```
> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([x, y, z, z1], [a, b, b1, c, d, e], M, verbose);
```

The following coordinates have been protected:

[x, y, z, z1, a, b, b1, c, d, e]

The following vector fields have been defined and protected:

[D_x, D_y, D_z, D_z1, D_a, D_b, D_b1, D_c, D_d, D_e]

The following differential 1-forms have been defined and protected:

[dx, dy, dz, dz1, da, db, db1, dc, dd, de]

frame name: M **(1)**

Par rapport à la session précédente, a := a*B^(1/2), c:=c*B^(1/2), d:=d*B^(1/2) et e:=e*B^(1/2).

Une procédure de dérivation:

```
> Der :=proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(Der(op(i, x)), i=1
      .. nops(x)) elif
          (type(x, `*`) = true) then expand((x/y)·Der(y) + y·Der(x/y)) elif
          (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1) ·Der(y) elif
          ((type(x, function) = true) or (type(x, symbol) = true)) then S(x)·W[1]
      + Tau(x)·W[2] + L(x)·W[3] + L^#(x)·W[4]
          else 0 fi end proc:
  derivation :=proc(x) : collect( Der(x), [W[1], W[2], W[3], W[4]]) : end proc:
>
```

On exprime le nouveau coframe initial (chapeau) en fonction du premier.
W[i] represente sigma_0 et V[1] represente sigma_0 chapeau. On a la relation V:=m.W.

```
> m := Matrix([[[B^(-1/2), 0, 0, 0], [0, 1, 0, 0], [0, 0, B^(-1/2), 0], [0, 0, 0, B^(1/2)]]) :
> minv := MatrixInverse(m) :
M > W := minv.Vector([V[1], V[2], V[3], V[4]]) :
M >
```

On donne ensuite la matrice de groupe:

```
M > g := Matrix([[a^3, 0, 0, 0], [c, a^2, 0, 0], [d, b, a, 0], [e, b1, 0, a]]);
```

$$g := \begin{bmatrix} a^3 & 0 & 0 & 0 \\ c & a^2 & 0 & 0 \\ d & b & a & 0 \\ e & b1 & 0 & a \end{bmatrix} \quad \textbf{(2)}$$

M > $h := MatrixInverse(g)$:

M > $Mat := map(evalDG, (ExteriorDerivative(g).h))$;

$$Mat := \begin{bmatrix} \frac{3\,da}{a} & 0\,dx & 0\,dx & 0\,dx \\ -\frac{2\,c\,da}{a^4} + \frac{dc}{a^3} & \frac{2\,da}{a} & 0\,dx & 0\,dx \\ -\frac{(d\,a^2 - b\,c)\,da}{a^6} - \frac{c\,db}{a^5} + \frac{dd}{a^3} & -\frac{b\,da}{a^3} + \frac{db}{a^2} & \frac{da}{a} & 0\,dx \\ -\frac{(e\,a^2 - b1\,c)\,da}{a^6} - \frac{c\,db1}{a^5} + \frac{de}{a^3} & -\frac{b1\,da}{a^3} + \frac{db1}{a^2} & 0\,dx & \frac{da}{a} \end{bmatrix} \quad (3)$$

On obtient ainsi la liste des formes de Maurer-Cartan:

M > $t[1] := \frac{da}{a}$:

M > $t[2] := -\frac{b\,da}{a^3} + \frac{db}{a^2}$:

M > $t[3] := -\frac{2\,c\,da}{a^4} + \frac{dc}{a^3}$:

M > $t[4] := -\frac{(d\,a^2 - b\,c)\,da}{a^6} - \frac{c\,db}{a^5} + \frac{dd}{a^3}$:

M > $t[5] := -\frac{(e\,a^2 - b1\,c)\,da}{a^6} - \frac{c\,db1}{a^5} + \frac{de}{a^3}$:

M > $t[6] := -\frac{b1\,da}{a^3} + \frac{db1}{a^2}$:

M > $FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], dx, dy, dz, dz1], N)$:
$DGsetup(FD, [E], [\text{alpha}[1], \text{alpha}[2], \text{alpha}[3], \text{alpha}[4], \text{alpha}[5], \alpha^{\#}[2], \text{sigma}, \text{rho}, \text{zeta}, \zeta^{\#}], verbose)$:

The following coordinates have been protected:

$[x, y, z, z1, a, b, b1, c, d, e]$

The following vector fields have been defined and protected:

$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10]$

The following differential 1-forms have been defined and protected:

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha^{\#}_2, \sigma, \rho, \zeta, \zeta^{\#}] \quad (4)$$

N > $T := Vector([\text{sigma}, \text{rho}, \text{zeta}, \zeta^{\#}])$:

N > $V := h.T$:

Les equations de courbure sur les W[i] sont connues:

M > $dW[1] := H\cdot(W[1]\&wedge\ W[2]) + F\cdot(W[1]\&wedge\ W[4]) + Q\cdot(W[1]\ \&wedge\ W[3]) + B\cdot(W[2]\ \&wedge\ W[4]) + (W[2]\ \&wedge\ W[3])$:

M > $dW[2] := G\cdot(W[1]\&wedge\ W[2]) + E\cdot(W[1]\&wedge\ W[4]) + P\cdot(W[1]\ \&wedge\ W[3]) + A\cdot(W[2]\&wedge\ W[4]) + I\cdot(W[3]\&wedge\ W[4])$:

On obtient ainsi les nouvelles equations de courbure initiales:

N > $dV[1] := evalDG\left(B^{\frac{-1}{2}}\cdot dW[1] + \left(Der\left(B^{-\frac{1}{2}}\right)\ \&wedge\ W[1]\right)\right)$:

N > $dV[2] := dW[2]$:

N > $dV[3] := Der\left(B^{-\frac{1}{2}}\right)\ \&wedge\ W[3]$:

N > $dV[4] := Der\left(B^{\frac{1}{2}}\right)\ \&wedge\ W[4]$:

On peut maintenant calculer les équations de courbure du coframe 'relevé' :

N > $\mathrm{Omega} := map(evalDG, g.Vector([dV[1], dV[2], dV[3], dV[4]]))$:

N > $Mat := map(evalDG, (ExteriorDerivative(g).h))$:

N > $Mat2 := Mat\ \&MatrixWedge\ T$:

N > $SE := map(evalDG, (Mat2\ \&MatrixPlus\ \mathrm{Omega}))$:

N > $List := GenerateForms([\mathrm{alpha}[1], \mathrm{alpha}[2], \mathrm{alpha}[3], \mathrm{alpha}[4], \mathrm{alpha}[5], \alpha^{\#}[2], \mathrm{sigma}, \mathrm{rho}, \mathrm{zeta}, \zeta^{\#}], 2)$:

N > $result :=$ **proc**(l) **local** $k, t, X; X := 0 : t := expand(GetComponents(l, List))$: **for** k **from** 1 **to** 45 **do** $X := X + t[k]\cdot List[k]$ **od**; X; **end proc**:

N > $result(SE[1])$;

$$3\,\alpha_1 \wedge \sigma + \left(-\frac{1}{2}\,\frac{L^{\#}(B)\,b1}{a^3\,B^{3/2}} - \frac{1}{2}\,\frac{L(B)\,b}{a^3\sqrt{B}} + \frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2\,B} + \frac{d}{a^3} + \frac{e}{a^3} - \frac{\sqrt{B}\,Q\,b}{a^3} - \frac{F\,b1}{a^3\sqrt{B}} + \frac{H}{a^2}\right)\sigma \wedge \rho + \left(\frac{1}{2}\,\frac{L(B)}{a\sqrt{B}} - \frac{c}{a^3} + \frac{\sqrt{B}\,Q}{a}\right)\sigma \wedge \zeta + \left(\frac{1}{2}\,\frac{L^{\#}(B)}{a\,B^{3/2}} - \frac{c}{a^3} + \frac{F}{a\sqrt{B}}\right)\sigma \wedge \zeta^{\#} + \rho \wedge \zeta + \rho \wedge \zeta^{\#} \quad \textbf{(5)}$$

