```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([l, x, y, z, z1], M, verbose);
```

The following coordinates have been protected:

$[l, x, y, z, z1]$

The following vector fields have been defined and protected:

$[D\_l, D\_x, D\_y, D\_z, D\_z1]$

The following differential 1-forms have been defined and protected:

$[dl, dx, dy, dz, dz1]$

frame name: M **(1)**

Une procédure de dérivation:

```
> Der :=proc(x) ; evalDG( (derive(x, 1) &wedge W[1]) + ( derive(x, 2)  &wedge W[2])
      + (derive(x, 3) &wedge W[3]) + (derive(x, 4) &wedge W[4]) + (derive(x, 5)
     &wedge W[5]) );end proc;
```

```
Der := proc(x)                                                              (2)
    DifferentialGeometry:-evalDG(DifferentialGeometry:-&wedge(derive(x, 1), W[1])
     + DifferentialGeometry:-&wedge(derive(x, 2), W[2]) + DifferentialGeometry:-
    &wedge(derive(x, 3), W[3]) + DifferentialGeometry:-&wedge(derive(x, 4), W[4])
     + DifferentialGeometry:-&wedge(derive(x, 5), W[5]))
end proc
```

```
M > derive :=proc(x, i) local y; y := op(1, x) : if (type(x,  `+`) = true)
        then  add(derive(op(j, x), i), j = 1 .. nops(x)) elif
                         (type(x,  `*`) = true) then expand(derive(y, i)·(x/y) + y·derive((x/y),
        i)) elif
                         (type(x,  `^`) = true) then op(2, x)·y^(op(2, x) − 1) ·derive(y, i)
        elif
                         (type(x, function) = true)  then  x[i]elif
                          (type(x, symbol) = true)    then x[i] else 0 fi end proc:
M > W := Vector([dl, dx, dy, dz, dz1]) :
M > List2 := GenerateForms([dl, dx, dy, dz, dz1], 2) : List1 := [dl, dx, dy, dz,  dz1] :
M > tr1(1) := 1 :
M >  tr1(2) := 1 :
M > tr1(3) := 1 :
M > tr1(4) := 1 :
M > tr1(5) := 2 :
M > tr1(6) := 2 :
M > tr1(7) := 2 :
M > tr1(8) := 3 :
M > tr1(9) := 3 :
M > tr1(10) := 4 :
M > tr2(1) := 2 :
M > tr2(2) := 3 :
```

```
M > tr2(3) := 4 :
M > tr2(4) := 5 :
M > tr2(5) := 3 :
M > tr2(6) := 4 :
M > tr2(7) := 5 :
M > tr2(8) := 4 :
M > tr2(9) := 5 :
M > tr2(10) := 5 :
M > DF :=proc(omega) local T, Res; T := GetComponents(omega, List2);
     Res[1] := evalDG(add(Der(T[i])&wedge List2[i], i=1..10));
     Res[2] := evalDG(add(T[i] &wedge dW[tr1(i)] &wedge W[tr2(i)], i=1..10));
     Res[3] := evalDG(add(T[i] &wedge W[tr1(i)] &wedge dW[tr2(i)], i=1..10) );
    evalDG(Res[1] + Res[2] - Res[3]);
     end proc:
M > BI :=proc(omega) local R, i; R := GetComponents(DF(omega), List3); for i from 1
       to 10 do print(R[i] = 0); od; end proc:
M > List3 := GenerateForms([dl, dx, dy, dz, dz1], 3);
```

$$List3 := [dl \wedge dx \wedge dy, dl \wedge dx \wedge dz, dl \wedge dx \wedge dz1, dl \wedge dy \wedge dz, dl \wedge dy \wedge dz1, dl \wedge dz \wedge dz1, dx \wedge dy \wedge dz, dx \wedge dy \wedge dz1, dx \wedge dz \wedge dz1, dy \wedge dz \wedge dz1] \quad \textbf{(3)}$$

