^4} \\
& - \frac{1}{6} \frac{L(L^\#(A)) L(B)}{B a^4} + \frac{1}{6} \frac{L(B)^2 L(L^\#(B))}{B^2 a^4} + \frac{1}{6} \frac{L(L^\#(L^\#(B))) Q}{B a^4} \\
& + \frac{2}{3} \frac{B^2 L(Q) P}{a^4} + \frac{1}{3} \frac{L^\#(B) L^\#(P)}{B a^4} - \frac{1}{3} \frac{AL^\#(P)}{a^4} + \frac{1}{3} \frac{L^\#(L^\#(B)) P}{B a^4}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3} \frac{L^\#(A) P}{a^4} - \frac{1}{3} \frac{L(L^\#(B)) L(A)}{B a^4} - \frac{1}{12} \frac{L^\#(L^\#(L(B))) Q}{B a^4} - \frac{1}{12} \frac{L^\#(L^\#(Q)) Q}{a^4} \\
& -\frac{1}{12} \frac{L^\#(A) Q^2}{a^4} + \frac{1}{12} \frac{L^\#(L^\#(B)) Q^2}{B a^4} + \frac{2}{3} \frac{L(L(B)) L(A)}{a^4} + \frac{2}{3} \frac{B L(L(B)) P}{a^4} \\
& + \frac{2}{3} \frac{B L(Q) L(A)}{a^4} + \frac{2}{3} \frac{Q L^\#(L(L(B)))}{a^4} + \frac{1}{6} \frac{A L^\#(L(Q))}{a^4} + \frac{4}{3} \frac{P L^\#(L(B))}{a^4} \\
& + \frac{2}{3} \frac{L^\#(L(Q)) L(B)}{a^4} + \frac{1}{3} \frac{Q L^\#(L(A))}{a^4} - \frac{1}{4} \frac{L^\#(Q) L(L(B))}{a^4} + \frac{13}{24} \frac{L^\#(L(B)) Q^2}{a^4} \\
& -\frac{1}{4} \frac{L(Q) L^\#(L(B))}{a^4} + \frac{1}{3} \frac{L(B) L^\#(P)}{a^4} + \frac{1}{6} \frac{L^\#(A) L(Q)}{a^4} - \frac{7}{6} \frac{L(A) L^\#(Q)}{a^4} \\
& -\frac{1}{4} \frac{B L(Q) L^\#(Q)}{a^4} + \frac{2}{3} \frac{B Q L^\#(L(Q))}{a^4} + \frac{1}{6} \frac{L(Q) L^\#(B)^2}{B^2 a^4} \\
& -\frac{1}{4} \frac{L(L(B)) L^\#(L(B))}{B a^4} - \frac{1}{6} \frac{L^\#(L^\#(B)) L(L(B))}{B^2 a^4} + \frac{1}{3} \frac{L^\#(L^\#(B)) L(B)^2}{B^3 a^4} \\
& -\frac{1}{6} \frac{L(A) L^\#(L(B))}{B a^4} + \frac{1}{6} \frac{L(L^\#(B)) L^\#(Q)}{B a^4} + \frac{2}{3} \frac{L^\#(L(A)) L(B)}{B a^4} \\
& + \frac{1}{6} \frac{L^\#(A) L(L(B))}{B a^4} - \frac{1}{12} \frac{L^\#(A) L(B)^2}{B^2 a^4} - \frac{1}{4} \frac{L^\#(L^\#(L(B))) L(B)}{B^2 a^4} \\
& -\frac{1}{4} \frac{L(B)^3 L^\#(B)}{B^3 a^4} + \frac{1}{3} \frac{L(L(B)) L^\#(B)^2}{B^3 a^4} - \frac{1}{6} \frac{L^\#(L^\#(B)) L(Q)}{B a^4} \\
& + \frac{1}{3} \frac{L^\#(B) L^\#(L(L(B)))}{B^2 a^4} - \frac{1}{2} \frac{L^\#(B) L(L^\#(L(B)))}{B^2 a^4} - \frac{1}{6} \frac{L^\#(B) L^\#(L(Q))}{B a^4} \\
& -\frac{1}{6} \frac{L^\#(L(L^\#(B))) Q}{B a^4} + \frac{1}{3} \frac{L^\#(L(L^\#(B))) L(B)}{B^2 a^4} - \frac{1}{3} \frac{L(L^\#(B)) L^\#(L(B))}{B^2 a^4} \\
& + \frac{1}{6} \frac{A L^\#(L(L(B)))}{B a^4} + \frac{2}{3} \frac{L(B) L^\#(L(L(B)))}{B a^4} + \frac{1}{8} \frac{L^\#(L(B)) L(B)^2}{B^2 a^4} \\
& -\frac{1}{12} \frac{L^\#(B)^2 Q^2}{B^2 a^4} - \frac{1}{3} \frac{L^\#(B)^2 P}{B^2 a^4} - \frac{1}{4} \frac{L^\#(L^\#(Q)) L(B)}{B a^4} + \frac{5}{24} \frac{L^\#(Q) L(B)^2}{B a^4}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{12} \frac{B Q^3 L(B)}{a^4} + \frac{1}{8} \frac{Q^2 L(B)^2}{a^4} + \frac{1}{12} \frac{L(A) L(B)^2}{B a^4} + \frac{1}{8} \frac{B Q^2 L(L(B))}{a^4} \\
& - \frac{1}{6} \frac{A^2 L(Q)}{a^4} - \frac{1}{6} \frac{P L(B)^2}{a^4} + \frac{1}{8} \frac{L(B)^2 L(Q)}{a^4} - \frac{1}{6} \frac{A^2 L(L(B))}{B a^4} \\
& + \frac{1}{8} \frac{B^2 Q^2 L(Q)}{a^4} - \frac{1}{24} \frac{L(B)^3 A}{B^2 a^4} + \frac{1}{4} \frac{B L(A) Q^2}{a^4} + \frac{1}{12} \frac{L(B)^2 A^2}{B^2 a^4} \\
& + \frac{1}{8} \frac{L(B)^2 L(L(B))}{B a^4} - \frac{1}{24} \frac{L^\#(B) Q^3}{a^4} - \frac{1}{8} \frac{B Q^2 L^\#(Q)}{a^4} + \frac{4}{3} \frac{B L(A) P}{a^4} \\
& - \frac{2}{3} \frac{B L^\#(Q) P}{a^4} - \frac{5}{6} \frac{L(B)^2 L^\#(B)^2}{B^4 a^4} + \frac{1}{12} \frac{A^2 Q^2}{a^4} + \frac{1}{3} \frac{A^2 P}{a^4} + \frac{1}{24} \frac{B A Q^3}{a^4} \\
& - \left. \frac{1}{6} \frac{IS(B) Q}{a^4} + \frac{1}{3} \frac{IS(L^\#(B))}{B a^4} - \frac{1}{6} \frac{IBS(Q)}{a^4} - \frac{1}{12} \frac{L^\#(B) L(B) A Q}{B^2 a^4} \right) \sigma \wedge \rho \\
& + \left(\frac{3}{8} \frac{IB^{3/2} QL(Q)}{a^3} + \frac{3}{8} \frac{I\sqrt{B} QL(L(B))}{a^3} - \frac{3}{4} \frac{IL(B)^2 L^\#(B)}{B^{5/2} a^3} + \frac{1}{8} \frac{IQ^2 L^\#(B)}{\sqrt{B} a^3} \right. \\
& + \frac{1}{8} \frac{IL^\#(Q) L(B)}{\sqrt{B} a^3} - \frac{1}{8} \frac{IL^\#(L(B)) Q}{\sqrt{B} a^3} - \frac{1}{4} \frac{IL(A) L(B)}{\sqrt{B} a^3} - \frac{1}{8} \frac{IAL(B)^2}{B^{3/2} a^3} \\
& + \frac{1}{4} \frac{I\sqrt{B} AL(Q)}{a^3} + \frac{1}{4} \frac{IAL(L(B))}{\sqrt{B} a^3} - \frac{1}{8} \frac{I\sqrt{B} Q^2 L(B)}{a^3} - \frac{1}{8} \frac{I\sqrt{B} A Q^2}{a^3} \\
& - \frac{1}{2} \frac{I\sqrt{B} AP}{a^3} + \frac{1}{4} \frac{IL(L^\#(B)) Q}{\sqrt{B} a^3} + \frac{1}{4} \frac{IL^\#(B) L(L(B))}{B^{3/2} a^3} - \frac{1}{4} \frac{IL^\#(B) L(Q)}{\sqrt{B} a^3} \\
& + \frac{1}{2} \frac{IPL^\#(B)}{\sqrt{B} a^3} - \frac{1}{2} \frac{IB^{3/2} PQ}{a^3} + \frac{3}{8} \frac{IL^\#(L(B)) L(B)}{B^{3/2} a^3} + \frac{3}{8} \frac{I\sqrt{B} L^\#(Q) Q}{a^3} \\
& + \frac{3}{8} \frac{I\sqrt{B} L(B) L(Q)}{a^3} - \frac{1}{4} \frac{IQL(B)^2}{\sqrt{B} a^3} + \frac{3}{8} \frac{IL(B) L(L(B))}{\sqrt{B} a^3} - \frac{1}{4} \frac{I\sqrt{B} L(A) Q}{a^3} \\
& \left. - \frac{1}{2} \frac{S(B)}{\sqrt{B} a^3} - \frac{1}{4} \frac{IL(B)^3}{B^{3/2} a^3} + \frac{1}{2} \frac{I\sqrt{B} L(L(A))}{a^3} + \frac{1}{2} \frac{IB^{3/2} L(P)}{a^3} \right)
\end{aligned}$$