N > $result(SE[2])$;

$$2\,\alpha_1 \wedge \rho + \alpha_3 \wedge \sigma + \left(-\frac{\mathrm{I}\,b\,e}{a^5} + \frac{\mathrm{I}\,b1\,d}{a^5} + \frac{A\,e}{\sqrt{B}\,a^4} - \frac{B\,P\,b}{a^4} - \frac{E\,b1}{a^4} + \frac{\sqrt{B}\,G}{a^3} - \frac{1}{2}\,\frac{L^{\#}(B)\,b1\,c}{B^{3/2}\,a^6} - \frac{1}{2}\,\frac{L(B)\,b\,c}{\sqrt{B}\,a^6} + \frac{1}{2}\,\frac{\mathrm{T}(B)\,c}{B\,a^5} + \frac{c\,d}{a^6} + \frac{c\,e}{a^6} - \frac{\sqrt{B}\,c\,Q\,b}{a^6} - \frac{c\,F\,b1}{\sqrt{B}\,a^6} + \frac{c\,H}{a^5}\right)\sigma \wedge \rho + \left(\frac{\mathrm{I}\,e}{a^3} - \frac{\mathrm{I}\,b1\,c}{a^5} + \frac{B\,P}{a^2} + \frac{1}{2}\,\frac{c\,L(B)}{\sqrt{B}\,a^4} - \frac{c^2}{a^6} + \frac{\sqrt{B}\,c\,Q}{a^4}\right)\sigma \wedge \zeta + \left(-\frac{\mathrm{I}\,d}{a^3} + \frac{\mathrm{I}\,b\,c}{a^5} - \frac{A\,c}{\sqrt{B}\,a^4} + \frac{E}{a^2} + \frac{1}{2}\,\frac{c\,L^{\#}(B)}{B^{3/2}\,a^4} - \frac{c^2}{a^6} \quad \textbf{(6)}$$

$$+\frac{c\,F}{\sqrt{B}\,a^4}\Bigg)\sigma\wedge\zeta^{\#}+\left(\frac{\mathrm{I}\,b1}{a^2}+\frac{c}{a^3}\right)\rho\wedge\zeta+\left(-\frac{\mathrm{I}\,b}{a^2}+\frac{A}{a\sqrt{B}}+\frac{c}{a^3}\right)\rho\wedge\zeta^{\#}+\mathrm{I}\,\zeta\wedge\zeta^{\#}$$

N > *result*(*SE*[3]);

$$\begin{aligned}
&\alpha_1\wedge\zeta+\alpha_2\wedge\rho+\alpha_4\wedge\sigma+\Bigg(-\frac{1}{2}\,\frac{b\,L^{\#}(B)\,e}{a^6B^{3/2}}+\frac{1}{2}\,\frac{b\,S(B)}{a^5\sqrt{B}}-\frac{\mathrm{I}\,b^2e}{a^7}+\frac{\mathrm{I}\,b\,b1\,d}{a^7}\\
&+\frac{b\,A\,e}{a^6\sqrt{B}}-\frac{B\,b^2P}{a^6}-\frac{b\,E\,b1}{a^6}+\frac{\sqrt{B}\,b\,G}{a^5}-\frac{1}{2}\,\frac{d\,L(B)\,b}{a^6\sqrt{B}}+\frac{d^2}{a^6}+\frac{e\,d}{a^6}\\
&-\frac{\sqrt{B}\,d\,Q\,b}{a^6}-\frac{d\,F\,b1}{a^6\sqrt{B}}+\frac{d\,H}{a^5}\Bigg)\sigma\wedge\rho+\Bigg(\frac{1}{2}\,\frac{e\,L^{\#}(B)}{a^4B^{3/2}}-\frac{1}{2}\,\frac{L^{\#}(B)\,b1\,c}{B^{3/2}a^6}\\
&+\frac{1}{2}\,\frac{\mathrm{T}(B)\,c}{B\,a^5}-\frac{1}{2}\,\frac{S(B)}{a^3\sqrt{B}}+\frac{\mathrm{I}\,b\,e}{a^5}-\frac{\mathrm{I}\,b1\,b\,c}{a^7}+\frac{B\,P\,b}{a^4}+\frac{1}{2}\,\frac{L(B)\,d}{a^4\sqrt{B}}-\frac{c\,d}{a^6}\\
&+\frac{\sqrt{B}\,d\,Q}{a^4}\Bigg)\sigma\wedge\zeta+\Bigg(\frac{1}{2}\,\frac{L^{\#}(B)\,b\,c}{a^6B^{3/2}}-\frac{\mathrm{I}\,b\,d}{a^5}+\frac{\mathrm{I}\,b^2c}{a^7}-\frac{b\,A\,c}{a^6\sqrt{B}}+\frac{b\,E}{a^4}-\frac{c\,d}{a^6}\\
&+\frac{d\,F}{a^4\sqrt{B}}\Bigg)\sigma\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L^{\#}(B)\,b1}{a^3B^{3/2}}-\frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2B}+\frac{\mathrm{I}\,b1\,b}{a^4}+\frac{d}{a^3}\right)\rho\wedge\zeta+\Bigg(\\
&-\frac{1}{2}\,\frac{L^{\#}(B)\,b}{B^{3/2}a^3}-\frac{\mathrm{I}\,b^2}{a^4}+\frac{b\,A}{\sqrt{B}\,a^3}+\frac{d}{a^3}\Bigg)\rho\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L^{\#}(B)}{a\,B^{3/2}}+\frac{\mathrm{I}\,b}{a^2}\right)\zeta\wedge\zeta^{\#}
\end{aligned}\tag{7}$$

