```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup( [x, y, z, z1], [a, d, d1], M, verbose);
```

The following coordinates have been protected:

$[x, y, z, z1, a, d, d1]$

The following vector fields have been defined and protected:

$[D\_x, D\_y, D\_z, D\_z1, D\_a, D\_d, D\_d1]$

The following differential 1-forms have been defined and protected:

$[dx, dy, dz, dz1, da, dd, dd1]$

frame name: M **(1)**

Une procédure de dérivation:

```
> Der :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Der(op(i, x)), i = 1
      .. nops(x)) elif
                    (type(x, `*`) = true) then expand( x/y ·Der(y) + y·Der( x/y ) ) elif
                    (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1) ·Der(y) elif
                  ( (type(x, function) = true) or (type(x, symbol) = true) ) then S(x)·W[1]
  + Tau(x)·W[2] + L(x)·W[3] + L^#(x)·W[4]
                    else 0 fi end proc:
  derivation :=proc(x) : collect( Der(x), [W[1], W[2], W[3], W[4]]) : end proc:
```

Les fonctions L et L^#:

```
> L :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
      .. nops(x)) elif
                    (type(x, `*`) = true) then expand( L(y)·x/y + y·L( x/y ) ) elif
                    (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1) ·L(y) elif
                    (type(x, function) = true) then 'L'(x) elif
                    (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:

> L^# :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L^#(op(i, x)), i = 1
      .. nops(x)) elif
                    (type(x, `*`) = true) then expand( L^#(y)·x/y + y·L^#( x/y ) ) elif
                    (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1) ·L^#(y) elif
                    (type(x, function) = true) then 'L^#(x)' elif
                    (type(x, symbol) = true) then 'L^#'(x) else 0 fi end proc:

> Tau :=proc(x) I·(L(L^#(x)) − L^#(L(x))) end proc:

N > S :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(S(op(i, x)), i = 1
      .. nops(x)) elif
                    (type(x, `*`) = true) then expand( S(y)·x/y + y·S( x/y ) ) elif
                    (type(x, `^`) = true) then op(2, x)·y^(op(2, x) − 1) ·S(y) elif
```

```
                    (type(x, function) = true)  then 'S'(x) elif
                    (type(x, symbol) = true)    then 'S'(x) else 0 fi end proc:
```

```
> S# :=
  proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(S#(op(i, x)), i = 1
      .. nops(x)) elif
                    (type(x, `*`) = true) then expand(S#(y)·x/y + y·S#(x/y)) elif
                    (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·S#(y) elif
                    (type(x, function) = true)  then 'S#'(x) elif
                    (type(x, symbol) = true)    then 'S#'(x) else 0 fi end proc:
```

Le nouveau coframe initial (chech, désigné par U), s'exprime en fonction du précédent (chapeau, désigné par V) par la relation U:=n.V.
où n est la matrice:

```
> n := Matrix([[1, 0, 0, 0], [f, 1, 0, 0], [0, h, 1, 0], [h1·A#·B^(1/2), h1, 0, 1]]) :
```

```
> ninv := MatrixInverse(n) :
> V := ninv.Vector([U[1], U[2], U[3], U[4]]) :
> A# := -B^(-1)·A :
```

Le coframe chapeau, désigné par V, s'exprime en fonction du coframe initial, W, par la relation V:=m. W.

```
> m := Matrix([[B^(-1/2), 0, 0, 0], [0, 1, 0, 0], [0, 0, B^(-1/2), 0], [0, 0, 0, B^(1/2)]]) :
> minv := MatrixInverse(m) :
> W := minv.Vector([V[1], V[2], V[3], V[4]]) :
```

On donne ensuite la matrice de groupe:

```
> g := Matrix([[a^3, 0, 0, 0], [0, a^2, 0, 0], [d, 0, a, 0], [d1, 0, 0, a]]);
```

$$g := \begin{bmatrix} a^3 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ d & 0 & a & 0 \\ d1 & 0 & 0 & a \end{bmatrix} \tag{2}$$

```
> ginv := MatrixInverse(g) :
> Mat := map(evalDG, (ExteriorDerivative(g).ginv));
```

$$Mat := \begin{bmatrix} \frac{3\,da}{a} & 0\,dx & 0\,dx & 0\,dx \\ 0\,dx & \frac{2\,da}{a} & 0\,dx & 0\,dx \\ -\frac{d\,da}{a^4} + \frac{dd}{a^3} & 0\,dx & \frac{da}{a} & 0\,dx \\ -\frac{d1\,da}{a^4} + \frac{dd1}{a^3} & 0\,dx & 0\,dx & \frac{da}{a} \end{bmatrix} \quad (3)$$

On obtient la liste des formes de Maurer Cartan:

```
> t[1] := da/a :
```

```
> t[2] := -d*da/a^4 + dd/a^3 :
```

```
> t[3] := -d1*da/a^4 + dd1/a^3 :
```

```
> FD := FrameData([t[1], t[2], t[3], dx, dy, dz, dz1], N) :
  DGsetup(FD, [E], [alpha[1], alpha[2], alpha[3], sigma, rho, zeta, ζ#], verbose) :
```

The following coordinates have been protected:

[x, y, z, z1, a, d, d1]

The following vector fields have been defined and protected:

[E1, E2, E3, E4, E5, E6, E7]

The following differential 1-forms have been defined and protected:

$[\alpha_1, \alpha_2, \alpha_3, \sigma, \rho, \zeta, \zeta^{\#}]$ (4)

Le coframe 'relevé' est noté T. Il est relié au coframe de base U par la relation T:=g.U.

```
> T := Vector([sigma, rho, zeta, ζ#]) :
> U := ginv.T :
```

Les equations de courbure de coframe W sont connues:

```
> dW[1] := H*(W[1]&wedge W[2]) + F*(W[1]&wedge W[4]) + Q*(W[1]
    &wedge W[3]) + B*(W[2]&wedge W[4]) + (W[2]&wedge W[3]) :
> dW[2] := G*(W[1]&wedge W[2]) + E*(W[1]&wedge W[4]) + P*(W[1]
    &wedge W[3]) + A*(W[2]&wedge W[4]) + I*(W[3]&wedge W[4]) :
```

Les equations de courbures du coframe V ont été calculées dans le fichier second step:

```
> dV[1] := evalDG(B^(-1/2)*dW[1] + (Der(B^(-1/2)) &wedge W[1])) :
```

> $dV[2] := dW[2]$:

> $dV[3] := Der\left(B^{-\frac{1}{2}}\right) \&wedge W[3]$:

> $dV[4] := Der\left(B^{\frac{1}{2}}\right) \&wedge W[4]$:

>

Les équations de courbures du coframe U s'en déduisent:

> $dU[1] := dV[1]$:

> $dU[2] := dV[2] + (Der(f) \&wedge V[1]) + f \cdot dV[1]$:

> $dU[3] := dV[3] + (Der(h) \&wedge V[2]) + h \cdot dV[2]$:

> $dU[4] := dV[4] + (Der(h1) \&wedge V[2]) + h1 \cdot dV[2] + \left(Der\left(h1 \cdot A^{\#} \cdot B^{\frac{1}{2}}\right) \&wedge V[1]\right) + h1 \cdot A^{\#} \cdot B^{\frac{1}{2}} \cdot dV[1]$:

On peut maintenant calculer les équations de courbure du coframe 'relevé' :

> Omega := $map(evalDG, g.Vector([dU[1], dU[2], dU[3], dU[4]]))$:

> $Mat := map(evalDG, (ExteriorDerivative(g).ginv))$:

> $Mat2 := Mat \,\&MatrixWedge\, T$:

> $SE := map(evalDG, (Mat2 \,\&MatrixPlus\, \text{Omega}))$:

> $List := GenerateForms([\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \text{sigma}, \text{rho}, \text{zeta}, \zeta^{\#}], 2)$:

> $result := \mathbf{proc}(l)\ \mathbf{local}\, k, t, X; X := 0 : t := expand(GetComponents(l, List)) : \mathbf{for}\, k\ \mathbf{from}\, 1\ \mathbf{to}\, 21\ \mathbf{do}\, X := X + t[k] \cdot List[k]\ \mathbf{od}; X; \mathbf{end\ proc}$:

> $Torsion := \mathbf{proc}(S, i, j)\ \mathbf{local}\, k, X;\ k := 7 \cdot (i-1) - \frac{i \cdot (i-1)}{2} + j - i;\ X := map(expand, GetComponents(S, List));\ X[k]; \mathbf{end\ proc}$:

> $B^{\#} := \frac{1}{B}$:

> $A^{\#} = -B^{\#} \cdot A$:

> $F := L(B) + B \cdot Q + A$:

> $h1 := \frac{I}{2} \cdot \left(\frac{L(B)}{B^{\frac{1}{2}}} + \frac{1}{3} \cdot B^{\frac{1}{2}} \cdot Q\right) : f := \frac{1}{2}\left(\frac{L(B)}{B^{\frac{1}{2}}} + B^{\frac{1}{2}} \cdot Q\right)$:

> $h := -\frac{I}{2} \cdot \left(L^{\#}(B^{\#}) \cdot B^{\frac{1}{2}} + \frac{1}{3} \cdot \frac{Q^{\#}}{B^{\frac{1}{2}}}\right) : h := -\frac{\frac{1}{6}\, \mathrm{I} \cdot \left(-2\, L^{\#}(B) + 2\, A\, B + B^2\, Q + L(B)\, B\right)}{B^{3/2}}$:

> $E := L(A) + B \cdot P$:

```
> H := expand(I·L(L(B)) + I·Q·L(B) + I·B·L(Q) + 2·I·L(A) - I·L#(Q));
  G := I· L(L(A)) + 2·I·P·L(B) - I·L#(P) - I·Q·L(A) + I·B·L(P);
```

$$H := \mathrm{I}L(L(B)) + \mathrm{I}QL(B) + \mathrm{I}BL(Q) + 2\,\mathrm{I}L(A) - \mathrm{I}L^{\#}(Q)$$

$$G := \mathrm{I}L(L(A)) + 2\,\mathrm{I}PL(B) - \mathrm{I}L^{\#}(P) - \mathrm{I}QL(A) + \mathrm{I}BL(P) \tag{5}$$

```
> result(SE[1]);
```

$$3\,\alpha_1 \wedge \sigma + \left(\frac{\frac{1}{2}\,\mathrm{I}L\left(L^{\#}(B)\right)}{a^2 B} - \frac{\frac{1}{2}\,\mathrm{I}L^{\#}(L(B))}{a^2 B} - \frac{\frac{5}{12}\,\mathrm{I}L^{\#}(B)\,L(B)}{a^2 B^2} - \frac{\frac{5}{12}\,\mathrm{I}L(B)^2}{a^2 B} \right.$$
$$+ \frac{\mathrm{I}L(L(B))}{a^2} + \frac{2\,\mathrm{I}L(A)}{a^2} - \frac{\mathrm{I}L^{\#}(Q)}{a^2} + \frac{dI}{a^3} + \frac{d}{a^3} - \frac{\frac{5}{12}\,\mathrm{I}L^{\#}(B)\,Q}{a^2 B} - \frac{\frac{5}{6}\,\mathrm{I}L(B)\,A}{a^2 B}$$
$$\left. + \frac{\frac{7}{12}\,\mathrm{I}L(B)\,Q}{a^2} + \frac{\mathrm{I}BL(Q)}{a^2} \right) \sigma \wedge \rho + \frac{1}{2}\,\frac{\sqrt{B}\,Q\sigma \wedge \zeta}{a} + \left(\frac{1}{2}\,\frac{L^{\#}(B)}{a\,B^{3/2}} + \frac{1}{2}\,\frac{L(B)}{a\sqrt{B}} \right.$$
$$\left. + \frac{1}{2}\,\frac{\sqrt{B}\,Q}{a} + \frac{A}{a\sqrt{B}} \right) \sigma \wedge \zeta^{\#} + \rho \wedge \zeta + \rho \wedge \zeta^{\#} \tag{6}$$

```
> result(SE[2]);
```

$$2\,\alpha_1 \wedge \rho + \left(-\frac{\frac{1}{18}\,\mathrm{I}A\,L^{\#}(B)\,Q}{a^3 B^{3/2}} - \frac{\frac{1}{4}\,\mathrm{I}L^{\#}(B)\,L(B)\,Q}{a^3 B^{3/2}} - \frac{\frac{7}{18}\,\mathrm{I}A\,L(B)\,Q}{a^3\sqrt{B}} \right.$$
$$- \frac{\frac{1}{6}\,\mathrm{I}A\,L^{\#}(B)\,L(B)}{a^3 B^{5/2}} + \frac{1}{3}\,\frac{\sqrt{B}\,d\,Q}{a^4} + \frac{1}{3}\,\frac{L^{\#}(B)\,dI}{a^4 B^{3/2}} + \frac{1}{3}\,\frac{\sqrt{B}\,dI\,Q}{a^4} + \frac{2}{3}\,\frac{A\,dI}{a^4\sqrt{B}}$$
$$+ \frac{1}{3}\,\frac{L(B)\,dI}{a^4\sqrt{B}} + \frac{\frac{1}{2}\,\mathrm{I}L^{\#}(L(L(B)))}{a^3\sqrt{B}} + \frac{\frac{1}{2}\,\mathrm{I}\sqrt{B}\,L^{\#}(L(Q))}{a^3} - \frac{\frac{1}{2}\,\mathrm{I}L\left(L^{\#}(L(B))\right)}{a^3\sqrt{B}}$$
$$- \frac{\frac{1}{2}\,\mathrm{I}\sqrt{B}\,L\left(L^{\#}(Q)\right)}{a^3} - \frac{\frac{1}{6}\,\mathrm{I}L(B)^3}{a^3 B^{3/2}} + \frac{\mathrm{I}\sqrt{B}\,L(L(A))}{a^3} - \frac{\mathrm{I}\sqrt{B}\,L^{\#}(P)}{a^3} + \frac{\mathrm{I}B^{3/2}L(P)}{a^3}$$
$$- \frac{\frac{5}{12}\,\mathrm{I}L(B)^2\,L^{\#}(B)}{a^3 B^{5/2}} + \frac{\frac{1}{2}\,\mathrm{I}L\left(L^{\#}(B)\right)L(B)}{a^3 B^{3/2}} + \frac{\frac{1}{2}\,\mathrm{I}L(B)\,L(A)}{a^3\sqrt{B}} + \frac{\frac{5}{12}\,\mathrm{I}B^{3/2}QL(Q)}{a^3}$$
$$+ \frac{\frac{5}{3}\,\mathrm{I}\sqrt{B}\,PL(B)}{a^3} - \frac{\frac{1}{6}\,\mathrm{I}L^{\#}(B)\,Q^2}{a^3\sqrt{B}} - \frac{\frac{1}{4}\,\mathrm{I}L(B)\,L^{\#}(L(B))}{a^3 B^{3/2}} - \frac{\frac{1}{4}\,\mathrm{I}L(B)\,L^{\#}(Q)}{a^3\sqrt{B}}$$
$$\left. + \frac{\frac{5}{12}\,\mathrm{I}L(L(B))\,L(B)}{a^3\sqrt{B}} + \frac{\frac{1}{12}\,\mathrm{I}QL(B)^2}{a^3\sqrt{B}} + \frac{\frac{5}{12}\,\mathrm{I}\sqrt{B}\,L(L(B))\,Q}{a^3} \right) \tag{7}$$