$$\begin{aligned}
& - \frac{\frac{1}{4} \text{I}\sqrt{B} L(L^\#(Q))}{a^3} - \frac{\frac{1}{4} \text{I}L(L^\#(L(B)))}{\sqrt{B} a^3} - \frac{\frac{1}{8} \text{I}B^{3/2} Q^3}{a^3} - \frac{\frac{1}{4} \text{I}\sqrt{B} L^\#(L(Q))}{a^3} \\
& + \frac{\frac{1}{2} \text{I}\sqrt{B} L^\#(P)}{a^3} - \frac{\frac{1}{4} \text{I}L^\#(L(L(B)))}{\sqrt{B} a^3} - \frac{\frac{1}{8} \text{I}L^\#(B) L(B) Q}{B^{3/2} a^3} \\
& + \frac{\frac{1}{2} \text{I}L(L^\#(B)) L(B)}{B^{3/2} a^3} \left) \sigma \wedge \zeta + \left(- \frac{\frac{1}{2} \text{I}B^{3/2} P Q}{a^3} - \frac{\frac{1}{8} \text{I}B^{3/2} Q^3}{a^3} - \frac{\frac{1}{2} \text{I}A Q L(B)}{\sqrt{B} a^3} \right. \\
& - \frac{\frac{1}{2} \text{I}A L^\#(B) L(B)}{B^{5/2} a^3} - \frac{\frac{1}{8} \text{I}L(B)^3}{B^{3/2} a^3} - \frac{\frac{1}{2} \text{I}L^\#(L^\#(L(B)))}{B^{3/2} a^3} - \frac{\frac{1}{2} \text{I}L^\#(L^\#(Q))}{\sqrt{B} a^3} \\
& + \frac{\text{I}\sqrt{B} L^\#(P)}{a^3} - \frac{\text{I}L(B) L^\#(B)^2}{B^{7/2} a^3} + \frac{\frac{3}{4} \text{I}L^\#(L(B)) L(B)}{B^{3/2} a^3} - \frac{\frac{1}{2} \text{I}L^\#(B) L(B)^2}{B^{5/2} a^3} \\
& + \frac{\frac{3}{4} \text{I}\sqrt{B} Q L^\#(Q)}{a^3} + \frac{\frac{3}{4} \text{I}L^\#(L(B)) Q}{\sqrt{B} a^3} + \frac{\frac{1}{4} \text{I}L^\#(B) Q^2}{\sqrt{B} a^3} + \frac{\text{I}L^\#(B) L^\#(L(B))}{B^{5/2} a^3} \\
& + \frac{\frac{1}{2} \text{I}A L^\#(Q)}{\sqrt{B} a^3} + \frac{\frac{3}{4} \text{I}L^\#(Q) L(B)}{\sqrt{B} a^3} + \frac{\text{I}L^\#(B) P}{\sqrt{B} a^3} + \frac{\frac{1}{2} \text{I}L^\#(L^\#(B)) L(B)}{B^{5/2} a^3} \\
& + \frac{\frac{1}{2} \text{I}A L^\#(L(B))}{B^{3/2} a^3} - \frac{\frac{1}{4} \text{I}A L(B)^2}{B^{3/2} a^3} - \frac{\frac{3}{8} \text{I}\sqrt{B} Q^2 L(B)}{a^3} - \frac{\frac{3}{8} \text{I}Q L(B)^2}{\sqrt{B} a^3} \\
& - \frac{\text{I}L(A) A}{\sqrt{B} a^3} - \frac{\frac{1}{2} \text{I}L(A) L(B)}{\sqrt{B} a^3} - \frac{\frac{1}{2} \text{I}\sqrt{B} P L(B)}{a^3} - \frac{\frac{1}{4} \text{I}\sqrt{B} A Q^2}{a^3} - \frac{\text{I}\sqrt{B} P A}{a^3} \\
& + \frac{\text{I}L^\#(L(A))}{\sqrt{B} a^3} - \frac{\frac{1}{4} \text{I}L^\#(B) L(B) Q}{B^{3/2} a^3} - \frac{\frac{1}{2} \text{I}\sqrt{B} L(A) Q}{a^3} \left) \sigma \wedge \zeta^\# + \left(- \frac{\frac{7}{18} \text{I}L(B) Q}{a^2} \right. \\
& + \frac{\frac{1}{9} \text{I}L^\#(B) Q}{B a^2} - \frac{\frac{1}{9} \text{I}A Q}{a^2} - \frac{\frac{1}{3} \text{I}A L(B)}{B a^2} + \frac{\text{I}L^\#(L(B))}{B a^2} - \frac{\frac{2}{3} \text{I}L(L^\#(B))}{B a^2} \\
& + \frac{\frac{1}{3} \text{I}B L(Q)}{a^2} + \frac{\frac{1}{3} \text{I}L(L(B))}{a^2} + \frac{\frac{1}{3} \text{I}L^\#(Q)}{a^2} - \frac{\frac{1}{3} \text{I}L(A)}{a^2} - \frac{\frac{5}{12} \text{I}L(B)^2}{B a^2} \\
& - \frac{\frac{11}{36} \text{I}B Q^2}{a^2} - \frac{\text{I}B P}{a^2} \left) \rho \wedge \zeta + \left(- \frac{\frac{4}{9} \text{I}L^\#(B) L(B)}{B^2 a^2} + \frac{\frac{2}{9} \text{I}L^\#(B) Q}{B a^2} \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{9} \frac{IL^\#(B) A}{B^2 a^2} - \frac{2}{9} \frac{IAL(B)}{B a^2} - \frac{11}{36} \frac{IL(B)^2}{B a^2} - \frac{IBP}{a^2} - \frac{2}{9} \frac{IAQ}{a^2} + \frac{1}{3} \frac{IL^\#(A)}{B a^2} \\
& + \frac{2}{3} \frac{IL^\#(Q)}{a^2} - \frac{IL(A)}{a^2} - \frac{11}{36} \frac{IBQ^2}{a^2} + \frac{2}{3} \frac{IL^\#(L(B))}{B a^2} - \frac{11}{18} \frac{IL(B) Q}{a^2} \\
& + \left. \frac{4}{9} \frac{IL^\#(B)^2}{B^3 a^2} - \frac{2}{9} \frac{IA^2}{B a^2} - \frac{1}{3} \frac{IL^\#(L^\#(B))}{B^2 a^2} \right) \rho \wedge \zeta^\#
\end{aligned}$$