N > *result*(*SE*[4]);

$$\begin{aligned}
&\alpha_1\wedge\zeta^{\#}+\alpha_5\wedge\sigma+\alpha^{\#}_2\wedge\rho+\Bigg(-\frac{e\,L(B)\,b}{a^6\sqrt{B}}+\frac{e\,\mathrm{T}(B)}{a^5B}+\frac{1}{2}\,\frac{b1\,L(B)\,d}{a^6\sqrt{B}}-\frac{1}{2}\,\frac{b1\,S(B)}{a^5\sqrt{B}}\\
&-\frac{\mathrm{I}\,b1\,b\,e}{a^7}+\frac{\mathrm{I}\,b1^2d}{a^7}+\frac{b1\,A\,e}{a^6\sqrt{B}}-\frac{B\,b1\,P\,b}{a^6}-\frac{b1^2E}{a^6}+\frac{\sqrt{B}\,b1\,G}{a^5}-\frac{1}{2}\,\frac{e\,L^{\#}(B)\,b1}{a^6B^{3/2}}\\
&+\frac{e\,d}{a^6}+\frac{e^2}{a^6}-\frac{\sqrt{B}\,e\,Q\,b}{a^6}-\frac{e\,F\,b1}{a^6\sqrt{B}}+\frac{e\,H}{a^5}\Bigg)\sigma\wedge\rho+\Bigg(\frac{e\,L(B)}{\sqrt{B}\,a^4}-\frac{1}{2}\,\frac{L(B)\,b1\,c}{\sqrt{B}\,a^6}\\
&+\frac{\mathrm{I}\,b1\,e}{a^5}-\frac{\mathrm{I}\,b1^2c}{a^7}+\frac{B\,b1\,P}{a^4}-\frac{c\,e}{a^6}+\frac{\sqrt{B}\,e\,Q}{a^4}\Bigg)\sigma\wedge\zeta+\Bigg(-\frac{1}{2}\,\frac{L(B)\,d}{a^4\sqrt{B}}\\
&+\frac{1}{2}\,\frac{L(B)\,b\,c}{\sqrt{B}\,a^6}-\frac{1}{2}\,\frac{\mathrm{T}(B)\,c}{B\,a^5}+\frac{1}{2}\,\frac{S(B)}{a^3\sqrt{B}}-\frac{\mathrm{I}\,b1\,d}{a^5}+\frac{\mathrm{I}\,b1\,b\,c}{a^7}-\frac{b1\,A\,c}{a^6\sqrt{B}}+\frac{E\,b1}{a^4}\\
&+\frac{1}{2}\,\frac{e\,L^{\#}(B)}{a^4B^{3/2}}-\frac{c\,e}{a^6}+\frac{e\,F}{a^4\sqrt{B}}\Bigg)\sigma\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L(B)\,b1}{\sqrt{B}\,a^3}+\frac{\mathrm{I}\,b1^2}{a^4}+\frac{e}{a^3}\right)\rho\wedge\zeta+\Bigg(\\
&-\frac{1}{2}\,\frac{L(B)\,b}{a^3\sqrt{B}}+\frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2B}-\frac{\mathrm{I}\,b1\,b}{a^4}+\frac{b1\,A}{\sqrt{B}\,a^3}+\frac{e}{a^3}\Bigg)\rho\wedge\zeta^{\#}+\Bigg(\frac{1}{2}\,\frac{L(B)}{a\sqrt{B}}\\
&+\frac{\mathrm{I}\,b1}{a^2}\Bigg)\zeta\wedge\zeta^{\#}
\end{aligned}\tag{8}$$