$$+\frac{\frac{1}{4}\,\mathrm{I}\sqrt{B}\,Q^2\,L(B)}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\sqrt{B}\,Q\,L(A)}{a^3}-\frac{\frac{5}{12}\,\mathrm{I}\sqrt{B}\,L^{\#}(Q)\,Q}{a^3}-\frac{\frac{1}{3}\,\mathrm{I}A^2\,L(B)}{a^3\,B^{3/2}}$$
$$-\frac{\frac{1}{9}\,\mathrm{I}A^2\,Q}{a^3\sqrt{B}}+\frac{\frac{1}{12}\,\mathrm{I}Q\,L^{\#}(L(B))}{a^3\sqrt{B}}-\frac{\frac{1}{4}\,\mathrm{I}A\,L(B)^2}{a^3\,B^{3/2}}+\frac{\frac{5}{12}\,\mathrm{I}\sqrt{B}\,L(B)\,L(Q)}{a^3}$$
$$-\frac{\frac{1}{6}\,\mathrm{I}L(L(B))\,A}{a^3\sqrt{B}}-\frac{\frac{1}{6}\,\mathrm{I}\sqrt{B}\,L(Q)\,A}{a^3}-\frac{\frac{1}{3}\,\mathrm{I}P\,L^{\#}(B)}{a^3\sqrt{B}}+\frac{\frac{1}{3}\,\mathrm{I}\sqrt{B}\,P\,A}{a^3}$$
$$+\frac{\frac{1}{36}\,\mathrm{I}\sqrt{B}\,A\,Q^2}{a^3}+\frac{\frac{1}{6}\,\mathrm{I}L^{\#}(B)\,L(L(B))}{a^3\,B^{3/2}}+\frac{\frac{1}{6}\,\mathrm{I}L^{\#}(B)\,L(Q)}{a^3\sqrt{B}}\Bigg)\sigma\wedge\rho+\Bigg(\frac{1}{2}\,\frac{L(B)^2}{a^2\,B}$$
$$+\frac{1}{3}\,\frac{L(B)\,Q}{a^2}+\frac{1}{3}\,\frac{B\,Q^2}{a^2}-\frac{1}{2}\,\frac{L(L(B))}{a^2}-\frac{1}{2}\,\frac{B\,L(Q)}{a^2}+\frac{\mathrm{I}dl}{a^3}+\frac{1}{2}\,\frac{L(B)\,A}{a^2\,B}$$
$$+\frac{1}{6}\,\frac{A\,Q}{a^2}+\frac{B\,P}{a^2}\Bigg)\sigma\wedge\zeta+\Bigg(\frac{1}{3}\,\frac{L^{\#}(B)\,L(B)}{a^2\,B^2}-\frac{1}{6}\,\frac{L^{\#}(B)\,Q}{a^2\,B}+\frac{2}{3}\,\frac{L(B)\,Q}{a^2}$$
$$+\frac{1}{3}\,\frac{L(B)^2}{a^2\,B}+\frac{1}{3}\,\frac{B\,Q^2}{a^2}-\frac{1}{2}\,\frac{L^{\#}(L(B))}{a^2\,B}-\frac{1}{2}\,\frac{L^{\#}(Q)}{a^2}+\frac{B\,P}{a^2}-\frac{\mathrm{I}d}{a^3}+\frac{L(A)}{a^2}$$
$$+\frac{1}{6}\,\frac{L(B)\,A}{a^2\,B}+\frac{1}{6}\,\frac{A\,Q}{a^2}\Bigg)\sigma\wedge\zeta^{\#}+\frac{1}{3}\,\frac{\sqrt{B}\,Q\rho\wedge\zeta}{a}+\Bigg(\frac{1}{3}\,\frac{L^{\#}(B)}{a\,B^{3/2}}+\frac{1}{3}\,\frac{L(B)}{a\sqrt{B}}$$
$$+\frac{1}{3}\,\frac{\sqrt{B}\,Q}{a}+\frac{2}{3}\,\frac{A}{a\sqrt{B}}\Bigg)\rho\wedge\zeta^{\#}+\mathrm{I}\zeta\wedge\zeta^{\#}$$

```
> result(SE[3]);
```

$$\alpha_1\wedge\zeta+\alpha_2\wedge\sigma+\Bigg(\frac{1}{108}\,\frac{A\,L^{\#}(B)\,Q\,L(B)}{B^2\,a^4}-\frac{\frac{1}{18}\,\mathrm{I}dl\,A\,L(B)}{B\,a^5}+\frac{\frac{1}{6}\,\mathrm{I}d\,L^{\#}(B)\,L(B)}{B^2\,a^5} \tag{8}$$
$$-\frac{\frac{5}{6}\,\mathrm{I}d\,L(B)\,A}{B\,a^5}+\frac{\frac{1}{18}\,\mathrm{I}dl\,L^{\#}(B)\,Q}{B\,a^5}-\frac{\frac{1}{9}\,\mathrm{I}dl\,L^{\#}(B)\,L(B)}{B^2\,a^5}+\frac{\frac{1}{9}\,\mathrm{I}dl\,A\,L^{\#}(B)}{B^2\,a^5}$$
$$-\frac{\frac{7}{18}\,\mathrm{I}d\,Q\,L^{\#}(B)}{B\,a^5}+\frac{d^2}{a^6}-\frac{\frac{1}{6}\,\mathrm{I}S(L(B))}{a^4}-\frac{\frac{1}{3}\,\mathrm{I}S(A)}{a^4}+\frac{2}{9}\,\frac{A\,L^{\#}(B)^2\,L(B)}{B^4\,a^4}$$
$$+\frac{1}{18}\,\frac{B\,P\,A\,Q}{a^4}-\frac{5}{18}\,\frac{Q\,L(B)\,L(A)}{a^4}-\frac{11}{18}\,\frac{P\,L^{\#}(B)\,L(B)}{B\,a^4}+\frac{5}{18}\,\frac{B\,P\,Q\,L(B)}{a^4}$$
$$+\frac{11}{18}\,\frac{P\,A\,L(B)}{a^4}-\frac{1}{27}\,\frac{A^2\,Q\,L(B)}{B\,a^4}+\frac{7}{216}\,\frac{A\,L(B)^2\,Q}{B\,a^4}+\frac{5}{216}\,\frac{A\,Q^2\,L(B)}{a^4}$$

$$
-\frac{1}{18}\,\frac{P\,L^{\#}(B)\,Q}{a^4}-\frac{2}{9}\,\frac{P\,L^{\#}(B)\,A}{B\,a^4}-\frac{7}{18}\,\frac{A\,Q\,L(A)}{a^4}-\frac{1}{6}\,\frac{A\,L(B)\,L(A)}{B\,a^4}
$$

$$
+\frac{1}{36}\,\frac{A\,L^{\#}(L(B))\,Q}{B\,a^4}+\frac{1}{18}\,\frac{A\,L^{\#}(A)\,Q}{B\,a^4}+\frac{1}{6}\,\frac{A\,L^{\#}(A)\,L(B)}{B^2\,a^4}-\frac{1}{18}\,\frac{A\,L^{\#}\left(L^{\#}(B)\right)Q}{B^2\,a^4}
$$

$$
-\frac{1}{6}\,\frac{A\,L^{\#}\left(L^{\#}(B)\right)L(B)}{B^3\,a^4}+\frac{7}{18}\,\frac{L^{\#}(B)\,Q\,L(A)}{B\,a^4}+\frac{2}{27}\,\frac{A\,L^{\#}(B)^2\,Q}{B^3\,a^4}
$$

