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> restart :
> with(DifferentialGeometry) :
> with(Tools) : with(LinearAlgebra) :
> DGsetup([w, x, y, z1, z], M, verbose);
      The following coordinates have been protected:
              [w, x, y, z1, z]
      The following vector fields have been defined and protected:
              [D_w, D_x, D_y, D_z1, D_z]
      The following differential 1-forms have been defined and protected:
              [dw, dx, dy, dz1, dz]
              frame name: M

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(1)

Une procédure de dérivation:

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> Der := proc(x) evalDG( S#(x) &wedge W[1] + S(x) &wedge W[2] + Tau(x) &wedge W[3]
+ L#(x) &wedge W[4] + L(x) &wedge W[5] ); end proc:

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> Tau := proc(x) I · (L(L#(x)) - L#(L(x))) end proc:

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M > S := proc(x) simplify(L(Tau(x)) - Tau(L(x))) end proc:

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> S# := proc(x) simplify(L#(Tau(x)) - Tau(L#(x))) end proc:

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> L := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(L(op(i, x)), i = 1
.. nops(x)) elif

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(type(x, '*' ) = true) then expand( L(y) ·  $\frac{x}{y}$  + y · L(  $\frac{x}{y}$  ) ) elif

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(type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · L(y) elif

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(type(x, function) = true) then 'L'(x) elif

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(type(x, symbol) = true) then 'L'(x) else 0 fi end proc:

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> L# := proc(x) local y; y := op(1, x) : if (type(x, '+' ) = true) then add(L#(op(i, x)), i = 1
.. nops(x)) elif

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(type(x, '*' ) = true) then expand( L#(y) ·  $\frac{x}{y}$  + y · L#(  $\frac{x}{y}$  ) ) elif

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(type(x, '^' ) = true) then op(2, x) · y(op(2, x) - 1) · L#(y) elif

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(type(x, function) = true) then 'L#'(x) elif

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(type(x, symbol) = true) then 'L#'(x) else 0 fi end proc:

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M > W := Vector([dw, dx, dy, dz1, dz]) :

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frame2 > dW[1] := evalDG(-K1 · (W[1] &wedge W[2]) + F1 · (W[1]
&wedge W[3]) + Q1 · (W[1] &wedge W[4]) + B1 · (W[1]
&wedge W[5]) + G · (W[2] &wedge W[3]) + B1 · (W[2]
&wedge W[4]) + R · (W[2] &wedge W[5]) + (W[3]
&wedge W[4])) :

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frame2 > dW[2] := evalDG(K · (W[1] &wedge W[2]) + G1 · (W[1] &wedge W[3])
+ R1 · (W[1] &wedge W[4]) + B · (W[1] &wedge W[5]) + F
· (W[2] &wedge W[3]) + B · (W[2] &wedge W[4]) + Q · (W[2]
&wedge W[5]) + (W[3] &wedge W[5])) :

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frame2 > dW[3] := evalDG(I·J·(W[1]&wedge W[2]) + E#·(W[1]
&wedge W[3]) + P1·(W[1]&wedge W[4]) + A·(W[1]
&wedge W[5]) + E·(W[2] &wedge W[3]) + A·(W[2]
&wedge W[4]) + P·(W[2] &wedge W[5]) - I·(W[4]
&wedge W[5])) :

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M > dW[4] := evalDG(0 &wedge dx &wedge dy);
dW4 := 0 dw ∧ dx

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(2)

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M > dW[5] := evalDG(0 &wedge dx &wedge dy);
dW5 := 0 dw ∧ dx

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(3)

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M > List2 := GenerateForms([dw, dx, dy, dz1, dz], 2) : List1 := [dw, dx, dy, dz1, dz] :

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M > tr1(1) := 1 :

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M > tr1(2) := 1 :

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M > tr1(3) := 1 :

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M > tr1(4) := 1 :

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M > tr1(5) := 2 :

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M > tr1(6) := 2 :

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M > tr1(7) := 2 :

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M > tr1(8) := 3 :

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M > tr1(9) := 3 :

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M > tr1(10) := 4 :