On peut remplacer les expressions E, F, G, et H par leurs valeurs, pour plus de précision:

Par exemple, pour E et F, qui ont des expressions simples, on a :

N > $E := L(A) + B \cdot P :$

N > $F := L(B) + B \cdot Q + A :$

N > $Res1 := result(SE[1]);$

$$Res1 := 3\,\alpha_1 \wedge \sigma + \left(-\frac{1}{2}\,\frac{L^{\#}(B)\,b1}{a^3\,B^{3/2}} - \frac{1}{2}\,\frac{L(B)\,b}{a^3\sqrt{B}} + \frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2\,B} + \frac{d}{a^3} + \frac{e}{a^3} - \frac{\sqrt{B}\,Q\,b}{a^3} - \frac{L(B)\,b1}{\sqrt{B}\,a^3} - \frac{\sqrt{B}\,b1\,Q}{a^3} - \frac{b1\,A}{\sqrt{B}\,a^3} + \frac{H}{a^2} \right) \sigma \wedge \rho + \left(\frac{1}{2}\,\frac{L(B)}{a\sqrt{B}} - \frac{c}{a^3} + \frac{\sqrt{B}\,Q}{a} \right) \sigma \wedge \zeta + \left(\frac{1}{2}\,\frac{L^{\#}(B)}{a\,B^{3/2}} - \frac{c}{a^3} + \frac{L(B)}{a\sqrt{B}} + \frac{\sqrt{B}\,Q}{a} + \frac{A}{a\sqrt{B}} \right) \sigma \wedge \zeta^{\#} + \rho \wedge \zeta + \rho \wedge \zeta^{\#} \qquad \mathbf{(9)}$$

N > $Res2 := result(SE[2]);$

$$Res2 := 2\,\alpha_1 \wedge \rho + \alpha_3 \wedge \sigma + \left(-\frac{\mathrm{I}\,b\,e}{a^5} + \frac{\mathrm{I}\,b1\,d}{a^5} + \frac{A\,e}{\sqrt{B}\,a^4} - \frac{B\,P\,b}{a^4} - \frac{b1\,L(A)}{a^4} - \frac{B\,b1\,P}{a^4} + \frac{\sqrt{B}\,G}{a^3} - \frac{1}{2}\,\frac{L^{\#}(B)\,b1\,c}{B^{3/2}\,a^6} - \frac{1}{2}\,\frac{L(B)\,b\,c}{\sqrt{B}\,a^6} + \frac{1}{2}\,\frac{\mathrm{T}(B)\,c}{B\,a^5} + \frac{c\,d}{a^6} + \frac{c\,e}{a^6} - \frac{\sqrt{B}\,c\,Q\,b}{a^6} - \frac{L(B)\,b1\,c}{\sqrt{B}\,a^6} - \frac{\sqrt{B}\,c\,b1\,Q}{a^6} - \frac{b1\,A\,c}{a^6\sqrt{B}} + \frac{c\,H}{a^5} \right) \sigma \wedge \rho + \left(\frac{\mathrm{I}\,e}{a^3} - \frac{\mathrm{I}\,b1\,c}{a^5} + \frac{B\,P}{a^2} + \frac{1}{2}\,\frac{c\,L(B)}{\sqrt{B}\,a^4} - \frac{c^2}{a^6} + \frac{\sqrt{B}\,c\,Q}{a^4} \right) \sigma \wedge \zeta + \left(-\frac{\mathrm{I}\,d}{a^3} + \frac{\mathrm{I}\,b\,c}{a^5} + \frac{L(A)}{a^2} + \frac{B\,P}{a^2} + \frac{1}{2}\,\frac{c\,L^{\#}(B)}{B^{3/2}\,a^4} - \frac{c^2}{a^6} + \frac{c\,L(B)}{\sqrt{B}\,a^4} + \frac{\sqrt{B}\,c\,Q}{a^4} \right) \sigma \wedge \zeta^{\#} + \left(\frac{\mathrm{I}\,b1}{a^2} + \frac{c}{a^3} \right) \rho \wedge \zeta + \left(-\frac{\mathrm{I}\,b}{a^2} + \frac{A}{a\sqrt{B}} + \frac{c}{a^3} \right) \rho \wedge \zeta^{\#} + \mathrm{I}\,\zeta \wedge \zeta^{\#} \qquad \mathbf{(10)}$$