$$
+\frac{1}{12}\,\frac{A\,L^{\#}(L(B))\,L(B)}{B^2\,a^4}+\frac{1}{36}\,\frac{A\,L^{\#}(Q)\,Q}{a^4}+\frac{1}{12}\,\frac{A\,L^{\#}(Q)\,L(B)}{B\,a^4}
$$

$$
+\frac{1}{6}\,\frac{L^{\#}(B)\,L(B)\,L(A)}{B^2\,a^4}-\frac{1}{18}\,\frac{A\,L^{\#}(B)\,L(B)^2}{B^3\,a^4}+\frac{1}{108}\,\frac{A\,L^{\#}(B)\,Q^2}{B\,a^4}
$$

$$
+\frac{1}{54}\,\frac{A^2\,L^{\#}(B)\,Q}{B^2\,a^4}+\frac{1}{18}\,\frac{A^2\,L^{\#}(B)\,L(B)}{B^3\,a^4}+\frac{\frac{7}{9}\,\mathrm{I}\,d\,Q\,L(B)}{a^5}+\frac{\frac{1}{18}\,\mathrm{I}\,d1\,Q\,L(B)}{a^5}
$$

$$
+\frac{\frac{1}{18}\,\mathrm{I}\,d\,Q\,A}{a^5}-\frac{\frac{1}{18}\,\mathrm{I}\,d1\,A\,Q}{a^5}-\frac{1}{3}\,\frac{A\,L^{\#}(P)}{a^4}-\frac{1}{3}\,\frac{L^{\#}(B)\,L(L(A))}{B\,a^4}
$$

$$
+\frac{1}{6}\,\frac{B\,L(B)\,L(P)}{a^4}-\frac{1}{6}\,\frac{B\,Q\,L^{\#}(P)}{a^4}+\frac{1}{3}\,\frac{B\,A\,L(P)}{a^4}+\frac{1}{6}\,\frac{B\,Q\,L(L(A))}{a^4}
$$

$$
-\frac{1}{6}\,\frac{L(B)\,L^{\#}(P)}{a^4}-\frac{1}{36}\,\frac{A^2\,L(B)^2}{B^2\,a^4}-\frac{1}{9}\,\frac{A^3\,L(B)}{B^2\,a^4}+\frac{1}{3}\,\frac{L^{\#}(B)\,L^{\#}(P)}{B\,a^4}+\frac{d1\,d}{a^6}
$$

$$
+\frac{\frac{7}{6}\,\mathrm{I}\,d\,L(L(B))}{a^5}-\frac{\mathrm{I}\,d\,L^{\#}(Q)}{a^5}+\frac{\frac{1}{6}\,\mathrm{I}\,d1\,L^{\#}(Q)}{a^5}-\frac{\frac{1}{6}\,\mathrm{I}\,S(B)\,Q}{a^4}+\frac{\frac{7}{3}\,\mathrm{I}\,d\,L(A)}{a^5}
$$

$$
+\frac{\frac{1}{3}\,\mathrm{I}\,S\left(L^{\#}(B)\right)}{B\,a^4}-\frac{\frac{1}{6}\,\mathrm{I}\,B\,S(Q)}{a^4}+\frac{\frac{7}{6}\,\mathrm{I}\,B\,d\,L(Q)}{a^5}+\frac{\frac{1}{36}\,\mathrm{I}\,B\,d\,Q^2}{a^5}-\frac{\frac{2}{9}\,\mathrm{I}\,d1\,A^2}{B\,a^5}
$$

$$
-\frac{\frac{5}{12}\,\mathrm{I}\,d\,L(B)^2}{B\,a^5}+\frac{\frac{1}{6}\,\mathrm{I}\,d1\,L^{\#}(L(B))}{B\,a^5}-\frac{\frac{1}{3}\,\mathrm{I}\,d\,L\left(L^{\#}(B)\right)}{B\,a^5}+\frac{\frac{1}{36}\,\mathrm{I}\,B\,d1\,Q^2}{a^5}
$$

$$
+\frac{\frac{1}{36}\,\mathrm{I}\,d1\,L(B)^2}{B\,a^5}-\frac{\frac{1}{3}\,\mathrm{I}\,d1\,L^{\#}\left(L^{\#}(B)\right)}{B^2\,a^5}+\frac{\frac{1}{3}\,\mathrm{I}\,d1\,L^{\#}(A)}{B\,a^5}-\frac{\frac{1}{3}\,\mathrm{I}\,S(B)\,L^{\#}(B)}{B^2\,a^4}
$$

$$
+\frac{\frac{4}{9}\,\mathrm{I}\,d1\,L^{\#}(B)^2}{B^3\,a^5}+\frac{1}{9}\,\frac{P\,A^2}{a^4}-\frac{1}{27}\,\frac{A^3\,Q}{B\,a^4}-\frac{1}{108}\,\frac{A^2\,Q^2}{a^4}+\frac{1}{216}\,\frac{B\,A\,Q^3}{a^4}
$$

$$+\frac{1}{6}\,\frac{B^2\,Q\,L(P)}{a^4}+\frac{1}{6}\,\frac{L(B)\,L(L(A))}{a^4}-\frac{7}{36}\,\frac{B\,Q^2\,L(A)}{a^4}-\frac{1}{12}\,\frac{L(B)^2\,L(A)}{B\,a^4}$$

$$+\frac{5}{18}\,\frac{P\,L(B)^2}{a^4}+\frac{1}{72}\,\frac{A\,L(B)^3}{B^2\,a^4}+\frac{1}{9}\,\frac{P\,L^{\#}(B)^2}{B^2\,a^4}-\frac{1}{3}\,\frac{L^{\#}(B)\,L(P)}{a^4}$$

$$+\frac{1}{3}\,\frac{A\,L(L(A))}{a^4}\Bigg)\,\sigma\wedge\rho+\Bigg(\frac{\frac{1}{3}\,\mathrm{I}\,P\,L^{\#}(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{3}\,\mathrm{I}\,\sqrt{B}\,P\,A}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,B^{3/2}\,P\,Q}{a^3}$$

$$-\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,P\,L(B)}{a^3}-\frac{1}{2}\,\frac{S(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,A\,L^{\#}(B)\,L(B)}{B^{5/2}\,a^3}-\frac{\frac{1}{36}\,\mathrm{I}\,A\,L^{\#}(B)\,Q}{B^{3/2}\,a^3}$$

$$-\frac{\frac{5}{36}\,\mathrm{I}\,A\,Q\,L(B)}{\sqrt{B}\,a^3}-\frac{\frac{11}{36}\,\mathrm{I}\,L^{\#}(B)\,L(B)\,Q}{B^{3/2}\,a^3}-\frac{\frac{1}{72}\,\mathrm{I}\,B^{3/2}\,Q^3}{a^3}+\frac{1}{2}\,\frac{\sqrt{B}\,d\,Q}{a^4}$$

$$+\frac{1}{6}\,\frac{L^{\#}(B)\,d\,I}{a^4\,B^{3/2}}+\frac{1}{6}\,\frac{\sqrt{B}\,d\,I\,Q}{a^4}+\frac{1}{3}\,\frac{A\,d\,I}{a^4\sqrt{B}}+\frac{1}{6}\,\frac{L(B)\,d\,I}{a^4\sqrt{B}}+\frac{\frac{5}{12}\,\mathrm{I}\,L\left(L^{\#}(B)\right)\,Q}{\sqrt{B}\,a^3}$$

$$-\frac{\frac{1}{72}\,\mathrm{I}\,L^{\#}(B)\,Q^2}{\sqrt{B}\,a^3}-\frac{\frac{1}{18}\,\mathrm{I}\,\sqrt{B}\,A\,Q^2}{a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,A\,L(B)^2}{B^{3/2}\,a^3}-\frac{\frac{1}{18}\,\mathrm{I}\,A^2\,Q}{\sqrt{B}\,a^3}$$