Equations de structures:

```
M > dW[1] := evalDG( I/2 ·I1· (W[2]&wedge W[5]) - I/2 ·I1#· (W[2]&wedge W[4]) - 1/3
       ·(I2 + I3#)·(W[3] &wedge W[4]) - 1/3 ·(I2# + I3)· (W[3] &wedge W[5]) + I0
       · (W[2] &wedge W[3]) );
```

$$dW_1 := I0\, dx \wedge dy - \frac{1}{2}\, \mathrm{I}\, I1^{\#}\, dx \wedge dz + \frac{1}{2}\, \mathrm{I}\, I1\, dx \wedge dz1 + \left(-\frac{1}{3}\, I2 - \frac{1}{3}\, I3^{\#}\right) dy \wedge dz + \left(-\frac{1}{3}\, I2^{\#} - \frac{1}{3}\, I3\right) dy \wedge dz1 \quad \textbf{(4)}$$

```
M > dW[2] := evalDG(3·(W[1] &wedge W[2]) + (W[3]&wedge W[4] ) + (W[3]
       &wedge W[5]));
```

$$dW_2 := 3\ dl \wedge dx + dy \wedge dz + dy \wedge dz1 \quad \textbf{(5)}$$

```
M > dW[4] := evalDG(W[1] &wedge W[4] + I1·(W[2]&wedge W[3]) + I2·(W[2]
       &wedge W[4]) + I3·(W[2]&wedge W[5]) + I4·(W[3] &wedge W[4]) + I5
       ·(W[3] &wedge W[5]));
```

$$dW_4 := dl \wedge dz + I1\, dx \wedge dy + I2\, dx \wedge dz + I3\, dx \wedge dz1 + I4\, dy \wedge dz + I5\, dy \wedge dz1 \quad \textbf{(6)}$$

M > $dW[5] := evalDG\big(W[1]\ \&wedge\ W[5] + I1^{\#}\cdot(W[2]\&wedge\ W[3]) + I2^{\#}\cdot(W[2]\ \&wedge\ W[5]) + I3^{\#}\cdot(W[2]\&wedge\ W[4]) + I4^{\#}\cdot(W[3]\ \&wedge\ W[5]) + I5^{\#}\cdot(W[3]\ \&wedge\ W[4])\big);$

$$dW_5 := dl \wedge dz1 + I1^{\#}\, dx \wedge dy + I3^{\#}\, dx \wedge dz + I2^{\#}\, dx \wedge dz1 + I5^{\#}\, dy \wedge dz + I4^{\#}\, dy \wedge dz1 \tag{7}$$

M > $dW[3] := evalDG(2\cdot(W[1]\ \&wedge\ W[3]) + I\cdot(W[4]\ \&wedge\ W[5]));$

$$dW_3 := 2\, dl \wedge dy + \mathrm{I}\, dz \wedge dz1 \tag{8}$$

M > $BI(dW[1]);$

$$5\, I0 + I0_1 = 0$$

$$-2\,\mathrm{I}\, I1^{\#} - \frac{1}{2}\,\mathrm{I}\, I1^{\#}{}_1 = 0$$

$$2\,\mathrm{I}\, I1 + \frac{1}{2}\,\mathrm{I}\, I1_1 = 0$$

$$-I2 - I3^{\#} - \frac{1}{3}\, I2_1 - \frac{1}{3}\, I3^{\#}{}_1 = 0$$

$$-I2^{\#} - I3 - \frac{1}{3}\, I2^{\#}{}_1 - \frac{1}{3}\, I3_1 = 0$$

$$0 = 0$$

$$-\frac{1}{3}\, I3^{\#}\, I2^{\#} - \frac{1}{3}\, I3\, I3^{\#} - \frac{1}{3}\, I2^2 - \frac{1}{3}\, I2\, I3^{\#} - \frac{1}{2}\,\mathrm{I}\, I1\, I5^{\#} + \frac{1}{2}\,\mathrm{I}\, I1^{\#}\, I4 - \frac{1}{3}\, I2_2 - \frac{1}{3}\, I3^{\#}{}_2 + \frac{1}{2}\,\mathrm{I}\, I1^{\#}{}_3 + I0_4 = 0$$