N > $result(SE[3]);$

$$\alpha_1 \wedge \zeta + \alpha_2 \wedge \rho + \alpha_4 \wedge \sigma + \left(-\frac{1}{2}\,\frac{b\,L^{\#}(B)\,e}{a^6\,B^{3/2}} + \frac{1}{2}\,\frac{b\,S(B)}{a^5\sqrt{B}} - \frac{\mathrm{I}\,b^2\,e}{a^7} + \frac{\mathrm{I}\,b\,b1\,d}{a^7} + \frac{b\,A\,e}{a^6\sqrt{B}} - \frac{B\,b^2\,P}{a^6} - \frac{b\,b1\,L(A)}{a^6} - \frac{B\,b1\,P\,b}{a^6} + \frac{\sqrt{B}\,b\,G}{a^5} - \frac{1}{2}\,\frac{d\,L(B)\,b}{a^6\sqrt{B}} + \frac{d^2}{a^6} + \frac{e\,d}{a^6} - \frac{\sqrt{B}\,d\,Q\,b}{a^6} - \frac{b1\,L(B)\,d}{a^6\sqrt{B}} - \frac{\sqrt{B}\,d\,b1\,Q}{a^6} - \frac{d\,b1\,A}{a^6\sqrt{B}} + \frac{d\,H}{a^5} \right) \sigma \wedge \rho + \left(\frac{1}{2}\,\frac{e\,L^{\#}(B)}{a^4\,B^{3/2}} - \frac{1}{2}\,\frac{L^{\#}(B)\,b1\,c}{B^{3/2}\,a^6} + \frac{1}{2}\,\frac{\mathrm{T}(B)\,c}{B\,a^5} - \frac{1}{2}\,\frac{S(B)}{a^3\sqrt{B}} + \frac{\mathrm{I}\,b\,e}{a^5} - \frac{\mathrm{I}\,b1\,b\,c}{a^7} + \frac{B\,P\,b}{a^4} + \frac{1}{2}\,\frac{L(B)\,d}{a^4\sqrt{B}} - \frac{c\,d}{a^6} + \frac{\sqrt{B}\,d\,Q}{a^4} \right) \sigma \wedge \zeta + \left(\frac{1}{2}\,\frac{L^{\#}(B)\,b\,c}{a^6\,B^{3/2}} - \frac{\mathrm{I}\,b\,d}{a^5} \right. \qquad \mathbf{(11)}$$

$$+\frac{\mathrm{I}\, b^2 c}{a^7}-\frac{b\, A\, c}{a^6\sqrt{B}}+\frac{b\, L(A)}{a^4}+\frac{B\, P\, b}{a^4}-\frac{c\, d}{a^6}+\frac{L(B)\, d}{a^4\sqrt{B}}+\frac{\sqrt{B}\, d\, Q}{a^4}+\frac{d\, A}{a^4\sqrt{B}}\Bigg)$$

$$\sigma\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L^{\#}(B)\, b1}{a^3 B^{3/2}}-\frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2 B}+\frac{\mathrm{I}\, b1\, b}{a^4}+\frac{d}{a^3}\right)\rho\wedge\zeta+\Bigg(-\frac{1}{2}\,\frac{L^{\#}(B)\, b}{B^{3/2} a^3}$$

$$-\frac{\mathrm{I}\, b^2}{a^4}+\frac{b\, A}{\sqrt{B}\, a^3}+\frac{d}{a^3}\Bigg)\rho\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L^{\#}(B)}{a\, B^{3/2}}+\frac{\mathrm{I}\, b}{a^2}\right)\zeta\wedge\zeta^{\#}$$

> *result*(*SE*[4]);

$$\alpha_1\wedge\zeta^{\#}+\alpha_5\wedge\sigma+\alpha^{\#}_2\wedge\rho+\Bigg(-\frac{e\, L(B)\, b}{a^6\sqrt{B}}+\frac{e\,\mathrm{T}(B)}{a^5 B}+\frac{1}{2}\,\frac{b1\, L(B)\, d}{a^6\sqrt{B}}-\frac{1}{2}\,\frac{b1\, S(B)}{a^5\sqrt{B}} \tag{12}$$