$$-\frac{\frac{1}{4}\,\mathrm{I}\,L(B)\,L^{\#}(L(B))}{a^3\,B^{3/2}}-\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,Q\,L(A)}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,L(A)\,L(B)}{\sqrt{B}\,a^3}$$

$$-\frac{\frac{1}{12}\,\mathrm{I}\,\sqrt{B}\,L(Q)\,L(B)}{a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,B^{3/2}\,L(Q)\,Q}{a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,L(L(B))\,L(B)}{\sqrt{B}\,a^3}$$

$$-\frac{\frac{1}{12}\,\mathrm{I}\,\sqrt{B}\,L(L(B))\,Q}{a^3}-\frac{\frac{7}{72}\,\mathrm{I}\,Q\,L(B)^2}{\sqrt{B}\,a^3}-\frac{\frac{1}{9}\,\mathrm{I}\,\sqrt{B}\,Q^2\,L(B)}{a^3}$$

$$+\frac{\frac{5}{12}\,\mathrm{I}\,L\left(L^{\#}(B)\right)\,L(B)}{B^{3/2}\,a^3}-\frac{\frac{1}{4}\,\mathrm{I}\,L^{\#}(L(B))\,Q}{\sqrt{B}\,a^3}-\frac{\frac{7}{24}\,\mathrm{I}\,L^{\#}(B)\,L(B)^2}{B^{5/2}\,a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,A^2\,L(B)}{B^{3/2}\,a^3}\Bigg)$$

$$\sigma\wedge\zeta+\Bigg(\frac{\frac{1}{3}\,\mathrm{I}\,P\,L^{\#}(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{3}\,\mathrm{I}\,\sqrt{B}\,P\,A}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,B^{3/2}\,P\,Q}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,P\,L(B)}{a^3}$$

$$+\frac{\frac{1}{18}\,\mathrm{I}\,A\,Q\,L(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{18}\,\mathrm{I}\,A\,L^{\#}(B)\,L(B)}{B^{5/2}\,a^3}+\frac{\frac{1}{36}\,\mathrm{I}\,L^{\#}(B)\,Q\,L(B)}{B^{3/2}\,a^3}-\frac{\frac{1}{72}\,\mathrm{I}\,B^{3/2}\,Q^3}{a^3}$$

$$
\begin{aligned}
&+\frac{1}{3}\,\frac{d\,L(B)}{\sqrt{B}\,a^4}+\frac{1}{3}\,\frac{d\,L^{\#}(B)}{B^{3/2}\,a^4}+\frac{2}{3}\,\frac{d\,A}{\sqrt{B}\,a^4}-\frac{\frac{1}{18}\,\mathrm{I}\,A\,L^{\#}(B)\,Q}{a^3\,B^{3/2}}+\frac{1}{3}\,\frac{\sqrt{B}\,d\,Q}{a^4}\\
&-\frac{\frac{1}{72}\,\mathrm{I}\,L(B)^3}{B^{3/2}\,a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,Q\,L(A)}{a^3}+\frac{\frac{1}{36}\,\mathrm{I}\,\sqrt{B}\,A\,Q^2}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,L(A)\,L(B)}{\sqrt{B}\,a^3}\\
&-\frac{\frac{1}{24}\,\mathrm{I}\,\sqrt{B}\,Q^2\,L(B)}{a^3}-\frac{\frac{1}{24}\,\mathrm{I}\,Q\,L(B)^2}{\sqrt{B}\,a^3}+\frac{\frac{1}{9}\,\mathrm{I}\,A^2\,L(B)}{B^{3/2}\,a^3}+\frac{\frac{1}{9}\,\mathrm{I}\,A^2\,Q}{\sqrt{B}\,a^3}\\
&+\frac{\frac{1}{36}\,\mathrm{I}\,A\,L(B)^2}{B^{3/2}\,a^3}-\frac{\frac{1}{36}\,\mathrm{I}\,L^{\#}(B)\,Q^2}{\sqrt{B}\,a^3}+\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}(B)\,L(A)}{B^{3/2}\,a^3}+\frac{\frac{1}{18}\,\mathrm{I}\,L^{\#}(B)\,L(B)^2}{B^{5/2}\,a^3}\\
&-\frac{\frac{1}{3}\,\mathrm{I}\,A\,L(A)}{\sqrt{B}\,a^3}+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}\left(L^{\#}(B)\right)\,L(B)}{B^{5/2}\,a^3}+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}\left(L^{\#}(B)\right)\,Q}{B^{3/2}\,a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(A)\,L(B)}{B^{3/2}\,a^3}\\
&-\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(A)\,Q}{\sqrt{B}\,a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,L^{\#}(Q)\,L(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,\sqrt{B}\,L^{\#}(Q)\,Q}{a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,L^{\#}(L(B))\,L(B)}{B^{3/2}\,a^3}\\
&-\frac{\frac{1}{12}\,\mathrm{I}\,L^{\#}(L(B))\,Q}{\sqrt{B}\,a^3}-\frac{\frac{2}{9}\,\mathrm{I}\,L^{\#}(B)^2\,Q}{B^{5/2}\,a^3}-\frac{\frac{2}{9}\,\mathrm{I}\,L^{\#}(B)^2\,L(B)}{B^{7/2}\,a^3}\Bigg)\,\sigma\wedge\zeta^{\#}\\
&+\Bigg(\frac{\frac{7}{12}\,\mathrm{I}\,L^{\#}(B)\,L(B)}{B^2\,a^2}+\frac{\frac{1}{36}\,\mathrm{I}\,L^{\#}(B)\,Q}{B\,a^2}+\frac{\frac{1}{6}\,\mathrm{I}\,B\,L(Q)}{a^2}+\frac{\frac{1}{18}\,\mathrm{I}\,A\,Q}{a^2}\\
&+\frac{\frac{7}{36}\,\mathrm{I}\,Q\,L(B)}{a^2}+\frac{\frac{1}{3}\,\mathrm{I}\,L(A)}{a^2}+\frac{\frac{1}{6}\,\mathrm{I}\,L(L(B))}{a^2}+\frac{\frac{1}{2}\,\mathrm{I}\,L^{\#}(L(B))}{B\,a^2}+\frac{\frac{1}{36}\,\mathrm{I}\,B\,Q^2}{a^2}\\
&-\frac{\frac{5}{6}\,\mathrm{I}\,L\left(L^{\#}(B)\right)}{B\,a^2}+\frac{d}{a^3}\Bigg)\,\rho\wedge\zeta+\Bigg(\frac{\frac{4}{9}\,\mathrm{I}\,L^{\#}(B)^2}{B^3\,a^2}-\frac{\frac{1}{9}\,\mathrm{I}\,L^{\#}(B)\,L(B)}{B^2\,a^2}\\
&+\frac{\frac{1}{18}\,\mathrm{I}\,L^{\#}(B)\,Q}{B\,a^2}-\frac{\frac{1}{18}\,\mathrm{I}\,A\,L(B)}{B\,a^2}+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(L(B))}{B\,a^2}-\frac{\frac{1}{18}\,\mathrm{I}\,A\,Q}{a^2}+\frac{\frac{1}{9}\,\mathrm{I}\,A\,L^{\#}(B)}{B^2\,a^2}\\
&+\frac{\frac{1}{6}\,\mathrm{I}\,L^{\#}(Q)}{a^2}+\frac{\frac{1}{18}\,\mathrm{I}\,Q\,L(B)}{a^2}+\frac{\frac{1}{36}\,\mathrm{I}\,L(B)^2}{B\,a^2}+\frac{\frac{1}{36}\,\mathrm{I}\,B\,Q^2}{a^2}-\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}\left(L^{\#}(B)\right)}{B^2\,a^2}\\
&-\frac{\frac{2}{9}\,\mathrm{I}\,A^2}{B\,a^2}+\frac{\frac{1}{3}\,\mathrm{I}\,L^{\#}(A)}{B\,a^2}+\frac{d}{a^3}\Bigg)\,\rho\wedge\zeta^{\#}+\Bigg(\frac{1}{6}\,\frac{L^{\#}(B)}{a\,B^{3/2}}+\frac{1}{3}\,\frac{A}{a\,\sqrt{B}}+\frac{1}{6}\,\frac{\sqrt{B}\,Q}{a}\\
&+\frac{1}{6}\,\frac{L(B)}{a\,\sqrt{B}}\Bigg)\,\zeta\wedge\zeta^{\#}
\end{aligned}
$$

```
> result(SE[4]);
```