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M > tr2(1) := 2 :

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M > tr2(2) := 3 :

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M > tr2(3) := 4 :

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M > tr2(4) := 5 :

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M > tr2(5) := 3 :

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M > tr2(6) := 4 :

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M > tr2(7) := 5 :

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M > tr2(8) := 4 :

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M > tr2(9) := 5 :

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M > tr2(10) := 5 :

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M > DF := proc(omega) local T, Res; T := GetComponents(omega, List2);
Res[1] := evalDG(add(Der(T[i]) &wedge List2[i], i = 1 ..10));
Res[2] := evalDG(add(T[i] &wedge dW[tr1(i)] &wedge W[tr2(i)], i = 1 ..10));
Res[3] := evalDG(add(T[i] &wedge W[tr1(i)] &wedge dW[tr2(i)], i = 1 ..10));
evalDG(Res[1] + Res[2] - Res[3]);
end proc;

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M > BI := proc(omega) local R, i; R := GetComponents(DF(omega), List3); for i from 1
to 10 do print(R[i] = 0); od; end proc;

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M > List3 := GenerateForms([dw, dx, dy, dz1, dz], 3) :

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N > BI(dW[1]);

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$$\begin{aligned}
& GE^{\#} - F1E + K1F + GK + 2IL^{\#}(L(L^{\#}(G))) - IL^{\#}(L^{\#}(L(G))) - IL(L^{\#}(L^{\#}(G))) \\
& - IL(L(L^{\#}(F1))) + 2IL(L^{\#}(L(F1))) - IL^{\#}(L(L(F1))) - IL(L^{\#}(K1)) + IL^{\#}(L(K1)) \\
& = 0
\end{aligned}$$

$$\begin{aligned}
& GP1 - F1A + K1B + IJ + B1K + 2IL^{\#}(L(L^{\#}(B1))) - IL^{\#}(L^{\#}(L(B1))) \\
& - IL(L^{\#}(L^{\#}(B1))) - IL(L(L^{\#}(Q1))) + 2IL(L^{\#}(L(Q1))) - IL^{\#}(L(L(Q1))) - L^{\#}(K1)
\end{aligned}$$

=0

$$\begin{aligned}
& GA - FIP + KI Q + RK + 2IL^\#(L(L^\#(R))) - IL^\#(L^\#(L(R))) - IL(L^\#(L^\#(R))) \\
& - IL(L(L^\#(BI))) + 2IL(L^\#(L(BI))) - IL^\#(L(L(BI))) - L(KI) = 0 \\
& E^\# + B I G I - G R I - IL(L^\#(QI)) + IL^\#(L(QI)) + L^\#(FI) = 0 \\
& KI + R G I - G B - IL(L^\#(BI)) + IL^\#(L(BI)) + L(FI) = 0 \\
& I F I - A + R R I - B I B - L^\#(BI) + L(QI) = 0 \\
& E + B I F - G B + Q I G - F I B I - K I - IL(L^\#(BI)) + IL^\#(L(BI)) + L^\#(G) = 0 \\
& R F - G Q + B I G - F I R - IL(L^\#(R)) + IL^\#(L(R)) + L(G) = 0 \\
& I G - P + R B - B I Q + B I^2 - Q I R - L^\#(R) + L(BI) = 0 \\
& 0 = 0
\end{aligned} \tag{4}$$