N > result(SE[4]);

$$\begin{aligned}
\beta_1 \wedge \zeta^\# & + \left(-\frac{1}{6} \frac{BAQP}{a^4} + \frac{7}{24} \frac{L^\#(B) L(B)^2 Q}{B^2 a^4} - \frac{1}{6} \frac{L^\#(B) L(B) P}{B a^4} \right. \\
& + \frac{1}{12} \frac{QL^\#(L(B)) L(B)}{B a^4} + \frac{1}{24} \frac{Q^2 L(B) A}{a^4} + \frac{1}{6} \frac{L(A) L(B) Q}{a^4} \\
& - \frac{1}{6} \frac{L^\#(B) L(L(B)) Q}{B a^4} + \frac{1}{12} \frac{QL(L(B)) A}{a^4} - \frac{1}{6} \frac{QL(L(B)) L(B)}{a^4} \\
& + \frac{1}{4} \frac{BAL(Q) Q}{a^4} - \frac{1}{12} \frac{AL(Q) L(B)}{a^4} + \frac{1}{6} \frac{PL(B) A}{a^4} - \frac{1}{2} \frac{BPL(B) Q}{a^4} \\
& - \frac{1}{12} \frac{BL(B) L(Q) Q}{a^4} - \frac{1}{3} \frac{L(L^\#(B)) QL(B)}{B a^4} + \frac{1}{4} \frac{AL(L(B)) L(B)}{B a^4} \\
& - \frac{1}{24} \frac{L(B)^2 A Q}{B a^4} + \frac{1}{6} \frac{L^\#(B) QP}{a^4} - \frac{3}{4} \frac{L(B) L(L(B)) L^\#(B)}{B^2 a^4} - \frac{1}{12} \frac{L(B) QL^\#(Q)}{a^4} \\
& - \frac{1}{4} \frac{L^\#(B) QL(Q)}{a^4} - \frac{BL^\#(L(P))}{a^4} + \frac{1}{8} \frac{L(B)^4}{B^2 a^4} + \frac{BL(L^\#(P))}{a^4} \\
& + \frac{1}{3} \frac{L(L^\#(Q)) L(B)}{a^4} + \frac{1}{3} \frac{QL(L^\#(L(B)))}{a^4} + \frac{2}{3} \frac{BQL(L^\#(Q))}{a^4} \\
& + \frac{L(B) L(L^\#(L(B)))}{B a^4} - \frac{L(B)^2 L(L^\#(B))}{B^2 a^4} - \frac{2}{3} \frac{B^2 L(Q) P}{a^4} - \frac{BL(L(B)) P}{a^4} \\
& + \frac{1}{3} \frac{BL(Q) L(A)}{a^4} - \frac{1}{4} \frac{QL^\#(L(L(B)))}{a^4} - \frac{1}{12} \frac{L^\#(L(Q)) L(B)}{a^4}
\end{aligned} \tag{9}$$