$$-\frac{1}{3}\, I2^{\#2} - \frac{1}{3}\, I2^{\#}\, I3 - \frac{1}{3}\, I3\, I2 - \frac{1}{3}\, I3\, I3^{\#} - \frac{1}{2}\,\mathrm{I}\, I1\, I4^{\#} + \frac{1}{2}\,\mathrm{I}\, I1^{\#}\, I5 - \frac{1}{3}\, I2^{\#}{}_2 - \frac{1}{3}\, I3_2 - \frac{1}{2}\,\mathrm{I}\, I1_3 + I0_5 = 0$$

$$-\mathrm{I}\, I0 - \frac{1}{2}\,\mathrm{I}\, I1_4 - \frac{1}{2}\,\mathrm{I}\, I1^{\#}{}_5 = 0$$

$$\frac{1}{2}\,\mathrm{I}\, I1 + \frac{1}{2}\,\mathrm{I}\, I1^{\#} + \frac{1}{3}\, I2^{\#}{}_4 + \frac{1}{3}\, I3_4 - \frac{1}{3}\, I2_5 - \frac{1}{3}\, I3^{\#}{}_5 = 0 \tag{9}$$

M > $BI(dW[2]);$

$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0$$
$$0 = 0 \tag{10}$$

M > $BI(dW[3]);$

$$0 = 0$$

$$0=0$$
$$0=0$$
$$0=0$$
$$0=0$$
$$0=0$$
$$0=0$$
$$0=0$$
$$\mathrm{I}\,I2^{\#}+\mathrm{I}\,I2=0$$
$$\mathrm{I}\,I4^{\#}+\mathrm{I}\,I4=0 \tag{11}$$

M > $BI(dW[4]);$

$$4\,I1+I1_{1}=0$$
$$3\,I2+I2_{1}=0$$
$$3\,I3+I3_{1}=0$$
$$2\,I4+I4_{1}=0$$
$$2\,I5+I5_{1}=0$$
$$0=0$$
$$I5\,I3^{\#}-I3\,I5^{\#}+I0+I4_{2}-I2_{3}+I1_{4}=0$$
$$I5\,I2^{\#}+I4\,I3-I3\,I4^{\#}-I2\,I5+I5_{2}-I3_{3}+I1_{5}=0$$
$$-\frac{3}{2}\,\mathrm{I}\,I1-I3_{4}+I2_{5}=0$$
$$\frac{4}{3}\,I3-I2+\frac{1}{3}\,I2^{\#}-I5_{4}+I4_{5}=0 \tag{12}$$

M > $BI(dW[5]):$

$$4\,I1^{\#}+I1^{\#}{}_{1}=0$$
$$3\,I3^{\#}+I3^{\#}{}_{1}=0$$
$$3\,I2^{\#}+I2^{\#}{}_{1}=0$$
$$2\,I5^{\#}+I5^{\#}{}_{1}=0$$
$$2\,I4^{\#}+I4^{\#}{}_{1}=0$$
$$0=0$$
$$I4^{\#}\,I3^{\#}+I5^{\#}\,I2-I2^{\#}\,I5^{\#}-I3^{\#}\,I4+I5^{\#}{}_{2}-I3^{\#}{}_{3}+I1^{\#}{}_{4}=0$$
$$-I5\,I3^{\#}+I3\,I5^{\#}+I0+I4^{\#}{}_{2}-I2^{\#}{}_{3}+I1^{\#}{}_{5}=0$$
$$-\frac{3}{2}\,\mathrm{I}\,I1^{\#}-I2^{\#}{}_{4}+I3^{\#}{}_{5}=0$$
$$I2^{\#}-\frac{4}{3}\,I3^{\#}-\frac{1}{3}\,I2-I4^{\#}{}_{4}+I5^{\#}{}_{5}=0 \tag{13}$$

M > $derive\left(\frac{I}{2}\cdot I1^{\#}\right);$

```
Error, invalid input: derive uses a 2nd argument, i, which is missing
```