Immeubles euclidiens : construction par les normes et étude des isométries

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Abstract

Let $\mathbb{F}$ be a field endowed with a valuation $v : \mathbb{F} \longrightarrow \mathbb{R} \cup \infty$. When v is discrete, the classical construction of the Bruhat-Tits building Δ associated with $\mathrm{GL}_n(\mathbb{F})$ relies on its simplicial complex structure, with vertices identified with homothety classes of lattices in $\mathbb{F}^n$. When the valuation is not discrete (dense or surjective), the building Δ is no longer simplicial. We first give a construction of Δ using ultrametric norms of $\mathbb{F}^n$, inspired by the work of Goldman-Iwahori dealing with locally compact fields $\mathbb{F}$. This approach allows to unify the cases where the valuation is discrete, dense or surjective and to give a concrete model for Δ.

After developping the basic properties of euclidean buildings, we prove the following result, in a purely geometric way. Let Δ be a complete Euclidean building. There exists a constant K depending only on the type of Δ such that for every isometry g of Δ preserving the type of the segments, we have

$$d(x, \mathrm{Min}_g) \leq K d(x, gx)$$

for all points x in Δ, where $\mathrm{Min}_g = \{x \in \Delta;\ d(x, gx) \text{ is minimal}\}$. In particular g either fixes a point or translates some geodesic.

The main difficulty lies in the case where $\mathbb{F}$ is not locally compact. An application to non archimedean representation of group with bounded generation is given.

Mots clefs : Immeuble euclidien, norme ultramétrique, corps valué, isométrie.
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