$$\alpha_1 \wedge \zeta^{\#} + \alpha_3 \wedge \sigma + \left(-\frac{1}{3}\,\frac{A\,L^{\#}(B)\,Q\,L(B)}{B^2\,a^4} + \frac{dl^2}{a^6} + \frac{\frac{1}{2}\,\mathrm{I}\,S(L(B))}{a^4} - \frac{1}{18}\,\frac{B\,P\,A\,Q}{a^4} \right. \tag{9}$$

$$+ \frac{5}{9}\,\frac{Q\,L(B)\,L(A)}{a^4} + \frac{1}{6}\,\frac{P\,L^{\#}(B)\,L(B)}{B\,a^4} - \frac{5}{18}\,\frac{B\,P\,Q\,L(B)}{a^4} - \frac{1}{6}\,\frac{P\,A\,L(B)}{a^4}$$

$$+ \frac{7}{24}\,\frac{A\,L(B)^2\,Q}{B\,a^4} + \frac{17}{216}\,\frac{A\,Q^2\,L(B)}{a^4} + \frac{1}{18}\,\frac{P\,L^{\#}(B)\,Q}{a^4} + \frac{5}{18}\,\frac{A\,Q\,L(A)}{a^4}$$

$$+ \frac{5}{6}\,\frac{A\,L(B)\,L(A)}{B\,a^4} - \frac{1}{6}\,\frac{A\,L^{\#}(L(B))\,Q}{B\,a^4} + \frac{1}{18}\,\frac{L^{\#}(B)\,Q\,L(A)}{B\,a^4} - \frac{A\,L^{\#}(L(B))\,L(B)}{B^2\,a^4}$$

$$- \frac{1}{6}\,\frac{A\,L^{\#}(Q)\,Q}{a^4} - \frac{1}{2}\,\frac{A\,L^{\#}(Q)\,L(B)}{B\,a^4} + \frac{1}{6}\,\frac{L^{\#}(B)\,L(B)\,L(A)}{B^2\,a^4} - \frac{1}{2}\,\frac{A\,L^{\#}(B)\,L(B)^2}{B^3\,a^4}$$

$$- \frac{2}{27}\,\frac{A\,L^{\#}(B)\,Q^2}{B\,a^4} - \frac{\frac{1}{18}\,\mathrm{I}\,dl\,A\,Q}{a^5} - \frac{1}{2}\,\frac{B\,L(B)\,L(P)}{a^4} + \frac{1}{6}\,\frac{B\,Q\,L^{\#}(P)}{a^4}$$

$$- \frac{1}{6}\,\frac{B\,Q\,L(L(A))}{a^4} + \frac{1}{2}\,\frac{L(B)\,L^{\#}(P)}{a^4} + \frac{dl\,d}{a^6} - \frac{\frac{5}{6}\,\mathrm{I}\,dl\,A\,L(B)}{B\,a^5} - \frac{\frac{1}{2}\,\mathrm{I}\,d\,L(B)\,A}{B\,a^5}$$

$$- \frac{\frac{4}{9}\,\mathrm{I}\,dl\,L^{\#}(B)\,Q}{B\,a^5} - \frac{\frac{1}{6}\,\mathrm{I}\,dl\,L^{\#}(B)\,L(B)}{B^2\,a^5} - \frac{1}{2}\,\frac{L(L^{\#}(A))\,L(B)}{B\,a^4} - \frac{1}{6}\,\frac{L(L^{\#}(A))\,Q}{a^4}$$

$$+ \frac{1}{2}\,\frac{L^{\#}(L(A))\,L(B)}{B\,a^4} + \frac{1}{6}\,\frac{L^{\#}(L(A))\,Q}{a^4} + \frac{1}{2}\,\frac{A\,L^{\#}(L(L(B)))}{B\,a^4} + \frac{1}{6}\,\frac{A\,L^{\#}(L(Q))}{a^4}$$

$$- \frac{1}{2}\,\frac{A\,L(L^{\#}(L(B)))}{B\,a^4} - \frac{1}{6}\,\frac{A\,L(L^{\#}(Q))}{a^4} - \frac{1}{6}\,\frac{A^2\,L(L(B))}{B\,a^4} - \frac{1}{18}\,\frac{A^2\,L(Q)}{a^4}$$

$$+ \frac{1}{4}\,\frac{L^{\#}(A)\,L(B)^2}{B^2\,a^4} + \frac{1}{36}\,\frac{L^{\#}(A)\,Q^2}{a^4} + \frac{\frac{1}{6}\,\mathrm{I}\,B\,S(Q)}{a^4} + \frac{\mathrm{I}\,dl\,L(L(B))}{a^5} + \frac{2\,\mathrm{I}\,dl\,L(A)}{a^5}$$

$$- \frac{\frac{7}{6}\,\mathrm{I}\,dl\,L^{\#}(Q)}{a^5} - \frac{\frac{1}{2}\,\mathrm{I}\,d\,L(L(B))}{a^5} + \frac{1}{54}\,\frac{A^2\,Q^2}{a^4} - \frac{1}{216}\,\frac{B\,A\,Q^3}{a^4} - \frac{1}{6}\,\frac{B^2\,Q\,L(P)}{a^4}$$

$$\left. - \frac{1}{2}\,\frac{L(B)\,L(L(A))}{a^4} + \frac{1}{6}\,\frac{B\,Q^2\,L(A)}{a^4} + \frac{1}{6}\,\frac{L(B)^2\,L(A)}{B\,a^4} - \frac{5}{6}\,\frac{P\,L(B)^2}{a^4} \right)$$

$$
-\frac{1}{8}\,\frac{A\,L(B)^3}{B^2\,a^4}+\frac{A\,L\left(L^{\#}(B)\right)\,L(B)}{B^2\,a^4}+\frac{1}{6}\,\frac{A\,L\left(L^{\#}(B)\right)\,Q}{B\,a^4}+\frac{1}{6}\,\frac{L^{\#}(A)\,L(B)\,Q}{B\,a^4}
$$

$$
+\frac{5}{12}\,\frac{A\,L(B)\,L(L(B))}{B\,a^4}+\frac{17}{36}\,\frac{A\,L(B)\,L(Q)}{a^4}+\frac{1}{12}\,\frac{A\,Q\,L(L(B))}{a^4}
$$

$$
+\frac{5}{36}\,\frac{B\,A\,Q\,L(Q)}{a^4}+\frac{1}{6}\,\frac{A\,L^{\#}(B)\,L(L(B))}{B^2\,a^4}+\frac{1}{18}\,\frac{A\,L^{\#}(B)\,L(Q)}{B\,a^4}-\frac{\frac{1}{6}\,\mathrm{I}\,d\,Q\,L(B)}{a^5}
$$

$$
+\frac{\frac{5}{9}\,\mathrm{I}\,d1\,Q\,L(B)}{a^5}-\frac{\frac{1}{6}\,\mathrm{I}\,d\,Q\,A}{a^5}+\frac{\mathrm{I}\,L\left(L^{\#}(B)\right)\,d1}{B\,a^5}+\frac{\mathrm{I}\,B\,d1\,L(Q)}{a^5}-\frac{\frac{1}{2}\,\mathrm{I}\,L(B)\,S(B)}{B\,a^4}
$$