$$\begin{aligned}
& -\frac{1}{12} \frac{L^\#(Q) L(L(B))}{a^4} - \frac{1}{24} \frac{L^\#(L(B)) Q^2}{a^4} + \frac{1}{12} \frac{L(Q) L^\#(L(B))}{a^4} \\
& -\frac{1}{4} \frac{BL(Q) L^\#(Q)}{a^4} - \frac{7}{12} \frac{BQL^\#(L(Q))}{a^4} - \frac{1}{4} \frac{L(L(B)) L^\#(L(B))}{Ba^4} \\
& + \frac{3}{4} \frac{L(B)^3 L^\#(B)}{B^3 a^4} - \frac{3}{4} \frac{L(B) L^\#(L(L(B)))}{Ba^4} + \frac{3}{8} \frac{L^\#(L(B)) L(B)^2}{B^2 a^4} \\
& + \frac{1}{24} \frac{L^\#(Q) L(B)^2}{Ba^4} - \frac{1}{12} \frac{BQ^3 L(B)}{a^4} + \frac{1}{24} \frac{Q^2 L(B)^2}{a^4} - \frac{7}{24} \frac{BQ^2 L(L(B))}{a^4} \\
& - \frac{1}{6} \frac{PL(B)^2}{a^4} - \frac{11}{24} \frac{L(B)^2 L(Q)}{a^4} - \frac{1}{8} \frac{B^2 Q^2 L(Q)}{a^4} - \frac{1}{8} \frac{L(B)^3 A}{B^2 a^4} \\
& - \frac{1}{6} \frac{BL(A) Q^2}{a^4} - \frac{3}{8} \frac{L(B)^2 L(L(B))}{Ba^4} + \frac{1}{24} \frac{L^\#(B) Q^3}{a^4} + \frac{1}{8} \frac{BQ^2 L^\#(Q)}{a^4} \\
& - \frac{BL(A) P}{a^4} + \frac{2}{3} \frac{BL^\#(Q) P}{a^4} - \frac{1}{24} \frac{BAQ^3}{a^4} - \frac{1}{2} \frac{BL(L^\#(L(Q)))}{a^4} \\
& - \frac{1}{2} \frac{L(L^\#(L(L(B))))}{a^4} + \frac{1}{2} \frac{BL^\#(L(L(Q)))}{a^4} + \frac{1}{2} \frac{L^\#(L(L(L(B))))}{a^4} \\
& + \frac{1}{4} \frac{L(L(B))^2}{a^4} + \frac{1}{4} \frac{B^2 L(Q)^2}{a^4} + \frac{1}{2} \frac{IS(L(B))}{a^4} - \frac{1}{2} \frac{IL(B) S(B)}{Ba^4} \\
& - \frac{1}{2} \frac{L(B) L(L(A))}{a^4} - \frac{1}{6} \frac{BQL(L(A))}{a^4} + \frac{1}{2} \frac{L(L^\#(B)) L(L(B))}{Ba^4} \\
& + \frac{1}{2} \frac{BL(L(B)) L(Q)}{a^4} - \frac{1}{6} \frac{BL(L(Q)) A}{a^4} - \frac{1}{12} \frac{B^2 L(L(Q)) Q}{a^4} \\
& - \frac{1}{12} \frac{BL(L(Q)) L(B)}{a^4} - \frac{1}{3} \frac{L^\#(B) L(P)}{a^4} + \frac{1}{6} \frac{L(L(L(B))) L^\#(B)}{Ba^4} \\
& - \frac{1}{6} \frac{L(L(L(B))) A}{a^4} - \frac{1}{12} \frac{BL(L(L(B))) Q}{a^4} - \frac{1}{12} \frac{L(L(L(B))) L(B)}{a^4} \\
& + \frac{1}{3} \frac{BL(P) A}{a^4} - \frac{1}{3} \frac{BL(P) L(B)}{a^4} + \frac{1}{6} \frac{L^\#(B) L(L(Q))}{a^4} + \frac{1}{6} \frac{IBS(Q)}{a^4} \Big) \sigma \wedge \rho
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{\frac{1}{8} IQL(B)^2}{\sqrt{B} a^3} - \frac{\frac{1}{8} I\sqrt{B} Q^2 L(B)}{a^3} - \frac{\frac{3}{4} IL(L(B)) L(B)}{\sqrt{B} a^3} + \frac{\frac{1}{4} I\sqrt{B} L(B) L(Q)}{a^3} \right. \\
& - \frac{\frac{1}{2} I\sqrt{B} PL(B)}{a^3} - \frac{\frac{1}{4} I\sqrt{B} QL(L(B))}{a^3} - \frac{\frac{3}{4} IB^{3/2} QL(Q)}{a^3} + \frac{\frac{1}{2} IB^{3/2} PQ}{a^3} \\
& + \frac{\frac{3}{8} IL(B)^3}{B^{3/2} a^3} + \frac{\frac{1}{8} IB^{3/2} Q^3}{a^3} + \frac{\frac{1}{2} IB^{3/2} L(L(Q))}{a^3} + \frac{\frac{1}{2} I\sqrt{B} L(L(L(B)))}{a^3} \\
& \left. - \frac{IB^{3/2} L(P)}{a^3} \right) \sigma \wedge \zeta + \left(-\frac{\frac{1}{8} IL^\#(Q) L(B)}{\sqrt{B} a^3} - \frac{\frac{1}{8} IL^\#(B) Q^2}{\sqrt{B} a^3} + \frac{\frac{1}{2} S(B)}{\sqrt{B} a^3} \right. \\
& + \frac{\frac{1}{8} IL^\#(B) L(B) Q}{B^{3/2} a^3} + \frac{\frac{3}{4} IL(B)^2 L^\#(B)}{B^{5/2} a^3} + \frac{\frac{1}{2} IB^{3/2} PQ}{a^3} + \frac{\frac{1}{8} IL^\#(L(B)) Q}{\sqrt{B} a^3} \\
& + \frac{\frac{1}{4} IL(A) L(B)}{\sqrt{B} a^3} + \frac{\frac{1}{8} IAL(B)^2}{B^{3/2} a^3} - \frac{\frac{1}{4} I\sqrt{B} AL(Q)}{a^3} - \frac{\frac{1}{4} IAL(L(B))}{\sqrt{B} a^3} \\
& + \frac{\frac{1}{8} I\sqrt{B} Q^2 L(B)}{a^3} + \frac{\frac{1}{8} I\sqrt{B} A Q^2}{a^3} + \frac{\frac{1}{2} I\sqrt{B} AP}{a^3} - \frac{\frac{1}{4} IL(L^\#(B)) Q}{\sqrt{B} a^3} \\
& - \frac{\frac{1}{4} IL^\#(B) L(L(B))}{B^{3/2} a^3} + \frac{\frac{1}{4} IL^\#(B) L(Q)}{\sqrt{B} a^3} - \frac{\frac{1}{2} IPL^\#(B)}{\sqrt{B} a^3} - \frac{\frac{3}{8} IL^\#(L(B)) L(B)}{B^{3/2} a^3} \\
& - \frac{\frac{3}{8} I\sqrt{B} L^\#(Q) Q}{a^3} - \frac{\frac{3}{8} I\sqrt{B} L(B) L(Q)}{a^3} - \frac{\frac{3}{8} IL(B) L(L(B))}{\sqrt{B} a^3} \\
& + \frac{\frac{1}{4} I\sqrt{B} L(A) Q}{a^3} - \frac{\frac{3}{8} IB^{3/2} QL(Q)}{a^3} - \frac{\frac{1}{2} IL(L^\#(B)) L(B)}{B^{3/2} a^3} \\
& - \frac{\frac{3}{8} I\sqrt{B} QL(L(B))}{a^3} + \frac{\frac{1}{4} IQL(B)^2}{\sqrt{B} a^3} + \frac{\frac{1}{8} IB^{3/2} Q^3}{a^3} + \frac{\frac{1}{4} IL(B)^3}{B^{3/2} a^3} \\
& - \frac{\frac{1}{2} I\sqrt{B} L(L(A))}{a^3} - \frac{\frac{1}{2} IB^{3/2} L(P)}{a^3} + \frac{\frac{1}{4} I\sqrt{B} L(L^\#(Q))}{a^3} + \frac{\frac{1}{4} IL(L^\#(L(B)))}{\sqrt{B} a^3} \\
& \left. + \frac{\frac{1}{4} I\sqrt{B} L^\#(L(Q))}{a^3} - \frac{\frac{1}{2} I\sqrt{B} L^\#(P)}{a^3} + \frac{\frac{1}{4} IL^\#(L(L(B)))}{\sqrt{B} a^3} \right) \sigma \wedge \zeta^\#
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{3}{4} \frac{IL(B)^2}{B a^2} + \frac{1}{6} \frac{IL(B) Q}{a^2} + \frac{11}{36} \frac{IB Q^2}{a^2} - \frac{IL(L(B))}{a^2} - \frac{2}{3} \frac{IBL(Q)}{a^2} \right. \\
& \left. + \frac{IBP}{a^2} \right) \rho \wedge \zeta + \left(\frac{7}{18} \frac{IL(B) Q}{a^2} - \frac{1}{9} \frac{IL^\#(B) Q}{B a^2} + \frac{1}{9} \frac{IA Q}{a^2} + \frac{1}{3} \frac{IAL(B)}{B a^2} \right. \\
& - \frac{IL^\#(L(B))}{B a^2} + \frac{2}{3} \frac{IL(L^\#(B))}{B a^2} - \frac{1}{3} \frac{IBL(Q)}{a^2} - \frac{1}{3} \frac{IL(L(B))}{a^2} - \frac{1}{3} \frac{IL^\#(Q)}{a^2} \\
& \left. + \frac{1}{3} \frac{IL(A)}{a^2} + \frac{5}{12} \frac{IL(B)^2}{B a^2} + \frac{11}{36} \frac{IB Q^2}{a^2} + \frac{IBP}{a^2} \right) \rho \wedge \zeta^\#
\end{aligned}$$