M > $BI(dW[2]);$

$$\begin{aligned}
& F E^\# - G I E - G I K I - K F I + 2IL^\#(L(L^\#(F))) - IL^\#(L^\#(L(F))) - IL(L^\#(L^\#(F))) \\
& - IL(L(L^\#(GI))) + 2IL(L^\#(L(GI))) - IL^\#(L(L(GI))) + IL(L^\#(K)) - IL^\#(L(K)) \\
& = 0 \\
& F P I - G I A - R I K I - K Q I + 2IL^\#(L(L^\#(B))) - IL^\#(L^\#(L(B))) - IL(L^\#(L^\#(B))) \\
& - IL(L(L^\#(RI))) + 2IL(L^\#(L(RI))) - IL^\#(L(L(RI))) + L^\#(K) = 0 \\
& F A - G I P + I J - K I B - B I K + 2IL^\#(L(L^\#(Q))) - IL^\#(L^\#(L(Q))) - IL(L^\#(L^\#(Q))) \\
& - IL(L(L^\#(B))) + 2IL(L^\#(L(B))) - IL^\#(L(L(B))) + L(K) = 0 \\
& B G I - F R I + R I F I - G I Q I - IL(L^\#(RI)) + IL^\#(L(RI)) + L^\#(GI) = 0 \\
& -K + E^\# + Q G I - F B + B F I - B I G I - IL(L^\#(B)) + IL^\#(L(B)) + L(GI) = 0 \\
& I G I + P I + Q R I - B^2 + B Q I - R I B I - L^\#(B) + L(RI) = 0 \\
& G R I - B I G I + K - IL(L^\#(B)) + IL^\#(L(B)) + L^\#(F) = 0 \\
& E + G B - R G I - IL(L^\#(Q)) + IL^\#(L(Q)) + L(F) = 0 \\
& I F + A + B I B - R R I - L^\#(Q) + L(B) = 0 \\
& 0 = 0
\end{aligned} \tag{5}$$

M > $BI(dW[3]);$

$$\begin{aligned}
& -I J F + E K - E^\# K I - I J F I + 2IL^\#(L(L^\#(E))) - IL^\#(L^\#(L(E))) - IL(L^\#(L^\#(E))) \\
& - IL(L(L^\#(E^\#))) + 2IL(L^\#(L(E^\#))) - IL^\#(L(L(E^\#))) - L(L^\#(J)) + L^\#(L(J)) = 0 \\
& E P I - E^\# A - I J B + A K - P I K I - I J Q I + 2IL^\#(L(L^\#(A))) - IL^\#(L^\#(L(A))) \\
& - IL(L^\#(L^\#(A))) - IL(L(L^\#(PI))) + 2IL(L^\#(L(PI))) - IL^\#(L(L(PI))) + IL^\#(J) \\
& = 0 \\
& E A - E^\# P - I J Q + P K - A K I - I J B I + 2IL^\#(L(L^\#(P))) - IL^\#(L^\#(L(P))) \\
& - IL(L^\#(L^\#(P))) - IL(L(L^\#(A))) + 2IL(L^\#(L(A))) - IL^\#(L(L(A))) + IL(J) = 0 \\
& G I A - E R I + P I F I - E^\# Q I - IL(L^\#(PI)) + IL^\#(L(PI)) + L^\#(E^\#) = 0 \\
& -I J + G I P - E B + F I A - E^\# B I - IL(L^\#(A)) + IL^\#(L(A)) + L(E^\#) = 0 \\
& I E^\# + P R I - A B + A Q I - P I B I - L^\#(A) + L(PI) = 0 \\
& F A - E B + G P I - E^\# B I + I J - IL(L^\#(A)) + IL^\#(L(A)) + L^\#(E) = 0
\end{aligned}$$

$$\begin{aligned}
PF - EQ + GA - E^\# R - IL(L^\#(P)) + IL^\#(L(P)) + L(E) &= 0 \\
IE + PB - AQ + ABI - PIR - L^\#(P) + L(A) &= 0 \\
0 &= 0
\end{aligned}$$

(6)

M > $T1 := \text{GetComponents}(DF(dW[1]), \text{List3}) :$

M > $T2 := \text{GetComponents}(DF(dW[2]), \text{List3}) :$

M > $T3 := \text{GetComponents}(DF(dW[3]), \text{List3}) :$

M > $\text{solve}(T3[9], E);$

$$I(PB - AQ + ABI - PIR - L^\#(P) + L(A))$$

(7)

M > $\text{solve}(T2[9], F);$

$$I(A + BIB - RRI - L^\#(Q) + L(B))$$

(8)

M > $\text{solve}(T1[9], G);$

$$I(-P + RB - BIQ + BI^2 - QIR - L^\#(R) + L(BI))$$

(9)

M >