$$
-\frac{\frac{5}{12}\,\mathrm{I}\,d1\,L(B)^2}{B\,a^5}-\frac{\frac{1}{36}\,\mathrm{I}\,B\,d1\,Q^2}{a^5}-\frac{\frac{3}{2}\,\mathrm{I}\,d1\,L^{\#}(L(B))}{B\,a^5}+\frac{\frac{1}{4}\,\mathrm{I}\,d\,L(B)^2}{B\,a^5}
$$

$$
-\frac{\frac{1}{36}\,\mathrm{I}\,B\,d\,Q^2}{a^5}-\frac{\frac{1}{6}\,\mathrm{I}\,B\,d\,L(Q)}{a^5}\Bigg)\,\sigma\wedge\rho+\Bigg(-\frac{\frac{1}{2}\,\mathrm{I}\,A\,L(B)^2}{B^{3/2}\,a^3}+\frac{\frac{1}{2}\,\mathrm{I}\,\sqrt{B}\,L(B)\,P}{a^3}
$$

$$
-\frac{\frac{1}{24}\,\mathrm{I}\,Q\,L(B)^2}{\sqrt{B}\,a^3}+\frac{\frac{1}{2}\,\mathrm{I}\,L(L(B))\,A}{\sqrt{B}\,a^3}+\frac{\frac{1}{4}\,\mathrm{I}\,L(B)\,L(L(B))}{\sqrt{B}\,a^3}+\frac{\frac{1}{12}\,\mathrm{I}\,B^{3/2}\,L(Q)\,Q}{a^3}
$$

$$
+\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,L(Q)\,A}{a^3}-\frac{\frac{1}{6}\,\mathrm{I}\,A\,Q\,L(B)}{\sqrt{B}\,a^3}-\frac{\frac{1}{8}\,\mathrm{I}\,L(B)^3}{B^{3/2}\,a^3}+\frac{\frac{1}{12}\,\mathrm{I}\,\sqrt{B}\,L(Q)\,L(B)}{a^3}
$$

$$
+\frac{\frac{1}{6}\,\mathrm{I}\,\sqrt{B}\,L(A)\,Q}{a^3}+\frac{\frac{1}{2}\,\mathrm{I}\,L(B)\,L(A)}{a^3\,\sqrt{B}}+\frac{\frac{7}{72}\,\mathrm{I}\,\sqrt{B}\,Q^2\,L(B)}{a^3}+\frac{\frac{1}{72}\,\mathrm{I}\,B^{3/2}\,Q^3}{a^3}
$$

$$
-\frac{\frac{1}{18}\,\mathrm{I}\,\sqrt{B}\,A\,Q^2}{a^3}+\frac{1}{3}\,\frac{\sqrt{B}\,d1\,Q}{a^4}+\frac{\frac{1}{6}\,\mathrm{I}\,B^{3/2}\,P\,Q}{a^3}+\frac{\frac{1}{4}\,\mathrm{I}\,\sqrt{B}\,Q\,L(L(B))}{a^3}\Bigg)\,\sigma\wedge\zeta
$$

$$
+\Bigg(\frac{1}{2}\,\frac{S(B)}{\sqrt{B}\,a^3}-\frac{\frac{3}{4}\,\mathrm{I}\,A\,L^{\#}(B)\,L(B)}{B^{5/2}\,a^3}-\frac{\frac{1}{12}\,\mathrm{I}\,A\,L^{\#}(B)\,Q}{B^{3/2}\,a^3}-\frac{\frac{23}{36}\,\mathrm{I}\,A\,Q\,L(B)}{\sqrt{B}\,a^3}
$$

$$
-\frac{\frac{1}{9}\,\mathrm{I}\,L^{\#}(B)\,L(B)\,Q}{B^{3/2}\,a^3}+\frac{1}{6}\,\frac{\sqrt{B}\,d\,Q}{a^4}+\frac{1}{2}\,\frac{L^{\#}(B)\,d1}{a^4\,B^{3/2}}+\frac{1}{2}\,\frac{\sqrt{B}\,d1\,Q}{a^4}+\frac{A\,d1}{a^4\,\sqrt{B}}
$$

$$
+\frac{1}{2}\,\frac{L(B)\,d1}{a^4\,\sqrt{B}}+\frac{\frac{1}{72}\,\mathrm{I}\,B^{3/2}\,Q^3}{a^3}+\frac{\frac{1}{2}\,\mathrm{I}\,L(B)\,L(A)}{a^3\,\sqrt{B}}-\frac{\frac{1}{2}\,\mathrm{I}\,A\,L(B)^2}{B^{3/2}\,a^3}
$$

$$+\frac{\frac{1}{2}\,\mathrm{I}\sqrt{B}\,L(B)\,P}{a^3}+\frac{\frac{1}{6}\,\mathrm{I}\sqrt{B}\,L(A)\,Q}{a^3}+\frac{\frac{1}{6}\,\mathrm{I}B^{3/2}PQ}{a^3}+\frac{\frac{1}{6}\,\mathrm{I}L^{\#}(A)\,Q}{\sqrt{B}\,a^3}$$
$$+\frac{\frac{1}{12}\,\mathrm{I}L^{\#}(Q)\,L(B)}{\sqrt{B}\,a^3}+\frac{\frac{1}{12}\,\mathrm{I}\sqrt{B}\,L^{\#}(Q)\,Q}{a^3}+\frac{\frac{1}{2}\,\mathrm{I}L^{\#}(A)\,L(B)}{B^{3/2}a^3}+\frac{\frac{1}{2}\,\mathrm{I}A\,L^{\#}(L(B))}{B^{3/2}a^3}$$
$$+\frac{\frac{1}{6}\,\mathrm{I}A\,L^{\#}(Q)}{\sqrt{B}\,a^3}+\frac{\frac{1}{72}\,\mathrm{I}QL(B)^2}{\sqrt{B}\,a^3}+\frac{\frac{1}{36}\,\mathrm{I}\sqrt{B}\,Q^2L(B)}{a^3}+\frac{\frac{1}{2}\,\mathrm{I}L^{\#}(L(B))\,L(B)}{B^{3/2}a^3}$$
$$+\frac{\frac{1}{2}\,\mathrm{I}L^{\#}(L(B))\,Q}{\sqrt{B}\,a^3}-\frac{\frac{1}{2}\,\mathrm{I}A^2L(B)}{B^{3/2}a^3}-\frac{\frac{1}{6}\,\mathrm{I}A^2Q}{\sqrt{B}\,a^3}-\frac{\frac{5}{36}\,\mathrm{I}\sqrt{B}\,AQ^2}{a^3}$$
$$-\frac{\frac{1}{4}\,\mathrm{I}L\left(L^{\#}(B)\right)Q}{\sqrt{B}\,a^3}-\frac{\frac{1}{4}\,\mathrm{I}L\left(L^{\#}(B)\right)L(B)}{B^{3/2}a^3}+\frac{\frac{1}{72}\,\mathrm{I}L^{\#}(B)\,Q^2}{\sqrt{B}\,a^3}-\frac{\frac{1}{8}\,\mathrm{I}L^{\#}(B)\,L(B)^2}{B^{5/2}a^3}\Bigg)$$
$$\sigma\wedge\zeta^{\#}+\left(-\frac{\frac{1}{2}\,\mathrm{I}AL(B)}{Ba^2}-\frac{\frac{1}{6}\,\mathrm{I}AQ}{a^2}-\frac{\frac{1}{6}\,\mathrm{I}QL(B)}{a^2}-\frac{\frac{1}{36}\,\mathrm{I}BQ^2}{a^2}+\frac{\frac{1}{4}\,\mathrm{I}L(B)^2}{Ba^2}\right.$$
$$\left.-\frac{\frac{1}{2}\,\mathrm{I}L(L(B))}{a^2}-\frac{\frac{1}{6}\,\mathrm{I}BL(Q)}{a^2}+\frac{dI}{a^3}\right)\rho\wedge\zeta+\left(-\frac{\frac{1}{36}\,\mathrm{I}L^{\#}(B)\,Q}{Ba^2}-\frac{\frac{1}{18}\,\mathrm{I}AQ}{a^2}\right.$$
$$-\frac{\frac{1}{36}\,\mathrm{I}BQ^2}{a^2}-\frac{\frac{1}{36}\,\mathrm{I}QL(B)}{a^2}+\frac{\frac{1}{4}\,\mathrm{I}L^{\#}(B)\,L(B)}{B^2a^2}-\frac{\mathrm{I}L^{\#}(L(B))}{Ba^2}-\frac{\frac{1}{6}\,\mathrm{I}L^{\#}(Q)}{a^2}$$
$$\left.+\frac{\frac{1}{2}\,\mathrm{I}L\left(L^{\#}(B)\right)}{a^2B}+\frac{dI}{a^3}\right)\rho\wedge\zeta^{\#}-\frac{1}{6}\,\frac{\sqrt{B}\,Q\zeta\wedge\zeta^{\#}}{a}$$

```
> Torsion(SE[2], 4, 6);
```