N > $I1 := \text{expand}(a^4 \cdot \text{Torsion}(SE[4], 2, 3)) : I2 := \text{expand}(a^3 \cdot \text{Torsion}(SE[4], 2, 4)) :$

N > $I3 := \text{expand}(a^3 \cdot \text{Torsion}(SE[4], 2, 5)) :$

N > $I4 := \text{expand}(a^2 \cdot \text{Torsion}(SE[4], 3, 4)) :$

N > $I5 := \text{expand}(a^2 \cdot \text{Torsion}(SE[4], 3, 5)) ;$

$$\begin{aligned}
I5 := & \frac{7}{18} \frac{IL(B) Q}{B} - \frac{1}{9} \frac{IL^\#(B) Q}{B} + \frac{1}{9} \frac{IA Q}{B} + \frac{1}{3} \frac{IAL(B)}{B} - \frac{IL^\#(L(B))}{B} \\
& + \frac{2}{3} \frac{IL(L^\#(B))}{B} - \frac{1}{3} \frac{IBL(Q)}{B} - \frac{1}{3} \frac{IL(L(B))}{B} - \frac{1}{3} \frac{IL^\#(Q)}{B} + \frac{1}{3} \frac{IL(A)}{B} \\
& + \frac{5}{12} \frac{IL(B)^2}{B} + \frac{11}{36} \frac{IB Q^2}{B} + IBP
\end{aligned} \tag{10}$$

> $B^\# := \frac{1}{B} : A^\# := -B^\# \cdot A :$

> $\text{expand}(\text{solve}(L(B) + B \cdot Q + A - B \cdot L^\#(B^\#) - Q^\# - A^\# \cdot B, A)) :$

> $Q^\# := \text{expand}(\text{solve}(L(B) + B \cdot Q + A - B \cdot L^\#(B^\#) - Q^\# - A^\# \cdot B, Q^\#)) :$

> $eq2 := -P^\# - A \cdot Q^\# + L^\#(A) + A^2 + B \cdot L(A) + B^2 \cdot P :$

> $P^\# := \text{expand}(\text{solve}(eq2, P^\#)) :$

N >

```

conj := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(conj(op(i,
x)), i = 1 .. nops(x)) elif
    (type(x, '*') = true) then simplify( conj(y) · conj(  $\left( \frac{x}{y} \right)$  ) ) elif
    (type(x, '^') = true) then conj(y)op(2, x) elif (type(x, 'complex')
= true) then conjugate(x) elif
    x = Q then Q# elif x = P then P# elif x = B then B# elif x = A then A#
    elif x = Q# then Q elif x = P# then P elif x = A# then A elif x = B#
then B elif x = a then a elif x = d then d elif x = dl then d elif (type(x, 'function')
= true) then if op(0, x) = L then (L#(conj(y))) elif op(0, x) = L#
then (L(conj(y))) elif op(0, x) = S then B · S(conj(y)) + A · Tau(conj(y)) end if

```

end if ;end proc:

N > *conjugue* := **proc**(*x*) ; *expand*(*Sub*(*conj*(*x*))) **end proc:**

> *substitution* := **proc**(*s*) ; *subs*($\text{sqrt}\left(\frac{1}{B}\right) = \frac{1}{\text{sqrt}(B)}, s$) **end proc:**

> *Sub* := **proc**(*s*) ; **if** (*type*(*s*, '+') = *true*) **then** *add*(*Sub*(*op*(*i*, *s*)), *i* = 1 .. *nops*(*s*))
elif (*type*(*s*, '^') **and** *op*(2, *s*) < 0) = *true*
then $\frac{1}{\text{substitution}(\text{op}(1, s)^{-\text{op}(2, s)})}$ **elif** (*type*(*s*, '*') = *true*) **then** *Sub*(*op*(1, *s*))
·Sub($\frac{s}{\text{op}(1, s)}$) **else** *substitution*(*s*) **fi** **end proc:**

N > *I1bar* := *expand*($a^4 \cdot \text{Torsion}(\text{SE}[3], 2, 3)$) :

N > *I3bar* := *expand*($a^3 \cdot \text{Torsion}(\text{SE}[3], 2, 4)$) :

N > *I2bar* := *expand*($a^3 \cdot \text{Torsion}(\text{SE}[3], 2, 5)$) :