$$-\frac{\mathrm{I}\, b1\, b\, e}{a^7}+\frac{\mathrm{I}\, b1^2 d}{a^7}-\frac{B\, b1\, P\, b}{a^6}-\frac{b1^2 L(A)}{a^6}-\frac{B\, b1^2 P}{a^6}+\frac{\sqrt{B}\, b1\, G}{a^5}$$

$$-\frac{1}{2}\,\frac{e\, L^{\#}(B)\, b1}{a^6 B^{3/2}}+\frac{e\, d}{a^6}+\frac{e^2}{a^6}-\frac{\sqrt{B}\, e\, Q\, b}{a^6}-\frac{b1\, e\, L(B)}{a^6\sqrt{B}}-\frac{\sqrt{B}\, b1\, e\, Q}{a^6}+\frac{e\, H}{a^5}\Bigg)$$

$$\sigma\wedge\rho+\Bigg(\frac{e\, L(B)}{\sqrt{B}\, a^4}-\frac{1}{2}\,\frac{L(B)\, b1\, c}{\sqrt{B}\, a^6}+\frac{\mathrm{I}\, b1\, e}{a^5}-\frac{\mathrm{I}\, b1^2 c}{a^7}+\frac{B\, b1\, P}{a^4}-\frac{c\, e}{a^6}$$

$$+\frac{\sqrt{B}\, e\, Q}{a^4}\Bigg)\sigma\wedge\zeta+\Bigg(-\frac{1}{2}\,\frac{L(B)\, d}{a^4\sqrt{B}}+\frac{1}{2}\,\frac{L(B)\, b\, c}{\sqrt{B}\, a^6}-\frac{1}{2}\,\frac{\mathrm{T}(B)\, c}{B\, a^5}+\frac{1}{2}\,\frac{S(B)}{a^3\sqrt{B}}$$

$$-\frac{\mathrm{I}\, b1\, d}{a^5}+\frac{\mathrm{I}\, b1\, b\, c}{a^7}-\frac{b1\, A\, c}{a^6\sqrt{B}}+\frac{b1\, L(A)}{a^4}+\frac{B\, b1\, P}{a^4}+\frac{1}{2}\,\frac{e\, L^{\#}(B)}{a^4 B^{3/2}}-\frac{c\, e}{a^6}$$

$$+\frac{e\, L(B)}{\sqrt{B}\, a^4}+\frac{\sqrt{B}\, e\, Q}{a^4}+\frac{A\, e}{\sqrt{B}\, a^4}\Bigg)\sigma\wedge\zeta^{\#}+\left(\frac{1}{2}\,\frac{L(B)\, b1}{\sqrt{B}\, a^3}+\frac{\mathrm{I}\, b1^2}{a^4}+\frac{e}{a^3}\right)\rho\wedge\zeta$$

$$+\left(-\frac{1}{2}\,\frac{L(B)\, b}{a^3\sqrt{B}}+\frac{1}{2}\,\frac{\mathrm{T}(B)}{a^2 B}-\frac{\mathrm{I}\, b1\, b}{a^4}+\frac{b1\, A}{\sqrt{B}\, a^3}+\frac{e}{a^3}\right)\rho\wedge\zeta^{\#}+\Bigg(\frac{1}{2}\,\frac{L(B)}{a\sqrt{B}}$$

$$+\frac{\mathrm{I}\, b1}{a^2}\Bigg)\zeta\wedge\zeta^{\#}$$

On obtient les equations d'absorptions suivantes:

> *Torsion* := **proc**(*S*, *i*, *j*) **local** *k*, *X*; $k := 10\cdot(i-1)-\frac{i\cdot(i-1)}{2}+j-i$; *X* := *GetComponents*(*S*, *List*); *expand*(*X*[*k*]); **end proc**:

> *Eq1* := 3·*U*[3]-*Torsion*(*SE*[1], 7, 9);

$$Eq1 := 3\, U_3-\frac{1}{2}\,\frac{L(B)}{a\sqrt{B}}+\frac{c}{a^3}-\frac{\sqrt{B}\, Q}{a} \tag{13}$$

> *Eq2* := 3·*U*[4]-*Torsion*(*SE*[1], 7, 10);

$$Eq2 := 3\, U_4-\frac{1}{2}\,\frac{L^{\#}(B)}{a\, B^{3/2}}+\frac{c}{a^3}-\frac{L(B)}{a\sqrt{B}}-\frac{\sqrt{B}\, Q}{a}-\frac{A}{a\sqrt{B}} \tag{14}$$