$$\frac{1}{2}\,\frac{L(B)^2}{a^2B}+\frac{1}{3}\,\frac{L(B)\,Q}{a^2}+\frac{1}{3}\,\frac{BQ^2}{a^2}-\frac{1}{2}\,\frac{L(L(B))}{a^2}-\frac{1}{2}\,\frac{BL(Q)}{a^2}+\frac{\mathrm{I}dI}{a^3}+\frac{1}{2}\,\frac{L(B)\,A}{a^2B}+\frac{1}{6}\,\frac{AQ}{a^2}+\frac{BP}{a^2} \tag{10}$$

```
> Torsion(SE[2], 4, 7);
```

$$\frac{1}{3}\,\frac{L^{\#}(B)\,L(B)}{a^2B^2}-\frac{1}{6}\,\frac{L^{\#}(B)\,Q}{a^2B}+\frac{2}{3}\,\frac{L(B)\,Q}{a^2}+\frac{1}{3}\,\frac{L(B)^2}{a^2B}+\frac{1}{3}\,\frac{BQ^2}{a^2}-\frac{1}{2}\,\frac{L^{\#}(L(B))}{a^2B}-\frac{1}{2}\,\frac{L^{\#}(Q)}{a^2}+\frac{BP}{a^2}-\frac{\mathrm{I}d}{a^3}+\frac{L(A)}{a^2}+\frac{1}{6}\,\frac{L(B)\,A}{a^2B}+\frac{1}{6}\,\frac{AQ}{a^2} \tag{11}$$

```
> expand(solve(Torsion(SE[2], 4, 6), dI));
```

$$\frac{\frac{1}{2}\,\mathrm{I}a\,L(B)^2}{B}+\frac{1}{3}\,\mathrm{I}a\,L(B)\,Q+\frac{1}{3}\,\mathrm{I}a\,BQ^2-\frac{1}{2}\,\mathrm{I}a\,L(L(B))-\frac{1}{2}\,\mathrm{I}a\,BL(Q) \tag{12}$$

$$+\frac{\frac{1}{2}\,\mathrm{I}\,a\,A\,L(B)}{B}+\frac{1}{6}\,\mathrm{I}\,a\,A\,Q+\mathrm{I}\,a\,B\,P$$

```
> expand(solve(Torsion(SE[2], 4, 7), d));
```

$$-\frac{2}{3}\,\mathrm{I}\,a\,Q\,L(B)-\frac{\frac{1}{6}\,\mathrm{I}\,a\,A\,L(B)}{B}-\frac{1}{6}\,\mathrm{I}\,a\,A\,Q+\frac{\frac{1}{6}\,\mathrm{I}\,a\,L^{\#}(B)\,Q}{B}-\frac{\frac{1}{3}\,\mathrm{I}\,a\,L(B)^{2}}{B}$$
$$-\frac{1}{3}\,\mathrm{I}\,a\,B\,Q^{2}-\mathrm{I}\,a\,L(A)-\frac{\frac{1}{3}\,\mathrm{I}\,a\,L^{\#}(B)\,L(B)}{B^{2}}+\frac{\frac{1}{2}\,\mathrm{I}\,a\,L^{\#}(L(B))}{B}+\frac{1}{2}\,\mathrm{I}\,a\,L^{\#}(Q)$$
$$-\mathrm{I}\,a\,B\,P \tag{13}$$

```
> expand(solve(Torsion(SE[4], 5, 6), d1));
```

$$\frac{\frac{1}{2}\,\mathrm{I}\,a\,A\,L(B)}{B}+\frac{1}{6}\,\mathrm{I}\,a\,A\,Q+\frac{1}{6}\,\mathrm{I}\,a\,L(B)\,Q+\frac{1}{36}\,\mathrm{I}\,a\,B\,Q^{2}-\frac{\frac{1}{4}\,\mathrm{I}\,a\,L(B)^{2}}{B}$$
$$+\frac{1}{2}\,\mathrm{I}\,a\,L(L(B))+\frac{1}{6}\,\mathrm{I}\,a\,B\,L(Q) \tag{14}$$

```
> expand(solve(Torsion(SE[3], 5, 7), d));
```

$$-\frac{\frac{4}{9}\,\mathrm{I}\,a\,L^{\#}(B)^{2}}{B^{3}}+\frac{\frac{1}{9}\,\mathrm{I}\,a\,L^{\#}(B)\,L(B)}{B^{2}}-\frac{\frac{1}{18}\,\mathrm{I}\,a\,L^{\#}(B)\,Q}{B}+\frac{\frac{1}{18}\,\mathrm{I}\,a\,A\,L(B)}{B}$$
$$-\frac{\frac{1}{6}\,\mathrm{I}\,a\,L^{\#}(L(B))}{B}+\frac{1}{18}\,\mathrm{I}\,a\,A\,Q-\frac{\frac{1}{9}\,\mathrm{I}\,a\,L^{\#}(B)\,A}{B^{2}}-\frac{1}{6}\,\mathrm{I}\,a\,L^{\#}(Q)-\frac{1}{18}\,\mathrm{I}\,a\,L(B)\,Q$$
$$-\frac{\frac{1}{36}\,\mathrm{I}\,a\,L(B)^{2}}{B}-\frac{1}{36}\,\mathrm{I}\,a\,B\,Q^{2}+\frac{\frac{1}{3}\,\mathrm{I}\,a\,L^{\#}\left(L^{\#}(B)\right)}{B^{2}}+\frac{\frac{2}{9}\,\mathrm{I}\,a\,A^{2}}{B}-\frac{\frac{1}{3}\,\mathrm{I}\,a\,L^{\#}(A)}{B} \tag{15}$$

```
> expand(solve(Torsion(SE[4], 5, 6), d1) - (solve(Torsion(SE[2], 4, 6), d1)));
```

$$-\frac{1}{6}\,\mathrm{I}\,a\,Q\,L(B)-\frac{11}{36}\,\mathrm{I}\,a\,B\,Q^{2}-\frac{\frac{3}{4}\,\mathrm{I}\,a\,L(B)^{2}}{B}+\mathrm{I}\,a\,L(L(B))+\frac{2}{3}\,\mathrm{I}\,a\,B\,L(Q)-\mathrm{I}\,a\,B\,P \tag{16}$$