N > *I5bar* := *expand*($a^2 \cdot \text{Torsion}(\text{SE}[3], 3, 4)$) :

N > *I4bar* := *expand*($a^2 \cdot \text{Torsion}(\text{SE}[3], 3, 5)$) :

N > *expand*(*I5bar* - *conjugue*(*I5*)) ;

$$0 \quad (11)$$

N > *expand*(*I4bar* - *conjugue*(*I4*)) ;

$$0 \quad (12)$$

N > *expand*(*I3bar* - *conjugue*(*I3*)) ;

$$0 \quad (13)$$

N > *expand*(*I2bar* - *conjugue*(*I2*)) ;

$$0 \quad (14)$$

N > *expand*(*I1bar* - *conjugue*(*I1*)) ;

$$0 \quad (15)$$

N > *I5*;

$$\begin{aligned} & \frac{7}{18} \text{IL}(B) Q - \frac{1}{9} \frac{\text{IL}^\#(B) Q}{B} + \frac{1}{9} \text{IA} Q + \frac{1}{3} \frac{\text{IAL}(B)}{B} - \frac{\text{IL}^\#(L(B))}{B} + \frac{2}{3} \frac{\text{IL}(L^\#(B))}{B} \\ & - \frac{1}{3} \text{IBL}(Q) - \frac{1}{3} \text{IL}(L(B)) - \frac{1}{3} \text{IL}^\#(Q) + \frac{1}{3} \text{IL}(A) + \frac{5}{12} \frac{\text{IL}(B)^2}{B} + \frac{11}{36} \text{IB} Q^2 \\ & + \text{IBP} \end{aligned} \quad (16)$$

N > *I4*;

$$\frac{3}{4} \frac{\text{IL}(B)^2}{B} + \frac{1}{6} \text{IL}(B) Q + \frac{11}{36} \text{IB} Q^2 - \text{IL}(L(B)) - \frac{2}{3} \text{IBL}(Q) + \text{IBP} \quad (17)$$

N > *I3*;

$$-\frac{1}{2} \frac{\text{IPL}^\#(B)}{\sqrt{B}} - \frac{3}{8} \frac{\text{IL}^\#(L(B)) L(B)}{B^{3/2}} - \frac{3}{8} \text{I}\sqrt{B} L^\#(Q) Q - \frac{3}{8} \text{I}\sqrt{B} L(B) L(Q) \quad (18)$$

$$\begin{aligned}
& -\frac{\frac{3}{8} IL(B) L(L(B))}{\sqrt{B}} + \frac{1}{4} I\sqrt{B} L(A) Q - \frac{3}{8} IB^{3/2} QL(Q) - \frac{\frac{1}{2} IL(L^\#(B)) L(B)}{B^{3/2}} \\
& -\frac{3}{8} I\sqrt{B} QL(L(B)) + \frac{\frac{1}{4} IQL(B)^2}{\sqrt{B}} + \frac{\frac{1}{4} IL(B)^3}{B^{3/2}} - \frac{1}{2} I\sqrt{B} L(L(A)) \\
& -\frac{1}{2} IB^{3/2} L(P) + \frac{1}{4} I\sqrt{B} L(L^\#(Q)) + \frac{\frac{1}{4} IL(L^\#(L(B)))}{\sqrt{B}} + \frac{1}{4} I\sqrt{B} L^\#(L(Q)) \\
& -\frac{1}{2} I\sqrt{B} L^\#(P) + \frac{\frac{1}{4} IL^\#(L(L(B)))}{\sqrt{B}} - \frac{\frac{1}{4} IL^\#(B) L(L(B))}{B^{3/2}} + \frac{1}{2} IB^{3/2} PQ \\
& + \frac{1}{2} \frac{S(B)}{\sqrt{B}} + \frac{1}{8} IB^{3/2} Q^3 + \frac{\frac{1}{8} IL^\#(B) L(B) Q}{B^{3/2}} + \frac{\frac{1}{4} IL^\#(B) L(Q)}{\sqrt{B}} + \frac{1}{2} I\sqrt{B} AP \\
& -\frac{\frac{1}{4} IL(L^\#(B)) Q}{\sqrt{B}} - \frac{1}{4} I\sqrt{B} AL(Q) - \frac{\frac{1}{4} IAL(L(B))}{\sqrt{B}} + \frac{1}{8} I\sqrt{B} Q^2 L(B) \\
& + \frac{1}{8} I\sqrt{B} A Q^2 - \frac{\frac{1}{8} IL^\#(Q) L(B)}{\sqrt{B}} - \frac{\frac{1}{8} IL^\#(B) Q^2}{\sqrt{B}} + \frac{\frac{3}{4} IL(B)^2 L^\#(B)}{B^{5/2}} \\
& + \frac{\frac{1}{8} IL^\#(L(B)) Q}{\sqrt{B}} + \frac{\frac{1}{4} IL(A) L(B)}{\sqrt{B}} + \frac{\frac{1}{8} IAL(B)^2}{B^{3/2}}
\end{aligned}$$

N > I2;

$$\begin{aligned}
& \frac{\frac{1}{8} IQL(B)^2}{\sqrt{B}} - \frac{1}{8} I\sqrt{B} Q^2 L(B) - \frac{\frac{3}{4} IL(L(B)) L(B)}{\sqrt{B}} + \frac{1}{4} I\sqrt{B} L(B) L(Q) \\
& -\frac{1}{2} I\sqrt{B} PL(B) - \frac{1}{4} I\sqrt{B} QL(L(B)) - \frac{3}{4} IB^{3/2} QL(Q) + \frac{1}{2} IB^{3/2} PQ \\
& + \frac{\frac{3}{8} IL(B)^3}{B^{3/2}} + \frac{1}{8} IB^{3/2} Q^3 + \frac{1}{2} IB^{3/2} L(L(Q)) + \frac{1}{2} I\sqrt{B} L(L(L(B))) - IB^{3/2} L(P)
\end{aligned} \tag{19}$$

N > expand(I4);

$$\frac{\frac{3}{4} IL(B)^2}{B} + \frac{1}{6} IL(B) Q + \frac{11}{36} IB Q^2 - IL(L(B)) - \frac{2}{3} IBL(Q) + IBP \tag{20}$$

N > conjugue(I5);

$$\frac{\frac{1}{9} IL^\#(B) Q}{B} - \frac{1}{9} IAQ - \frac{\frac{1}{3} IL(B) A}{B} + \frac{IL^\#(L(B))}{B} - \frac{\frac{2}{3} IL(L^\#(B))}{B} + \frac{1}{3} IBL(Q) \tag{21}$$

$$\begin{aligned}
& + \frac{1}{3} \text{IL}(L(B)) + \frac{1}{3} \text{IL}^\#(Q) - \frac{1}{3} \text{IL}(A) - \frac{\frac{5}{12} \text{IL}(B)^2}{B} - \frac{11}{36} \text{IB} Q^2 - \text{IB} P \\
& - \frac{7}{18} \text{IL}(B) Q
\end{aligned}$$

