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[> restart :
[> with(DifferentialGeometry) :
[> with(Tools) : with(LinearAlgebra) :
[> DGsetup([x, y, z, z1], [a, a1, b, b1, c, d, e], M, verbose);
      The following coordinates have been protected:
          [x, y, z, z1, a, a1, b, b1, c, d, e]
      The following vector fields have been defined and protected:
          [D_x, D_y, D_z, D_z1, D_a, D_a1, D_b, D_b1, D_c, D_d, D_e]
      The following differential 1-forms have been defined and protected:
          [dx, dy, dz, dz1, da, da1, db, db1, dc, dd, de]
          frame name: M
(1)

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> g := Matrix([[a^2*a1, 0, 0, 0], [c, a*a1, 0, 0], [d, b, a, 0], [e, b1, 0, a1]]);

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$$g := \begin{bmatrix} a^2 a1 & 0 & 0 & 0 \\ c & a a1 & 0 & 0 \\ d & b & a & 0 \\ e & b1 & 0 & a1 \end{bmatrix} \quad (2)$$

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> h := MatrixInverse(g) :

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> Mat := map(evalDG, (ExteriorDerivative(g).h));

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$$Mat := \begin{bmatrix} \frac{2 da}{a} + \frac{da1}{a1} & 0 dx & 0 dx & 0 dx \\ -\frac{c da}{a1 a^3} - \frac{c da1}{a1^2 a^2} + \frac{dc}{a^2 a1} & \frac{da}{a} + \frac{da1}{a1} & 0 dx & 0 dx \\ -\frac{(d a a1 - b c) da}{a^4 a1^2} - \frac{c db}{a^3 a1^2} + \frac{dd}{a^2 a1} & -\frac{b da}{a^2 a1} + \frac{db}{a a1} & \frac{da}{a} & 0 dx \\ -\frac{(e a a1 - b1 c) da1}{a^3 a1^3} - \frac{c db1}{a^3 a1^2} + \frac{de}{a^2 a1} & -\frac{b1 da1}{a a1^2} + \frac{db1}{a a1} & 0 dx & \frac{da1}{a1} \end{bmatrix} \quad (3)$$

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> t[1] := \frac{da}{a} :

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M > t[2] := -\frac{b da}{a^2 a1} + \frac{db}{a a1} :

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M > t[3] := -\frac{c da}{a1 a^3} - \frac{c da1}{a1^2 a^2} + \frac{dc}{a^2 a1} :

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M > t[4] := -\frac{(d a a1 - b c) da}{a^4 a1^2} - \frac{c db}{a^3 a1^2} + \frac{dd}{a^2 a1} :

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M > t[5] := -\frac{(e a a1 - b1 c) da1}{a^3 a1^3} - \frac{c db1}{a^3 a1^2} + \frac{de}{a^2 a1} :

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M > t[6] := \frac{da1}{a1} :

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$$\mathbf{M} > t[7] := -\frac{bl\ da1}{a\ a1^2} + \frac{db1}{a\ a1} :$$

$\mathbf{M} > FD := FrameData([t[1], t[2], t[3], t[4], t[5], t[6], t[7], dx, dy, dz, dz1], N) :$
 $DGsetup(FD, [E], [\alpha[1], \alpha[2], \alpha[3], \alpha[4], \alpha[5], \alpha^{\#}[1], \alpha^{\#}[2], \sigma, \rho, \zeta, \zeta^{\#}], \text{rho, zeta, } \zeta^{\#}, \text{verbose});$

The following coordinates have been protected:

$[x, y, z, z1, a, a1, b, b1, c, d, e]$

The following vector fields have been defined and protected:

$[E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11]$

The following differential 1-forms have been defined and protected:

$[\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha^{\#}_1, \alpha^{\#}_2, \sigma, \rho, \zeta, \zeta^{\#}]$

frame name: N

(4)

$\mathbf{M} > T := Vector([\sigma, \rho, \zeta, \zeta^{\#}]) :$
 $W := h.T :$

$Eq1 := H \cdot (W[1] \wedge W[2]) + F \cdot (W[1] \wedge W[4]) + Q \cdot (W[1] \wedge W[3]) + B \cdot (W[2] \wedge W[4]) + (W[2] \wedge W[3]);$

$$\frac{H\sigma \wedge \rho}{a^3 a1^2} + F \left(-\frac{bl\sigma \wedge \rho}{a^3 a1^3} + \frac{\sigma \wedge \zeta^{\#}}{a1^2 a^2} \right) + Q \left(-\frac{b\sigma \wedge \rho}{a^4 a1^2} + \frac{\sigma \wedge \zeta}{a1 a^3} \right) + B \left(\frac{e\sigma \wedge \rho}{a^3 a1^3} - \frac{c\sigma \wedge \zeta^{\#}}{a^3 a1^3} + \frac{\rho \wedge \zeta^{\#}}{a a1^2} \right) + \frac{d\sigma \wedge \rho}{a^4 a1^2} - \frac{c\sigma \wedge \zeta}{a^4 a1^2} + \frac{\rho \wedge \zeta}{a^2 a1} \quad (5)$$