> *Eq3* := 2·*U*[3]-*Torsion*(*SE*[2], 8, 9);

(15)

$$Eq3 := 2\, U_3 - \frac{\mathrm{I}\, b1}{a^2} - \frac{c}{a^3} \qquad \textbf{(15)}$$

```
> Eq4 := 2·U[4]-Torsion(SE[2], 8, 10);
```

$$Eq4 := 2\, U_4 + \frac{\mathrm{I}\, b}{a^2} - \frac{A}{a\sqrt{B}} - \frac{c}{a^3} \qquad \textbf{(16)}$$

```
> Eq5 := U[4] - Torsion(SE[3], 9, 10);
```

$$Eq5 := U_4 - \frac{1}{2}\, \frac{L^{\#}(B)}{a\, B^{3/2}} - \frac{\mathrm{I}\, b}{a^2} \qquad \textbf{(17)}$$

```
> Eq6 := U[3] + Torsion(SE[4], 9, 10);
```

$$Eq6 := U_3 + \frac{1}{2}\, \frac{L(B)}{a\sqrt{B}} + \frac{\mathrm{I}\, b1}{a^2} \qquad \textbf{(18)}$$

```
> solve({Eq1, Eq3, Eq6}, {c, b1, U[3]});
```

$$\left\{ b1 = \frac{\frac{1}{6}\, \mathrm{I}\, a\, (B\, Q + 3\, L(B))}{\sqrt{B}},\ c = \frac{1}{2}\, \frac{a^2\, (B\, Q + L(B))}{\sqrt{B}},\ U_3 = \frac{1}{6}\, \frac{\sqrt{B}\, Q}{a} \right\} \qquad \textbf{(19)}$$

```
N > solve({Eq2, Eq4, Eq5}, {c, b, U[4]});
```

$$\left\{ b = -\frac{\frac{1}{6}\, \mathrm{I}\, a\, \left(-2\, L^{\#}(B) + 2\, A\, B + B^2\, Q + L(B)\, B\right)}{B^{3/2}},\ c = \frac{1}{2}\, \frac{a^2\, (B\, Q + L(B))}{\sqrt{B}},\ U_4 = \frac{1}{6}\, \frac{L^{\#}(B) + 2\, A\, B + B^2\, Q + L(B)\, B}{a\, B^{3/2}} \right\} \qquad \textbf{(20)}$$

```
N > exprB# := (1/6 I (-2 L(B^-1) + 2 A# B^-1 + (B^-1)^2 Q# + L#(B^-1) B^-1)) / B^(-3/2) :
```

$$exprB^{\#} := \frac{\frac{1}{6}\, \mathrm{I}\, \left(-2\, L(B^{-1}) + 2\, A^{\#}\, B^{-1} + (B^{-1})^2\, Q^{\#} + L^{\#}(B^{-1})\, B^{-1}\right)}{B^{-3/2}} :$$

```
> A# := -B^-1·A :
>
> L := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L(op(i, x)), i = 1
      .. nops(x)) elif
        (type(x, `*`) = true) then expand(L(y)·x/y + y·L(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·L(y) elif
        (type(x, function) = true) then 'L'(x) elif
        (type(x, symbol) = true) then 'L'(x) else 0 fi end proc:

> L# := proc(x) local y; y := op(1, x) : if (type(x, `+`) = true) then add(L#(op(i, x)), i = 1
      .. nops(x)) elif
        (type(x, `*`) = true) then expand(L#(y)·x/y + y·L#(x/y)) elif
        (type(x, `^`) = true) then op(2, x)·y^(op(2, x) - 1)·L#(y) elif
        (type(x, function) = true) then 'L#(x)' elif
        (type(x, symbol) = true) then 'L#'(x) else 0 fi end proc:
```

>

> $A := solve\left(L(B) + B \cdot Q + A = B \cdot L^{\#}\left(B^{-1}\right) + Q^{\#} - A, A\right);$

$$A := -\frac{1}{2}\,\frac{L^{\#}(B) - Q^{\#}\,B + L(B)\,B + B^2\,Q}{B} \tag{21}$$

> $expand\left(exprB^{\#}\right);$

$$\frac{\frac{1}{2}\,\mathrm{I}\,L(B)}{\sqrt{B}} + \frac{1}{6}\,\mathrm{I}\sqrt{B}\,Q \tag{22}$$

>

N >