N > I3;

$$\begin{aligned}
& - \frac{\frac{1}{2} \text{IPL}^\#(B)}{\sqrt{B}} - \frac{\frac{3}{8} \text{IL}^\#(L(B)) L(B)}{B^{3/2}} - \frac{3}{8} \text{I}\sqrt{B} L^\#(Q) Q - \frac{3}{8} \text{I}\sqrt{B} L(B) L(Q) \\
& - \frac{\frac{3}{8} \text{IL}(B) L(L(B))}{\sqrt{B}} + \frac{1}{4} \text{I}\sqrt{B} L(A) Q - \frac{3}{8} \text{IB}^{3/2} Q L(Q) - \frac{\frac{1}{2} \text{IL}(L^\#(B)) L(B)}{B^{3/2}} \\
& - \frac{3}{8} \text{I}\sqrt{B} Q L(L(B)) + \frac{\frac{1}{4} \text{I}QL(B)^2}{\sqrt{B}} + \frac{\frac{1}{4} \text{IL}(B)^3}{B^{3/2}} - \frac{1}{2} \text{I}\sqrt{B} L(L(A)) \\
& - \frac{1}{2} \text{IB}^{3/2} L(P) + \frac{1}{4} \text{I}\sqrt{B} L(L^\#(Q)) + \frac{\frac{1}{4} \text{IL}(L^\#(L(B)))}{\sqrt{B}} + \frac{1}{4} \text{I}\sqrt{B} L^\#(L(Q)) \\
& - \frac{1}{2} \text{I}\sqrt{B} L^\#(P) + \frac{\frac{1}{4} \text{IL}^\#(L(L(B)))}{\sqrt{B}} - \frac{\frac{1}{4} \text{IL}^\#(B) L(L(B))}{B^{3/2}} + \frac{1}{2} \text{IB}^{3/2} P Q \\
& + \frac{1}{2} \frac{S(B)}{\sqrt{B}} + \frac{1}{8} \text{IB}^{3/2} Q^3 + \frac{\frac{1}{8} \text{IL}^\#(B) L(B) Q}{B^{3/2}} + \frac{\frac{1}{4} \text{IL}^\#(B) L(Q)}{\sqrt{B}} + \frac{1}{2} \text{I}\sqrt{B} A P \\
& - \frac{\frac{1}{4} \text{IL}(L^\#(B)) Q}{\sqrt{B}} - \frac{1}{4} \text{I}\sqrt{B} A L(Q) - \frac{\frac{1}{4} \text{IAL}(L(B))}{\sqrt{B}} + \frac{1}{8} \text{I}\sqrt{B} Q^2 L(B) \\
& + \frac{1}{8} \text{I}\sqrt{B} A Q^2 - \frac{\frac{1}{8} \text{IL}^\#(Q) L(B)}{\sqrt{B}} - \frac{\frac{1}{8} \text{IL}^\#(B) Q^2}{\sqrt{B}} + \frac{\frac{3}{4} \text{IL}(B)^2 L^\#(B)}{B^{5/2}} \\
& + \frac{\frac{1}{8} \text{IL}^\#(L(B)) Q}{\sqrt{B}} + \frac{\frac{1}{4} \text{IL}(A) L(B)}{\sqrt{B}} + \frac{\frac{1}{8} \text{IAL}(B)^2}{B^{3/2}}
\end{aligned} \tag{22}$$

N > eq2 := IL(Q) L^\#(B) - IB L^\#(P) + IL^\#(Q) L(B) - IAL(Q) B + 2IBL(B) P
+ IB L(L(A)) + 2IQ L^\#(L(B)) - IL(L^\#(Q)) B + IB^2 L(P) + 2IL^\#(L(Q)) B
- IL^\#(L^\#(Q)) - IAL(Q) L(B) - IB Q L(A) - IQ L(L^\#(B)) + 2IL^\#(L(L(B)))
+ IAL^\#(Q) - IAL(L(B)) - IL(L^\#(A)) - 2IAL(A) - IL(L^\#(L(B)))
+ 3IL^\#(L(A)) + BP + S(B) :

N > I3mod := expand(subs(S(B) = solve(eq2, S(B)), I3));

$$\begin{aligned}
I3mod := & -\frac{1}{2} \sqrt{B} P - \frac{\frac{1}{4} IL^\#(B) L(Q)}{\sqrt{B}} + \frac{\frac{1}{4} IL(L^\#(B)) Q}{\sqrt{B}} + \frac{\frac{1}{4} IAL(L(B))}{\sqrt{B}} \\
& + \frac{1}{4} I\sqrt{B} AL(Q) - \frac{\frac{1}{2} IPL^\#(B)}{\sqrt{B}} - \frac{\frac{3}{8} IL^\#(L(B)) L(B)}{B^{3/2}} - \frac{3}{8} I\sqrt{B} L^\#(Q) Q \\
& - \frac{3}{8} I\sqrt{B} L(B) L(Q) - \frac{\frac{3}{8} IL(B) L(L(B))}{\sqrt{B}} - \frac{3}{8} IB^{3/2} QL(Q) \\
& - \frac{\frac{1}{2} IL(L^\#(B)) L(B)}{B^{3/2}} - \frac{3}{8} I\sqrt{B} QL(L(B)) + \frac{\frac{1}{4} IQL(B)^2}{\sqrt{B}} + \frac{\frac{1}{4} IL(B)^3}{B^{3/2}} \\
& - \frac{\frac{1}{4} IL^\#(B) L(L(B))}{B^{3/2}} - I\sqrt{B} L(L(A)) + \frac{\frac{1}{2} IL(L^\#(A))}{\sqrt{B}} + \frac{\frac{1}{2} IL^\#(L^\#(Q))}{\sqrt{B}} \\
& - \frac{\frac{3}{2} IL^\#(L(A))}{\sqrt{B}} + \frac{3}{4} I\sqrt{B} L(L^\#(Q)) + \frac{\frac{3}{4} IL(L^\#(L(B)))}{\sqrt{B}} - \frac{3}{4} I\sqrt{B} L^\#(L(Q)) \\
& - \frac{\frac{3}{4} IL^\#(L(L(B)))}{\sqrt{B}} - I\sqrt{B} L(B) P + \frac{IAL(A)}{\sqrt{B}} - \frac{\frac{1}{2} IAL^\#(Q)}{\sqrt{B}} + \frac{3}{4} I\sqrt{B} L(A) Q \\
& - \frac{\frac{5}{8} IL^\#(Q) L(B)}{\sqrt{B}} - \frac{\frac{7}{8} IL^\#(L(B)) Q}{\sqrt{B}} + \frac{1}{2} IB^{3/2} PQ + \frac{1}{8} IB^{3/2} Q^3 - IB^{3/2} L(P) \\
& + \frac{\frac{1}{8} IL^\#(B) L(B) Q}{B^{3/2}} + \frac{1}{2} I\sqrt{B} AP + \frac{1}{8} I\sqrt{B} Q^2 L(B) + \frac{1}{8} I\sqrt{B} A Q^2 \\
& - \frac{\frac{1}{8} IL^\#(B) Q^2}{\sqrt{B}} + \frac{\frac{3}{4} IL(B)^2 L^\#(B)}{B^{5/2}} + \frac{\frac{1}{4} IL(A) L(B)}{\sqrt{B}} + \frac{\frac{1}{8} IAL(B)^2}{B^{3/2}} \\
& + \frac{\frac{1}{2} IAL(Q) L(B)}{\sqrt{B}}
\end{aligned} \tag{23}$$