$\mathbf{M} > Eq2 := G \cdot (W[1] \wedge W[2]) + E \cdot (W[1] \wedge W[4]) + P \cdot (W[1] \wedge W[3]) + A \cdot (W[2] \wedge W[4]) + I \cdot (W[3] \wedge W[4]);$

$$Eq2 := \frac{G\sigma \wedge \rho}{a^3 a1^2} + E \left(-\frac{bl\sigma \wedge \rho}{a^3 a1^3} + \frac{\sigma \wedge \zeta^{\#}}{a1^2 a^2} \right) + P \left(-\frac{b\sigma \wedge \rho}{a^4 a1^2} + \frac{\sigma \wedge \zeta}{a1 a^3} \right) + A \left(\frac{e\sigma \wedge \rho}{a^3 a1^3} - \frac{c\sigma \wedge \zeta^{\#}}{a^3 a1^3} + \frac{\rho \wedge \zeta^{\#}}{a a1^2} \right) + I \left(-\frac{(be - bl d)\sigma \wedge \rho}{a^4 a1^3} + \frac{(e a a1 - bl c)\sigma \wedge \zeta}{a^4 a1^3} - \frac{(d a a1 - b c)\sigma \wedge \zeta^{\#}}{a^4 a1^3} + \frac{bl\rho \wedge \zeta}{a^2 a1^2} - \frac{b\rho \wedge \zeta^{\#}}{a^2 a1^2} + \frac{\zeta \wedge \zeta^{\#}}{a a1} \right) \quad (6)$$

$\mathbf{N} > Eq3 := 0 :$

$\mathbf{N} > Eq4 := 0 :$

$\mathbf{N} > \omega := Vector([Eq1, Eq2, Eq3, Eq4]) :$

$\mathbf{N} > \Omega := map(evalDG, g.\omega) :$

$Mat := map(evalDG, (ExteriorDerivative(g).h)) :$

$Mat2 := Mat \&MatrixWedge T :$

$\mathbf{N} > SE := map(evalDG, (Mat2 \&MatrixPlus \Omega)) :$

N > $List := GenerateForms([alpha[1], alpha[2], alpha[3], alpha[4], alpha[5], \alpha^{\#}[1], \alpha^{\#}[2], sigma, rho, zeta, \zeta^{\#}], 2) :$

N > $Torsion := \mathbf{proc}(S, i, j) \mathbf{local} k, X; k := 11 \cdot (i - 1) - \frac{i \cdot (i - 1)}{2} + j - i; X := GetComponents(S, List); X[k]; \mathbf{end} \mathbf{proc} :$

N > $result := \mathbf{proc}(l) \mathbf{local} k, t, X; X := 0 : t := expand(GetComponents(l, List)) : \mathbf{for} k \mathbf{from} 1 \mathbf{to} 55 \mathbf{do} X := X + t[k] \cdot List[k] \mathbf{od}; X; \mathbf{end} \mathbf{proc} :$

N > $Res1 := result(SE[1]);$

$$Res1 := 2 \alpha_1 \wedge \sigma + \alpha_1^{\#} \wedge \sigma + \left(\frac{d}{a^2 a l} + \frac{e B}{a a l^2} - \frac{Q b}{a l a^2} - \frac{F b l}{a l^2 a} + \frac{H}{a l a} \right) \sigma \wedge \rho + \left(-\frac{c}{a^2 a l} + \frac{Q}{a} \right) \sigma \wedge \zeta + \left(-\frac{c B}{a a l^2} + \frac{F}{a l} \right) \sigma \wedge \zeta^{\#} + \rho \wedge \zeta + \frac{a B \rho \wedge \zeta^{\#}}{a l} \quad (7)$$

N > $Res2 := result(SE[2]);$

$$Res2 := \alpha_1 \wedge \rho + \alpha_3 \wedge \sigma + \alpha_1^{\#} \wedge \rho + \left(-\frac{I b e}{a^3 a l^2} + \frac{I b l d}{a^3 a l^2} + \frac{A e}{a^2 a l^2} - \frac{b P}{a^3 a l} - \frac{b l E}{a^2 a l^2} + \frac{G}{a^2 a l} + \frac{d c}{a^4 a l^2} + \frac{e B c}{a^3 a l^3} - \frac{c Q b}{a^4 a l^2} - \frac{c F b l}{a^3 a l^3} + \frac{c H}{a^3 a l^2} \right) \sigma \wedge \rho + \left(\frac{I e}{a^2 a l} - \frac{I b l c}{a^3 a l^2} + \frac{P}{a^2} - \frac{c^2}{a^4 a l^2} + \frac{c Q}{a^3 a l} \right) \sigma \wedge \zeta + \left(-\frac{I d}{a^2 a l} + \frac{I b c}{a^3 a l^2} - \frac{A c}{a^2 a l^2} + \frac{E}{a a l} - \frac{B c^2}{a^3 a l^3} + \frac{c F}{a^2 a l^2} \right) \sigma \wedge \zeta^{\#} + \left(\frac{I b l}{a a l} + \frac{c}{a^2 a l} \right) \rho \wedge \zeta + \left(-\frac{I b}{a a l} + \frac{A}{a l} + \frac{c B}{a a l^2} \right) \rho \wedge \zeta^{\#} + I \zeta \wedge \zeta^{\#} \quad (8)$$

N > $Res3 := result(SE[3]);$

$$Res3 := \alpha_1 \wedge \zeta + \alpha_2 \wedge \rho + \alpha_4 \wedge \sigma + \left(-\frac{I b^2 e}{a^4 a l^3} + \frac{I b b l d}{a^4 a l^3} + \frac{b A e}{a^3 a l^3} - \frac{P b^2}{a^4 a l^2} - \frac{b E b l}{a^3 a l^3} + \frac{b G}{a^3 a l^2} + \frac{d^2}{a^4 a l^2} + \frac{d B e}{a^3 a l^3} - \frac{d Q b}{a^4 a l^2} - \frac{d F b l}{a^3 a l^3} + \frac{d H}{a^3 a l^2} \right) \sigma \wedge \rho + \left(\frac{I b e}{a^3 a l^2} - \frac{I b l b c}{a^4 a l^3} + \frac{b P}{a^3 a l} - \frac{d c}{a^4 a l^2} + \frac{d Q}{a^3 a l} \right) \sigma \wedge \zeta + \left(-\frac{I b d}{a^3 a l^2} + \frac{I b^2 c}{a^4 a l^3} - \frac{b A c}{a^3 a l^3} + \frac{b E}{a^2 a l^2} - \frac{d B c}{a^3 a l^3} + \frac{d F}{a^2 a l^2} \right) \sigma \wedge \zeta^{\#} + \left(\frac{I b b l}{a^2 a l^2} + \frac{d}{a^2 a l} \right) \rho \wedge \zeta + \left(-\frac{I b^2}{a l^2 a^2} + \frac{b A}{a l^2 a} + \frac{d B}{a a l^2} \right) \rho \wedge \zeta^{\#} + \frac{I b \zeta \wedge \zeta^{\#}}{a a l} \quad (9)$$

N > $Res4 := result(SE[4]);$

$$Res4 := \alpha_5 \wedge \sigma + \alpha_1^{\#} \wedge \zeta^{\#} + \alpha_2^{\#} \wedge \rho + \left(-\frac{I b l b e}{a^4 a l^3} + \frac{I b l^2 d}{a^4 a l^3} + \frac{b l A e}{a^3 a l^3} - \frac{b l P b}{a^4 a l^2} - \frac{E b l^2}{a^3 a l^3} + \frac{b l G}{a^3 a l^2} + \frac{e d}{a^4 a l^2} + \frac{B e^2}{a^3 a l^3} - \frac{e Q b}{a^4 a l^2} - \frac{e F b l}{a^3 a l^3} + \frac{e H}{a^3 a l^2} \right) \sigma \wedge \rho \quad (10)$$

$$\begin{aligned}
& + \left(\frac{Ibl e}{a^3 al^2} - \frac{Ibl^2 c}{a^4 al^3} + \frac{bl P}{a^3 al} - \frac{ec}{a^4 al^2} + \frac{eQ}{a^3 al} \right) \sigma \wedge \zeta + \left(-\frac{Ibl d}{a^3 al^2} + \frac{Ibl bc}{a^4 al^3} \right. \\
& - \frac{bl Ac}{a^3 al^3} + \frac{bl E}{a^2 al^2} - \frac{eBc}{a^3 al^3} + \frac{eF}{a^2 al^2} \left. \right) \sigma \wedge \zeta^\# + \left(\frac{Ibl^2}{al^2 a^2} + \frac{e}{al a^2} \right) \rho \wedge \zeta + \left(\right. \\
& \left. - \frac{Ibl b}{al^2 a^2} + \frac{bl A}{al^2 a} + \frac{eB}{a al^2} \right) \rho \wedge \zeta^\# + \frac{Ibl \zeta \wedge \zeta^\#}{a al}
\end{aligned}$$

N >