N > `expand(I2 - conjugue(I2bar));`

$$0 \tag{24}$$

> `expand(I4 + I4bar);`

$$-\frac{4}{9} IL(B) Q - \frac{\frac{1}{3} IL^\#(L^\#(B))}{B^2} - \frac{\frac{4}{9} IL^\#(B) L(B)}{B^2} - \frac{\frac{2}{9} IA^2}{B} + \frac{\frac{1}{3} IL^\#(A)}{B} - IL(L(B)) \tag{25}$$

$$\begin{aligned}
& -\frac{2}{9} \frac{IL(B) A}{B} + \frac{1}{9} \frac{IL^\#(B) A}{B^2} - \frac{2}{3} IB L(Q) - \frac{2}{9} IA Q + \frac{2}{3} IL^\#(Q) - IL(A) \\
& + \frac{4}{9} \frac{IL(B)^2}{B} + \frac{2}{9} \frac{IL^\#(B) Q}{B} + \frac{2}{3} \frac{IL^\#(L(B))}{B} + \frac{4}{9} \frac{IL^\#(B)^2}{B^3}
\end{aligned}$$

N > I4;

$$\frac{3}{4} \frac{IL(B)^2}{B} + \frac{1}{6} IL(B) Q + \frac{11}{36} IB Q^2 - IL(L(B)) - \frac{2}{3} IB L(Q) + IB P \quad (26)$$

N > *expand(I2 + conjugue(I2))*;

$$\begin{aligned}
& -\frac{1}{4} \frac{IL^\#(B) L(B) Q}{B^{3/2}} - \frac{1}{2} \frac{IAL^\#(B) L(B)}{B^{5/2}} - \frac{1}{2} \frac{IA QL(B)}{\sqrt{B}} - \frac{1}{4} \frac{IQL(B)^2}{\sqrt{B}} \\
& + \frac{1}{4} \frac{IL(B)^3}{B^{3/2}} + \frac{IL^\#(L(A))}{\sqrt{B}} - \frac{1}{2} \frac{IL^\#(L^\#(Q))}{\sqrt{B}} + I\sqrt{B} L^\#(P) - \frac{1}{2} \frac{IL^\#(L^\#(L(B)))}{B^{3/2}} \\
& - I\sqrt{B} L(B) P - \frac{1}{2} I\sqrt{B} Q^2 L(B) - \frac{3}{4} \frac{IL(L(B)) L(B)}{\sqrt{B}} + \frac{1}{4} I\sqrt{B} L(B) L(Q) \\
& - \frac{1}{4} I\sqrt{B} QL(L(B)) - \frac{3}{4} IB^{3/2} QL(Q) - \frac{IL(B) L^\#(B)^2}{B^{7/2}} + \frac{3}{4} \frac{IL^\#(L(B)) L(B)}{B^{3/2}} \\
& - \frac{1}{2} \frac{IL^\#(B) L(B)^2}{B^{5/2}} - \frac{1}{4} \frac{IAL(B)^2}{B^{3/2}} + \frac{3}{4} I\sqrt{B} QL^\#(Q) + \frac{3}{4} \frac{IL^\#(L(B)) Q}{\sqrt{B}} \\
& + \frac{1}{2} \frac{IL^\#(L^\#(B)) L(B)}{B^{5/2}} + \frac{1}{4} \frac{IL^\#(B) Q^2}{\sqrt{B}} + \frac{1}{2} \frac{IAL^\#(Q)}{\sqrt{B}} + \frac{3}{4} \frac{IL^\#(Q) L(B)}{\sqrt{B}} \\
& + \frac{IL^\#(B) P}{\sqrt{B}} + \frac{IL^\#(B) L^\#(L(B))}{B^{5/2}} + \frac{1}{2} \frac{IAL^\#(L(B))}{B^{3/2}} - \frac{IL(A) A}{\sqrt{B}} - \frac{1}{2} \frac{IL(A) L(B)}{\sqrt{B}} \\
& - \frac{1}{4} I\sqrt{B} A Q^2 - I\sqrt{B} PA - \frac{1}{2} I\sqrt{B} L(A) Q + \frac{1}{2} IB^{3/2} L(L(Q)) \\
& + \frac{1}{2} I\sqrt{B} L(L(L(B))) - IB^{3/2} L(P)
\end{aligned} \quad (27)$$

N > *expand(I4 + conjugue(I4))*;

$$-\frac{4}{9} IL(B) Q - \frac{1}{3} \frac{IL^\#(L^\#(B))}{B^2} - \frac{4}{9} \frac{IL^\#(B) L(B)}{B^2} - \frac{2}{9} \frac{IA^2}{B} + \frac{1}{3} \frac{IL^\#(A)}{B} - IL(L(B)) \quad (28)$$

$$\begin{aligned}
& -\frac{\frac{2}{9} \text{IL}(B) A}{B} + \frac{\frac{1}{9} \text{IL}^\#(B) A}{B^2} - \frac{2}{3} \text{IBL}(Q) - \frac{2}{9} \text{IAQ} + \frac{2}{3} \text{IL}^\#(Q) - \text{IL}(A) \\
& + \frac{\frac{4}{9} \text{IL}(B)^2}{B} + \frac{\frac{2}{9} \text{IL}^\#(B) Q}{B} + \frac{\frac{2}{3} \text{IL}^\#(L(B))}{B} + \frac{\frac{4}{9} \text{IL}^\#(B)^2}{B^3}
\end{aligned}$$